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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN No. 74 APRIL, 1914

THE TRACTIVE RESISTANCE OF A 28-TON ELECTRIC CAR

BY HAROLD H. DUNN,
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THE TRACTIVE RESISTANCE OF A 28-TON ELECTRIC CAR.

I. Introduction.

1. *Purpose.*—Three series of tests have been conducted by the Railway Engineering Department of the University of Illinois to determine the resistance offered to the motion of a 28-ton electric interurban car running on straight, level track, in still air at uniform speed; and to ascertain the relation existing between that resistance and the speed of the car. The first part of this bulletin describes the purpose, methods, and final results of the tests, but all details not essential to an understanding of the general methods and results are excluded. In the three appendices details are given concerning the apparatus, the methods of calculation, the test data, and the intermediate results.

2. *Definition of Terms.*—By the terms "resistance," "train resistance," and "car resistance" used in this bulletin is meant the number of pounds of tractive effort required at the rims of the driving wheels for each ton of train or car weight, to keep it moving at uniform speed on straight level track in still air. Such resistance data is employed in determining railway motor capacity, power consumption, and possible schedule speeds of a car or train of cars.

3. *Acknowledgments.*—The interest and co-operation of the officials of the Illinois Traction System have made it possible to carry out these tests over the lines of that road, and members of the Railway Engineering Department staff who have been concerned with this investigation thoroughly appreciate such interest and assistance.

The tests were planned by Professor Edward C. Schmidt, in charge of the Department of Railway Engineering, and they have been carried out under his general direction and supervision. He has also directed the work of the final computations, and reviewed the manuscript of the report. The tests of groups A and B were made under the direct supervision of Mr. Edgar I. Wenger, formerly Associate in the Department of Railway Engineering, and the preliminary calculations for these tests were made under his direction. The tests of group C and their preliminary calculations were made with the assistance of Mr. Daniel C. Faber, formerly a Research Fellow in the Engineering Experiment Station.

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II. Summary.

Eighteen tests made on selected sections of tangent track are described. The track upon which the tests were made is of good construction, such as one would expect to find on the better grade of interurban electric railways. The weather during the tests was generally fair. The average temperature was not below 25° F., and the wind velocity did not exceed 26 miles per hour. The test car used is described in detail in Appendix I.

The general plan was to run the test car backward and forward over the chosen track section at a variety of speeds, but maintaining approximately the same constant speed during each pair of opposing runs. The tests comprise in all 269 resistance determinations of which 131 have been calculated from runs during which the wind opposed the motion of the car, 130 from runs during which the wind helped the car, and 8 from runs made when no wind was blowing. During the two tests of group A, comprising 45 resistance determinations, opposing runs have been separated by a considerable time interval during which wind and weather conditions might change; but through changes made in the method of making the tests, opposing runs for the remaining tests have been separated by a small time interval.

Each test, consisting of a number of "runs," resulted in a number of values of average net car resistance which have been plotted with the corresponding average speed, giving diagrams such as Fig. 3. Through the two groups of points representing the results of runs in opposing directions, curves have been drawn between which a final mean curve has been decided upon. By this method wind resistance has been eliminated as nearly as may be from the results. The curves thus determined have been accepted as the results desired from the individual tests and have been grouped in Fig. 1. A final mean curve has been drawn there and repeated as Fig. 2, which represents the average relation between resistance and speed for all the tests.

The results of the individual tests are shown in Figs. 10 to 19 and Tables 3 to 12, inclusive, in Appendix III. The final results of the investigation are presented in Fig. 1, Table 2, and in formulae 1 and 2, page 20.

It is believed that Fig. 1, Table 2, and formulae 1 and 2 provide expressions for the mean relation between the speed and the resistance of the car to which these tests apply, when it runs on tangent level track of good construction, at uniform speed, and in still air. The

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greatest variation from these mean values under these conditions was about 9 per cent.

If application is made of these results to predict the resistance of any car of similar design and weight it should be borne in mind that the results apply to tests made with a car whose running-gear was in excellent condition, and that the tests were run in weather when the temperature was above 25° F.

III. Means Employed in Conducting the Tests.

4. *The Electric Test Car.*—The data for this report have been obtained by means of the electric test car belonging to the Railway Engineering Department of the University of Illinois. It is a standard 45-foot interurban car of the double end type, weighing, complete with equipment, 55 150 pounds, subject to correction on account of changes in the equipment at different times. The cross sectional area of the car body and trucks is 90 square feet.

The motor equipment consists of four 50-horsepower, axle mounted, number 101-D Westinghouse direct current motors, geared to the axle in the ratio of 22 to 62. The Westinghouse unit switch system of multiple control is used.

The trucks, which are of the C-60 type of the Standard Motor Truck Company, are placed 23 ft. 3 in. between centers and have a wheel base of 6 ft. 4 in. The wheels are 33 in. in diameter and have the standard Master Car Builders' tread and flange, and the journals conform to the 4¼ in. × 8 in. standard of the same association. A more complete description of the car will be found in Appendix I.

5. *The Recording Apparatus.*—By means of the apparatus within the car a continuous graphical record of current, voltage, speed, time by 5 second intervals, distance traversed, brake cylinder pressure, and location on the road may be kept. These records are drawn upon a chart 40 inches wide which is made to pass under the recording pens at a rate proportional to time or distance. Other data taken during this investigation, but not recorded upon the chart, are described on page 11. Fig. 9 shows a copy of a portion of the chart from test 121. The transverse lines which mark one of the sections selected for calculation and some of the explanatory lettering do not appear on the original record, and the records as shown in Fig. 9 are grouped more closely together than on the original chart. A more complete description of the recording apparatus is to be found in Appendix I.

6. *The Track.*—This investigation was carried out on the tracks

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of the Illinois Traction System between Danville, Urbana, Champaign, Decatur, and Springfield—a single track line of good construction over which an hourly service is maintained between the points named.

That part of the line between Danville and Urbana, 32 miles in length, was laid during 1903 and 1904 with 70-pound A. S. C. E. section rails except for 9 miles near Urbana, where 60-pound A. S. C. E. 1901 section rails were in use at the time these tests were made. All rails were laid on oak cross-ties spaced about 24 inches between centers and ballasted with gravel and cinders. This track had between 2 and 3 years in which to become settled before the tests were started. The track between Champaign and Decatur was laid in 1906 and 1907 with 70-pound A. S. C. E. section rails laid on oak, elm, and chestnut cross-ties spaced about 24 inches between centers and ballasted with gravel. When tests were made over this track it was well settled and in first-class condition. The line between Decatur and Springfield was laid in 1905 and is in all respects similar to that between Champaign and Decatur, except that part of the track is ballasted with cinders.

A survey of the Danville-Urbana line was made by the Railway Engineering Department of the University immediately preceding these tests and the results expressed in a profile drawn to a scale of ¼-inch to 100 feet. Elevations were taken on the north rail to 0.1 ft. at stations 300 feet apart, and turning points were taken at every fourth station where levels were read to 0.01 ft. The exact distances between all trolley line poles were recorded and incorporated in a table from which all distances, such as those between section limits, were determined.

For the purpose of this investigation certain long sections of comparatively level and well ballasted tangent track on the Champaign-Decatur-Springfield line were selected and surveyed by the Railway Engineering Department of the University. The results of these surveys were expressed in profiles, drawn to a scale of ¼-inch to 100 feet, which were used in making the calculations.

IV. Test Conditions and Methods of Test.

7. *Test Conditions.*—The tests were all run during moderate weather. With four exceptions they were made on clear days only and on dry track. Data in regard to wind velocity and direction were obtained from United States Meteorological stations at Urbana, Springfield and Peoria, and during one group of tests, designated as group "C," these data were supplemented by wind determinations made by means of a portable wind vane and anemometer set up beside the test track. Tables 3 to 12 inclusive in Appendix III show that the approximate

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average wind velocities prevailing during the tests varied from 0 to 26 miles per hour. The lowest average air temperature recorded during any test was 25° F., and the highest recorded temperature was 80° F.

TABLE 1

A SUMMARY OF TEST CONDITIONS

1	2	3	4	5	6	7
Test No.	Weight of Car and Load	Approximate Average Air Temperature	Rail Condition	Approximate Wind Data		
				Average Direction	Average Velocity	Average Velocity Parallel to Test Track
	Pounds	Deg. F.			M. P. H.	M. P. H.
15-16	56 600	40	Dry	S. of W.	7.2	7
23-24	56 250	70	Dry	S. W.	15.1	11
73-74	56 650	40	Wet	N. of W.	10.0	9
77-78	56 800	40	Dry	N. W.	12.0	12
91-92	56 350	45	Dry	S. W.	26.0	24
95-96	56 350	65	Dry	N. W.	16.0	15
109-110	56 950	55	Wet	S. W.	18.0	13
111-112	56 350	55	Wet	S. W.	18.0	13
113-114	56 200	55	Dry	S. W.	10.0	7
117-118	56 200	25	Wet	S. W.	12.0	9
119-120	56 750	45	Dry	W. of S.	3.5	2
121-122	56 750	30	Dry	E. of N.	3.0	2
123-124	56 750	30	Dry	S. E.	4.0	3
125-126	57 350	35	Dry	S. W.	15.0	10
127-128	57 500	25	Wet		0	0
129-130	57 350	65	Dry	S. E.	15.0	10
141-142	57 800	40	Dry	W.	3.8	3
153-154	57 900	60	Dry		0	0

The track sections selected for the tests are all straight and they vary in length from 250 to 10 000 feet. The majority of the tests were made on sections varying in length from 1000 to 1500 feet. Other requirements to be met by the sections were that the grade should be comparatively light and uniform, and that the track should be well ballasted and in good condition. While on the road the car was always in charge of a regular train crew provided by the Illinois Traction System.

8. *Calibrations.*—Preceding the commencement of the tests, all instruments were calibrated and at intervals during the investigation check readings were taken by means of indicating instruments in the same circuits as the recording instruments. Wherever possible, the results of these calibrations were expressed in the form of equations or tables as well as curves, and in making the calculations one of the two former was used in place of taking readings from curves. Owing to changes made in the apparatus during the tests, more than one calibration was necessary for several of the instruments.

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9. *General Plan of the Tests.*—The test car records give quantities from which the gross resistance offered to motion of the car over a given section of track must first be calculated. The result desired from each run is that element of gross resistance which is always operating to retard a moving car or train; namely, the net resistance on straight and level track, at uniform speed, in still air. The other elements, one or more or none of which may be acting with the net resistance to form gross resistance, are: Resistance due to grade, resistance due to acceleration, curve resistance, and wind resistance (as distinguished from still air resistance). The tractive effort consumed by grade and acceleration may readily be determined by calculation, and the tests herein reported were so planned as to eliminate the two remaining elements. Curve resistance has been eliminated by selecting for calculation only those sections of chart made while the car was running on straight track. It is thought that by the procedure explained below the effect of wind resistance upon the final result has been reduced to a minimum.

The car was first run in one direction and then in the other over a given track section, at as nearly uniform speed as possible. A series of such runs, made at speeds varying through as wide a range as possible. constituted a test. Each run results in a value of average net car resistance which, when plotted with the corresponding average speed, becomes one point on a resistance-speed plot such as is shown in Fig. 3. By the to and fro motion of the car the wind alternately helps and opposes its motion and thereby decreases or increases its resistance. The resistance values from each test therefore fall into two groups, one of which represents values of resistance running with the wind, while the other represents values running against the wind.

In the figures, those points applying to runs with the wind are shown as solid dots, while the points applying to runs against the wind are shown as open circles. By methods which are described on pages 13 and 14, a mean curve has been drawn in each figure which represents the values of resistance with the influence of wind eliminated. These mean curves have been accepted as the desired resistance curves for the individual tests and have been grouped together as Fig. 1. An average curve obtained from this group, by the method described on page 20, has been repeated as Fig. 2 and accepted as the final curve of resistance for this 28-ton electric car under the conditions described.

As a result of slight variations in the methods of conducting the tests, they fall into three groups designated as A, B and C, which differ in the effect their conditions have on the assumptions regarding wind

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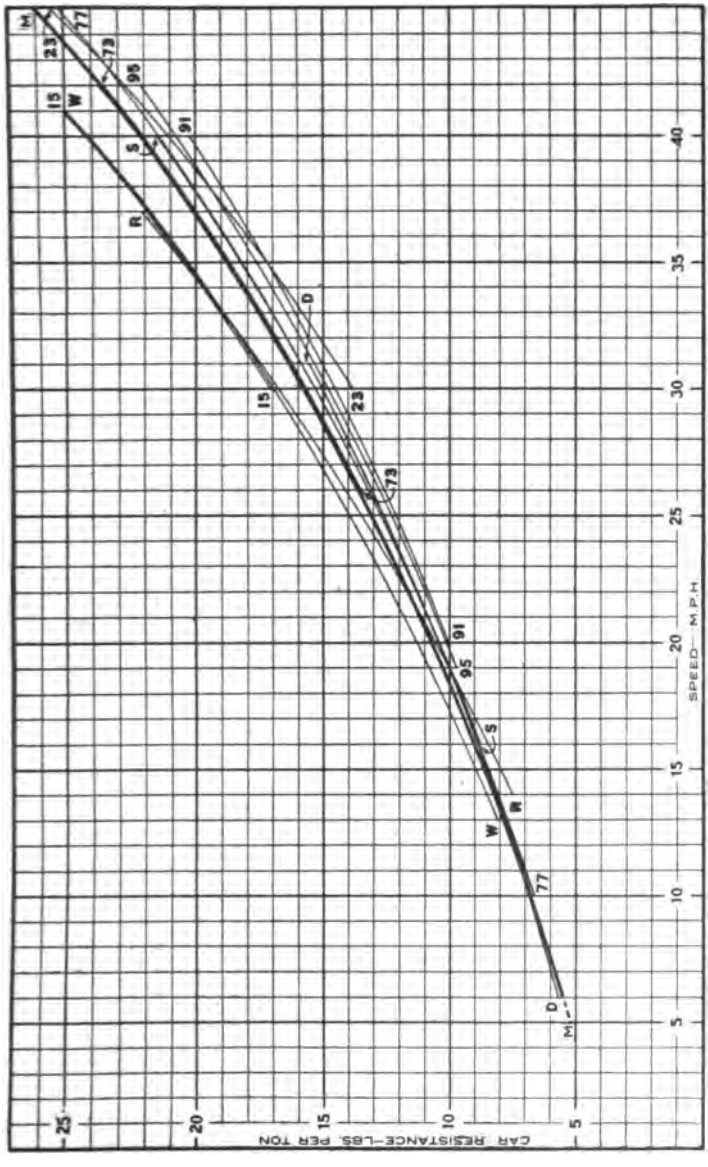

FIG. 1. THE RELATION OF RESISTANCE TO SPEED FOR ALL THE TESTS.

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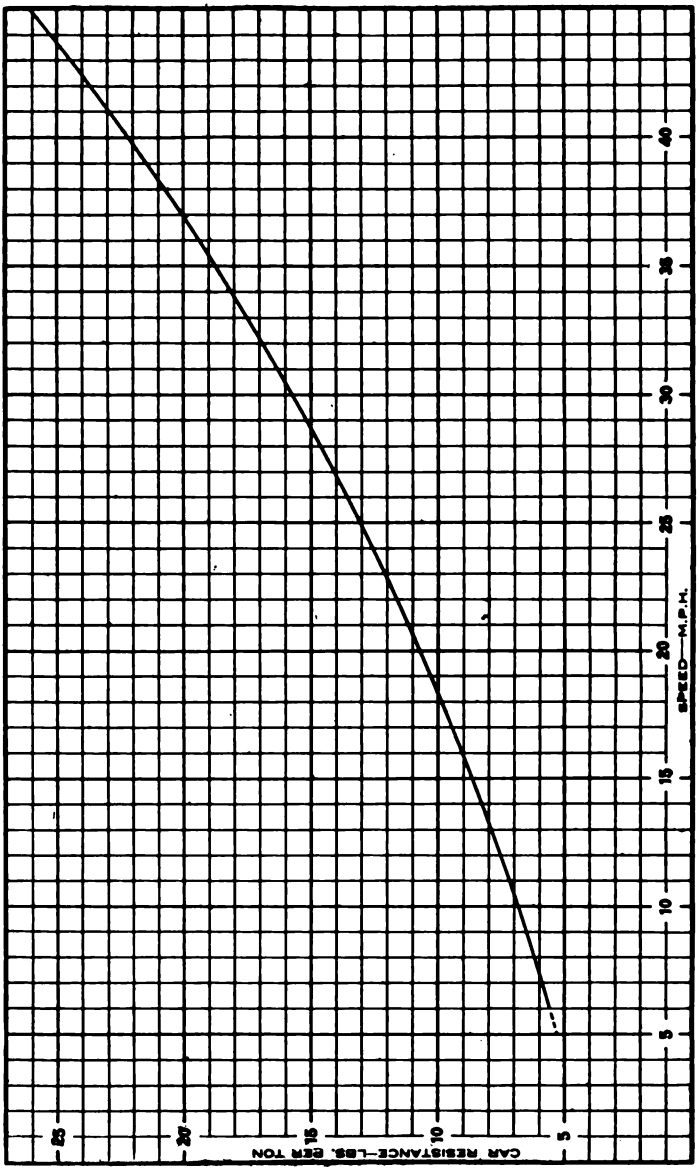

FIG. 2. THE MEAN RELATION BETWEEN RESISTANCE AND SPEED FOR ALL THE TESTS.

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resistance, in the methods of obtaining certain data, and in the speed range.

10. *Group A.*—Those tests comprised in group A were through runs from Urbana to Danville, 32 miles, and return. During any test of this group only one set of opposing runs was made on any given section and such opposing runs were separated by a considerable time interval, since it took over an hour to run from Urbana to Danville or to return.

Test 23-24 is characteristic of this group. The results are shown in Table 4 and Fig. 11, which has here been repeated as Fig. 3. The recording apparatus was kept in operation throughout the trip and the following data recorded graphically upon the chart:

a. Current used by the motors.
b. Trolley line voltage.
c. Speed of the car by electrical recorder. (See App. I.)
d. Time by 5 second intervals.
e. Location upon the road.
f. Distance by 50 ft. intervals.
g. Brake cylinder pressure.

The following data were taken but not recorded graphically:

h. Average wind velocity and direction.
k. Air temperature.
l. Rail condition.
m. Weight of car and load.

Item *e,* the location of the car on the road, by means of which it is possible to correlate any position of the car with the profile of the road, was made by marking upon the chart the position of numbered trolley line poles, stations, or other markers as they were passed by a given point on the car. The brake cylinder pressure, item *g,* was recorded in order to make it possible to distinguish and to avoid those periods during the tests when the brakes were applied. Data regarding average wind velocity and direction, item *h,* were obtained from United States Weather Bureau Stations. Fig. 9, page 27, has been reproduced from a tracing of a portion of the chart for test No. 121.

Sections of chart were selected for calculation by comparison of the profile and chart as described in Appendix II. In general, the requirements to be met by the chosen chart sections are that the car shall be running on straight track, that no brake applications be made, that there be no heavy grades between the section limits, and that there be no considerable voltage, current or speed variation. They were chosen

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so that the speed range covered was as large as possible and so that the number of sections from an east bound trip (Urbana to Danville) would be balanced by the same or approximately the same number from the west bound trip. In this way the number of resistance determinations made from data taken during runs against the wind were balanced by an approximately equal number from runs with the wind. From test 23-24, for example, fourteen sections were chosen from the east bound and thirteen from the west bound trip.

In all the tests reported in this bulletin the variations in speed in passing the track sections have exceeded 2 miles per hour in only 28 per cent of the total number of resistance determinations, and in only 24 cases out of 269 has this speed variation exceeded 5 miles per hour. The maximum variation over any section was 9.95 miles per hour.

Each of the chosen chart sections has afforded, through calculations made with the data there recorded, a value of average net car resistance. The calculations are described in Appendix II. The steps in the process are here stated briefly. The average values of current and voltage were found for each section by determining the average ordinates of the curves. This data, with the time required for the car to pass the track section, and the efficiency of motors and gears at the above current and voltage, made it possible to calculate the energy in foot pounds delivered by the current to the driving wheels of the car. The energy thus determined was then corrected for grade and acceleration resistances, thereby giving the net energy absorbed by car resistance, from which the average net car resistance in pounds per ton was readily obtainable. In order to make the grade correction it was first necessary to determine the elevation of the center of gravity of the car as it entered and again as it left the track section. The correction for acceleration was calculated from the speeds determined by the heights of the speed curve at the points of entrance and exit. For this purpose determinations were made of the force required to produce accelerations in the rotation of the revolving parts as well as of the force required to produce the acceleration in the motion of translation of the car as a whole.

From each test a number of chart sections have been chosen and, as has been stated, each section has resulted in a value of average net car resistance which has been plotted with the corresponding average speed, thereby giving diagrams such as Fig. 3. Due to the method of selecting chart sections from opposing runs, the points on each such resistance-speed plot fall into two groups as previously explained. The

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numbers beside the points are the item numbers from the tables in Appendix III.

In drawing the curve to represent the results of the individual tests the two groups of points were first considered separately and a curve drawn for each. Thus, for example, curve *A*, Fig. 3, was drawn to represent the relation between resistance and speed for all the runs of test 23-24 during which the wind opposed the motion of the car, and curve *B* for the opposite condition. In order to draw these curves the plotted points were considered as being subdivided into a number of smaller groups, for each of which averages of speed and resistance were determined and plotted. Through the points representing these averages

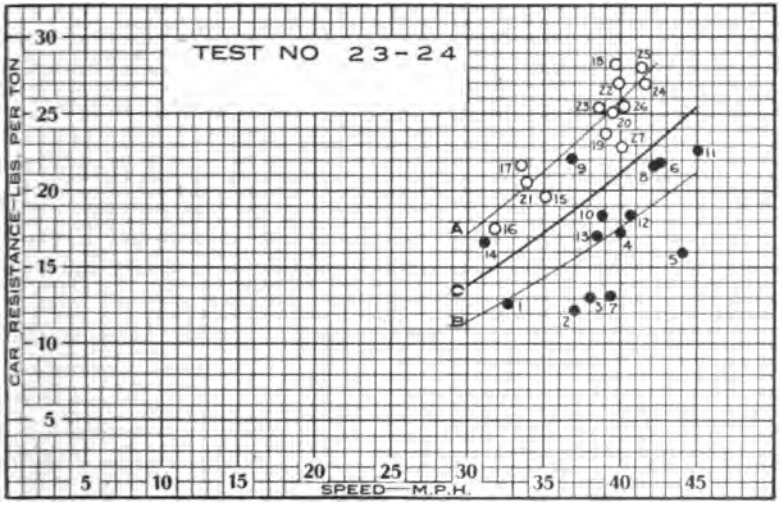

FIG. 3. THE RELATION OF RESISTANCE TO SPEED FOR TEST 23-24.

the curves were drawn. The groups of points were chosen so that the resulting average points would be distributed at about equal intervals throughout the speed range. The next step was to draw a mean curve such as *C*, Fig. 3, which would represent the relation existing between resistance and speed in still air. Since investigations indicate that the resistance offered by wind varies with the square of the speed, an arithmetical mean would not satisfy the conditions. Accordingly, an equation taking account of this fact has been derived (see Appendix II, page 34) and by means of it the ordinates of points on the mean curve have been calculated from the ordinates of the *A* and *B* curves at the same speed. The mean curves drawn by this method agree much more closely

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than do those curves drawn as arithmetical means between the *A* and *B* curves, and it has consequently been assumed that the method used has resulted in curves more accurately representing the resistance in still air than would have been obtained by using the arithmetical means.

The six curves thus obtained from tests of groups A and B, and the four from group C which were drawn by another method, were then superimposed upon each other in order to determine the characteristic form, and the individual curves were then modified to conform to that general shape. The curves drawn in this manner have been accepted as the results desired from the individual tests.

In group A there are two tests, namely 15-16 and 23-24, comprising together a total of 45 resistance determinations. The results are shown in Figs. 10 and 11 and Tables 3 and 4 in Appendix III.

11. *Group B.*—The tests belonging in group B differ from those in group A in the method of making the tests and in the effect this change in method has upon the assumptions regarding wind resistance. Whereas opposing runs from tests of group A were separated by a considerable time interval during which wind conditions might change, the tests of group B were planned so that the time interval between opposing runs would be as small as possible. A well kept section of track 1495 feet in length and located on the Urbana-Danville line was chosen for this purpose. The limits of this section, between which there was a difference in elevation of 0.56 feet, were defined by markers and offsets were made in the location record on the test chart as they were passed by the car. These points located the chart section to be calculated for each run. During any test the car was run back and forth over this section at a variety of speeds, each pair of opposing runs being made at approximately the same uniform speed. Data exactly similar to that obtained during the tests of group A were taken during each run. The reasons enumerated on pages 11 and 30 for the rejection of any chart section apply also to the runs of this group of tests. Through calculations made in the same way as has been described for the previous group, each run has resulted in an average value of net car resistance for the average speed at which the car passed the track section. These have been plotted and curves drawn by the method described for group A.

There are four tests in group B, namely, 73-74, 77-78, 91-92 and 95-96. The results are given in Figs. 12 to 15 and Tables 5 to 8, inclusive, in Appendix III.

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12. *Group C.*—The principal points of difference between the tests of this and of preceding groups lie in the method of obtaining certain data, in the plan of grouping the results from two or more tests made on the same track section, and in the method of drawing the final curves.

Track sections, each comprising two or more tangents varying in length from 338 to 1320 feet, were chosen so as to meet the same requirements as those prescribed for the tests heretofore described. They have been designated by letters as follows:

D: Approximately 2680 feet long, located near Urbana on the Danville-Urbana line.

W: About 3700 feet in length, located near White Heath on the Champaign-Decatur line.

S: About 4100 feet long, located near White Heath on the Champaign-Decatur line.

R No. 1: Approximately 3000 feet long, located near Riverton on the Decatur-Springfield line.

R No. 2: About 4500 feet in length, located near Bement on the Champaign-Decatur line.

During each test the car was run back and forth over the chosen track section, as during the tests of group B, so that opposing runs were separated by a small time interval as in that group. Throughout each run the recording apparatus was kept in operation and data taken which were similar to those obtained during the tests that have been described, although the method of taking them was in some cases different. Changes made in the instrument equipment were as follows:

For the tests of groups A and B line voltage was recorded; but during the tests of group C connections were changed so that the record was that of voltage across the terminals of one motor. This change was made to facilitate calculations.

A Boyer mechanical speed recorder was installed in addition to the electric recorder described in Appendix I, and the record made by it has been used in the calculations. Both instruments were driven from the same shaft.

A portable windvane and anemometer were used during these tests to determine wind direction and velocity. These instruments were set up beside the test track and readings were made at intervals during the tests. By this means much more accurate determinations were made than could be obtained from the Weather Bureau reports.

It was found upon the completion of the tests of groups A and B

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that a low enough speed range had not been obtained and steps were therefore taken to supply this deficiency on subsequent tests. Two water rheostats were connected in the trolley circuit for the control of the motor voltage and the speed was held as nearly as possible at a constant predetermined value by the operator, who had for his guidance an ammeter in series with the speed-recording ammeter.

From the record of each run made over the test track, chart sections were chosen for calculation in accordance with the requirements stated on page 14 to conform to the characteristic shape. The resistance curves groups A and B. Each such chart section has resulted in one point on a resistance-speed plot such as Fig. 16, Appendix III. The results of all tests made on any given track section have been grouped together as, for example, track section D, on which tests 109-110, 111-112 and 113-114 were made. Inasmuch as the resistance-speed points in Fig. 16 have been derived from the results of tests made under somewhat different weather and wind conditions, it was found impracticable to draw the final curve in the same manner as for groups A and B. Consequently, in place of following the method described on page 13, the points on each resistance-speed plot, irrespective of the wind effect, were assumed to be divided into a number of groups, for each of which arithmetical averages of the values of speed and resistance were determined and plotted. These groups of points were chosen so that the resulting averages would be distributed at about equal intervals throughout the speed range. Through them the curve representing the relation between resistance and speed was drawn and later modified as described on page 14 to conform to the characteristic shape. The resistance curves thus derived, which are shown in Figs. 16 to 19 inclusive, have been accepted as the results desired from this group of tests.

It is recognized that this method of drawing the resistance curves is a deviation from the general plan followed for the preceding tests, but Fig. 1 shows that this deviation has brought about no greater variation of the individual curves from the mean curve *MM* than is found among those drawn as described on page 13. This may be accounted for by the fact that the wind velocities prevailing during the tests in question (see column 7, Table 1) were on the whole less than those prevailing during the preceding tests, and that consequently the resistance-speed points for runs with and against the wind were closely interwoven. Had the tests of group C lent themselves to the plan described on page 13, the curves with and against the wind would have

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been quite close together, and the mean curves determined by the two methods would not have differed greatly.

It will be noted that the numbers of the tests here included are not consecutive. Some of the tests bearing the intervening numbers were made for other purposes, but a considerable number were made for the purpose of this investigation and were later discarded for various reasons. Of the latter the greater number were omitted because an insufficient number of runs were made over the track sections to accurately determine a curve. This was occasioned by the necessity of running the car on sidings to allow regular cars to pass. Others were discarded because it was impossible to choose a sufficient number of chart sections that would meet the requirements, and still others were in the nature of preliminary runs.

V. Results of the Tests.

13. *Results of the Individual Tests.*—The results of the individual tests are shown in the form of resistance-speed curves in Figs. 10 to 19 and Tables 3 to 12, inclusive, in Appendix III.

These tests, without exception, show a marked increase in resistance as the speed increases. The effect of wind resistance is clearly shown by a comparison of Figs. 10 to 15 inclusive, which have been derived from tests of groups A and B. For those tests during which a heavy wind alternately opposed and helped the motion of the car, the two curves, such as *A* and *B* (Fig. 3), drawn to represent the results of runs in opposite directions, are some distance apart, while for tests during which the wind was light these curves are quite close together. Figs. 15 and 10 illustrate this difference. Fig. 15 has been derived from test 95-96 during which the component of the average wind velocity parallel to the track was 15 miles per hour while Fig. 10 has been derived from test 15-16 during which that component was 7 miles per hour.

Had the wind velocity and direction remained precisely alike during all runs of a test, then the components of the wind velocity parallel to the track would have been of like value and the process of drawing the mean curve for the test would be valid in so far as the fundamental assumption underlying this process is in itself valid. The wind velocity and direction did not remain precisely constant during all runs of a test, however, and the resulting mean curves are consequently slightly in error. No method of eliminating this error or of evaluating it has presented itself.

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The component of the wind velocity which is perpendicular to the test track increases gross resistance, irrespective of the direction in which the car is running, by pressing the flanges of the car wheels against the head of the rail, thereby increasing the frictional resistance. No method has been found for evaluating its effect upon the final resistance values and it is therefore included in the accepted results. Obviously it is one cause for the variations among the resistance curves as well as for the variations among the individual resistance-speed points.

Due in part to the reasons just mentioned, the points in any of the resistance-speed plots may vary considerably from the final mean curve for that test. The maximum variation of any point from the mean is approximately 90 per cent. Any comparison of variations from the mean made to determine the agreement of the results among themselves should, however, be confined to the results obtained from runs in one direction only. Such a comparison among the resistance determinations for tests of groups A and B shows variations which, though large in some cases, are not unusual for this class of experimental work. The maximum variation from the mean for the first two groups of tests occurred during test 23-24 (Fig. 3). Point 9 varies 42.5 per cent from mean curve *B*. The next largest variation is 24 per cent, while the average is much less than that. A very small percentage of this variation may be due to accumulated errors in instruments or in the calculations, although all reasonable precautions have been taken to avoid such errors. Each step in the process of making the calculations and producing the tables and curves has been duplicated at a different time and usually by a different person, and in all cases where the calculated value of resistance differed greatly from the mean all calculations leading thereto were repeated. The influence of such variable and uncontrollable elements of net resistance as flange friction, journal friction, instantaneous changes in velocity and direction of the wind, and changes in the lubrication of bearings and gears, is believed to be sufficient to account for the differences shown in the curves here presented. The variations of the points in Figs. 16 to 19, inclusive, are chargeable to the differences in the test conditions that are represented on those resistance-speed plots as well as to the above-mentioned causes.

14. *Results of All the Tests.*—The six resistance curves for the individual tests of groups A and B, and those representing the results obtained on the four track sections of group C, have all been brought together in Fig. 1.* In view of the number and character of the elements

*Each curve has been marked with the test number or letter corresponding thereto.

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of net resistance that can not be controlled, the differences among the curves there shown are not greater than might be expected. For speeds up to 20 miles per hour the maximum variation among the resistance values from the different curves is about 13 per cent, between 20 and 30 miles per hour this variation is approximately 17 per cent, while above 30 miles per hour it approximates only 20 per cent.

Table 2

Values of Resistance at Various Speeds, Derived From the Final Curve and the Curves for the Individual Tests. This Table Provides the Co-ordinates of the Curves in Figure 2.

Speed — Miles Per Hour	Car Resistance—Pounds Per Ton										
	Final Mean Curve	Test 15–16	Test 23–24	Test 73–74	Test 77–78	Test 91–92	Test 95–96	Tests on Section D	Tests on Section W	Tests on Section S	Tests on Section R
5	5.25										
6	5.52							5.65			
7	5.81										
8	6.12										
9	6.48										
10	6.80				6.62			6.87			
11	7.12										
12	7.50										
13	7.87								8.12		
14	8.25										7.55
15	8.62				8.48			8.62	9.00		8.02
16	9.05									8.75	
17	9.48										
18	9.87										
19	10.32						9.75				
20	10.75				10.50	10.02	10.18	10.55	11.30	10.44	10.60
21	11.22										
22	11.66										
23	12.13										
24	12.62										
25	13.03				12.75	12.13	12.37	12.65	13.93	12.75	13.45
26	13.62			13.55							
27	14.12										
28	14.65										
29	15.22										
30	15.75	17.10	13.84	15.73	15.34	14.54	14.85	15.00	16.88	15.30	16.70
31	16.36							15.50			
32	16.90										
33	17.50										
34	18.12										
35	18.75	20.44	17.15	18.75	18.22	17.22	17.62		20.25	18.21	20.47
36	19.37										
37	20.04										22.13
38	20.75										
39	21.44										
40	22.13	24.20	21.00	22.16	21.45	20.25	20.75		24.15	21.50	
41	22.87	25.02							25.00		
42	23.66			23.68			22.12				
43	24.45										
44	25.26										
45	26.12		25.56		25.25						

The mean curve *MM* shown in Fig. 1 and reproduced as Fig. 2 has been drawn to express the average relation between resistance and

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speed for all the tests. To determine this curve, the resistances corresponding to a given speed, say 10 m.p.h., were taken from the individual curves of Fig. 2 and averaged. In this process each resistance value was given a weight corresponding to the total number of resistance determinations represented by the curve from which it was taken. These averages were calculated at speeds varying by steps of 5 miles per hour and plotted in Figs. 1 and 2. Through them the mean curve *MM* has been drawn. The curve thus determined represents the final results obtained from this investigation. The values of resistance at various speeds have been determined from all the curves shown in Fig. 1 and they are presented in Table 2. The data there given are sufficient to accurately reproduce all the final resistance-speed curves.

The final results have also been expressed in the form of an equation which is given as formula 1. This equation makes it possible to calculate resistance values that do not vary more than one-half of one per cent from those obtained from the curve *MM*. In this formula R is the resistance expressed in pounds per ton and S is the speed in miles per hour.

$$R = 4 + 0.222\,S + 0.00582\,S^2 \ldots\ldots\ldots (1)$$

This equation has been modified as shown in formula 2 to take more definite account of air resistance. In formula 2, A is the cross-sectional area of the car expressed in square feet and W is the car weight in tons.

$$R = 4 + 0.222\,S + 0.00181\,\frac{A}{W}\,S^2 \ldots\ldots (2)$$

These formulae are the equations of a parabola which corresponds closely to the curve *MM*. In using these formulae care should be taken not to extend their use to speeds much beyond the limits of the curve *MM*.

15. *Discussion of the Final Results.*—The final results of this investigation are presented in Fig. 2, Table 2, and formulae 1 and 2, all of which express the mean relation between speed and resistance for the 28-ton car when running at uniform speed on tangent and level track of good construction, during weather when the temperature is not lower than 25° F., and when the wind velocity does not exceed about 26 miles per hour. The curve *MM* (Figs. 1 and 2) is an average of curves derived from ten individual tests that have been run under conditions which, though they differ somewhat, are not at all unusual. Notwithstanding this difference in test conditions, the maximum variation of the individual curves from the mean curve *MM* is only 12 per cent (see Fig. 1). With the exception of that portion of the curve for

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test 23-24, which lies between 30 and 34 miles per hour, the maximum variation of the individual curves from the mean curve is only 9 per cent for all speeds above 30 miles per hour. At speeds below 30 miles per hour this variation is considerably less than 9 per cent. In other words, all curves from the individual tests lie within a narrow belt. All points within this belt may very properly be considered to represent a relation between car resistance and speed which is true under some usual conditions.

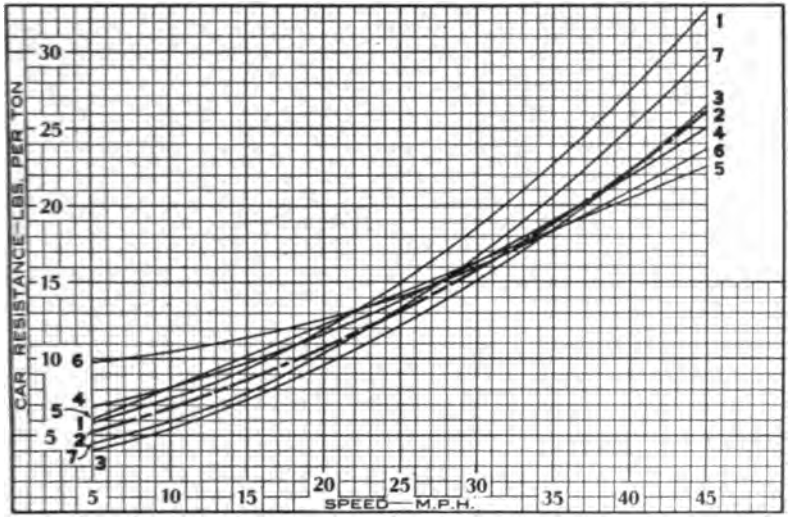

FIG. 4. COMPARATIVE DIAGRAM OF RESISTANCE-SPEED CURVES CALCULATED BY MEANS OF SEVEN FORMULAE APPLICABLE TO ELECTRIC RAILWAY CONDITIONS.

W = 28 TONS. A = 90 SQ. FT. N = 1.

1. DAVIS.
2. UNIVERSITY OF ILLINOIS.
3. SMITH.
4. BLOOD.
5. LUNDIE.
6. ARMSTRONG.
7. MAILLOUX.

If it is desired to estimate the resistance of any car which in weight and general design is similar to the car used during these tests, it is believed that the curve *MM* offers a basis for the prediction of such resistance, the accuracy of which will lie within the limits just stated; that is, it is believed that by the use of the curve *MM* net resistance may be predicted within about 9 per cent. Obviously, in using the curve for such a purpose the conditions surrounding these tests must be fully appreciated.

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In addition to the expression given for the resistance in still air, the tables and curves in Appendix III provide information as to the excess resistance due to wind. They show that during moderate weather and for wind velocities up to 24 miles per hour which directly oppose the motion of the car, the resistance may exceed that in still air by as much as 70 per cent at speeds below 30 miles per hour, and by 50 per cent at speeds above 30 miles per hour.

16. *Comparison with Other Experiments.*—For purposes of comparison with the results of other investigations the following formulae are given, together with Fig. 4:

Davis[1] $$R = 5 + 0.13\,S + \frac{0.3\,S^2}{W}[1 + 0.1\,(N-1)]$$

University of Illinois $$R = 4 + 0.222\,S + 0.00582\,S^2.$$

$$R = 4 + 0.222\,S + 0.00181\frac{A}{W}S^2.$$

Smith[2] $$R = 3 + 0.167\,S + 0.0025\,\frac{A}{W}\,S^2.$$

Blood[3] $$R = 6 + 0.13\,S + (0.0014 + \frac{0.35}{W}\cdot)\,S^{1.8}.$$

Lundie[4] $$R = 4 + 0.24\,S + \frac{4.8\,S}{W}$$

Armstrong[5] $$R = \frac{50}{\sqrt{W}} + 0.03\,S + \frac{0.002AS^2}{W}[1 + 0.1\,(N-1)]$$

Mailloux[6] $$R = 3.5 + 0.15\,S + \frac{0.02\,N + 0.25}{N\,W}S^2$$

Notation:—R= Car resistance in pounds per ton.
S= Speed of the car in miles per hour.
W= Weight of the car in tons.
N= Number of cars in the train.
A= Effective cross-sectional area of the car body and trucks in sq. ft.

1. Street Railway Journal, 1904, v. 24, p. 1003.
2. Proc. Amer. Inst. Elec. Eng., 1904, v. 23, p. 696.
3. Proc. Amer. Soc. Mech. Eng., v. 24, p. 945.
4. Street Railway Journal, 1902, v. 19, p. 557.
5. Standard Handbook for Electrical Engineers, Section on Electric Traction, by A. H. Armstrong.
6. Proc. Amer. Inst. Elec. Eng., 1904, v. 23, p. 731.

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APPENDIX I.

The Electric Test Car.

The electric test car, by means of which the tests reported in this bulletin were made, was built under the direction of the University of Illinois and placed in operation in 1906. The equipment, which is for 500-volt direct current operation, was placed so as to provide facilities for the instruction of students as well as for conducting investigations concerning various features of electric car operation. For this purpose the switch group, circuit breaker, limit switch and certain other parts of the equipment were placed within the car where their action under all conditions might readily be seen. The weight of the car with its equipment, subject to certain corrections, is 55 150 pounds. The corrections referred to are necessitated by additions to the equipment and changes made therein at different times. The general design of the car is shown in Figs. 5, 6 and 7.

Car Body.—The car body, which is of the double-end type commonly used in moderately heavy interurban service, was built by the Jewett Car Co., of Newark, Ohio. Its principal dimensions are given in Fig. 7. The cross-sectional area of the body and trucks is 90 square feet. The vestibules are of the round end type. Ball bearing center plates were installed between tests 29 and 30.

Trucks.—The car body is mounted upon Standard Motor Truck Company C-60 type trucks whose main dimensions are:—

Distance from center to center of trucks 23 ft. 3 in.

Wheel base, 6 ft. 4 in.

Lateral play of axle journals, 3/16 in.

Axle journals, 4¼ in. × 8 in.

Diameter of wheels, 33 in.

Weight of truck exclusive of electrical equipment, 7824 pounds.

On one truck there are four rolled steel wheels, while those on the other truck are chilled cast iron. Both sets have the standard M. C. B. tread and flange.

Motors.—On each truck are two number 101-D Westinghouse 500-volt, direct current motors, each one having a commercial rating of 50 H. P. They are mounted on the axles and geared thereto in the ratio of 22:62. The characteristic curves are shown in Fig. 8.

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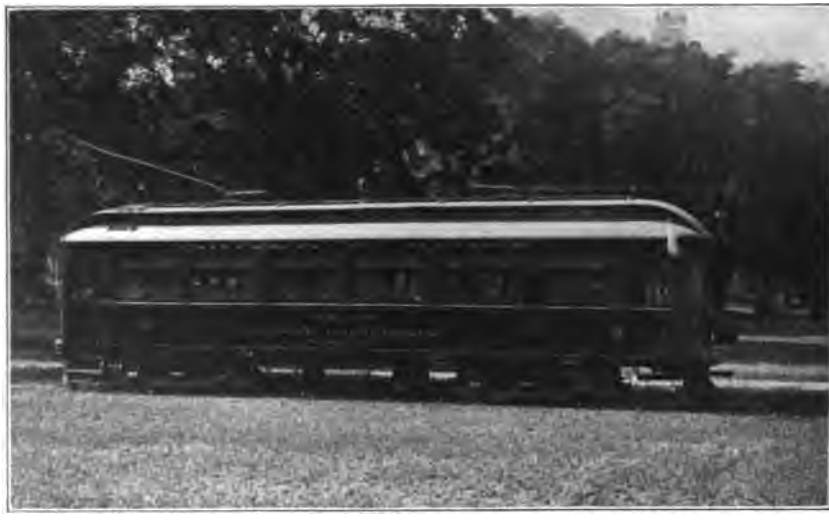

FIG. 5. THE ELECTRIC RAILWAY TEST CAR.

FIG. 6. INTERIOR OF ELECTRIC RAILWAY TEST CAR.

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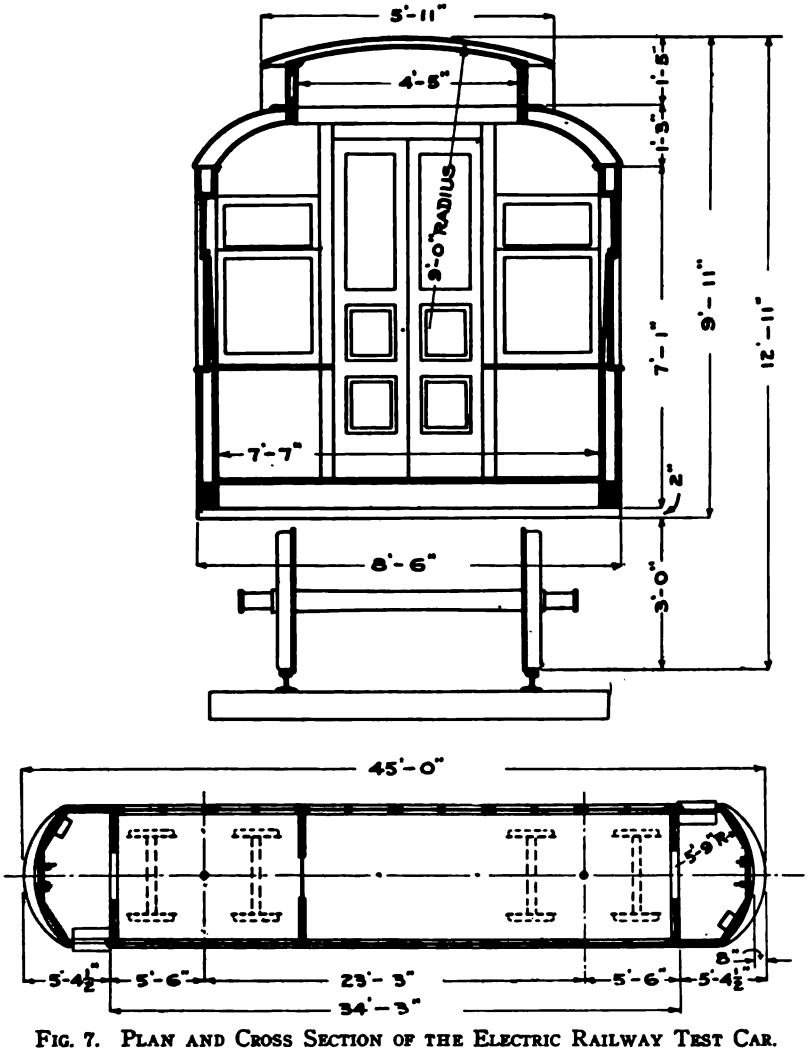

FIG. 7. PLAN AND CROSS SECTION OF THE ELECTRIC RAILWAY TEST CAR.

Control Equipment.—The motor control equipment is that known as the Westinghouse unit switch system of multiple control. In this type of control the switches which make the necessary connections and short circuit the starting resistances are operated by compressed air from the brake reservoir. The valves which control the air are operated from the master controller through the medium of magnets and a low voltage storage battery circuit. Acceleration is governed by a limit switch the armature of which is weighted to give the desired rate.

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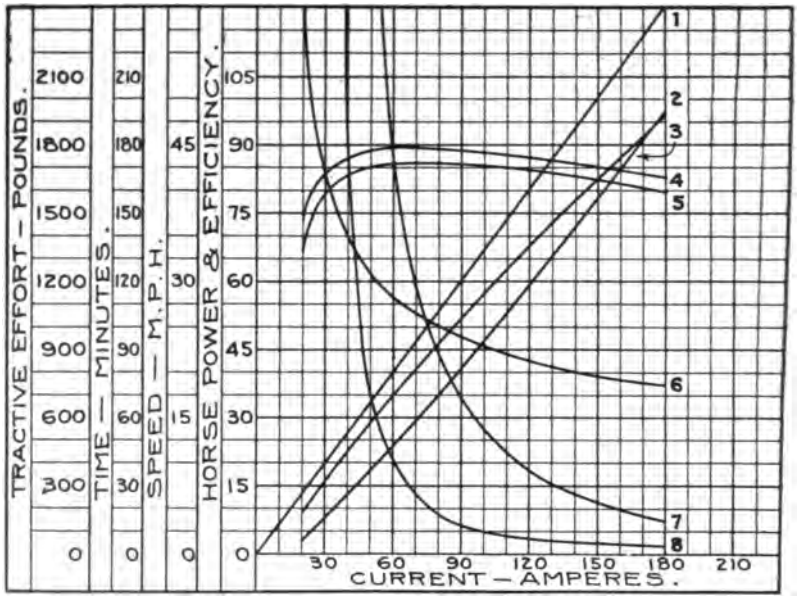

FIG. 8. THE CHARACTERISTIC CURVES OF THE TEST CAR MOTORS.

WESTINGHOUSE NO. 101-D, 500 VOLT D. C. RAILWAY MOTOR.
GEAR RATIO 22:62. 33 INCH WHEELS.
CONTINUOUS CAPACITY 46 AMPERES AT 300 VOLTS.
CONTINUOUS CAPACITY 42 AMPERES AT 400 VOLTS.

1. ELECTRICAL HORSE POWER.
2. BRAKE HORSE POWER WITH GEARS.
3. TRACTIVE EFFORT.
4. EFFICIENCY WITHOUT GEARS.
5. APPROXIMATE EFFICIENCY WITH GEARS.
6. SPEED.
7. TIME TO RISE 75° C. FROM 25° C.
8. SAFE TIME FOR LOAD IN SERVICE FOR 20° C. RISE FROM 75° C.

THE RECORDING APPARATUS.

Within the car there is apparatus by means of which a continuous graphical record may be kept of motor current, voltage, speed, time, distance, location, and brake cylinder pressure upon a chart 40 inches in width.

The Chart.—Fig. 9 has been reproduced from a tracing of a portion of the chart for test No. 121. The records there shown are grouped more closely together than in the original record, and the transverse lines which mark one of the sections selected for calculation and some of the explanatory lettering do not appear thereon. This chart is

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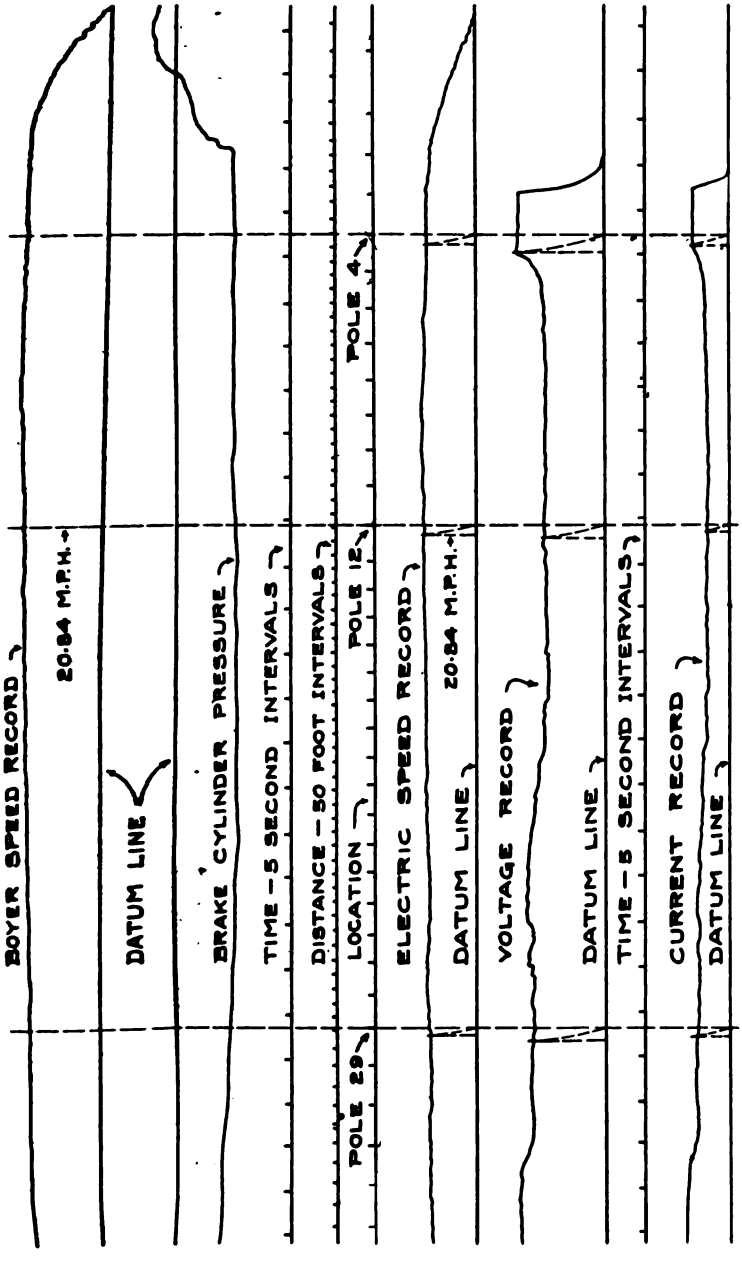

FIG. 9. A PORTION OF THE CHART FROM TEST 121.

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caused to pass from a supply roll, over a series of tension and guide rolls and the recording table, and is then rewound upon a detachable record roll. As the paper travels over the table the instruments draw on it continuous records of the above mentioned data. The paper may be driven by a motor, thus making the records on a time base, or by a flexible shaft geared to the axle, thereby making them on a distance base.

Current Record.—The current taken by the motors is recorded by a General Electric graphic recording ammeter which is excited from a storage battery. A rheostat is provided for the adjustment of the exciting current which should be kept at a constant predetermined value. Check readings may be made by means of an indicating ammeter in the same circuit as the recording meter.

Voltage Record.—The voltage record is made by a General Electric graphic recording voltmeter which is likewise excited by a constant current from the storage battery. The same provision is made for making check readings as described for the current record.

Prior to test 121-122 these instruments were connected so as to give a record of line voltage, but for that and all subsequent tests the connections were changed so that the record is that of voltage across the terminals of one motor.

Speed Record.—One speed record is made by a General Electric graphic recording ammeter connected in the armature circuit of a ½ kilowatt low voltage generator which is excited by a constant storage battery current and driven from the axle by a flexible shaft. The current generated is proportional to speed and therefore the recording ammeter when properly calibrated gives a record of speed.

A Boyer mechanical speed recorder was installed just prior to test 103-104 and its record has been used on all subsequent tests.

Instantaneous values of speed are obtained from an "Autometer," which also is driven from the flexible shaft.

Time Record.—By means of a contacting device on a clock, electrical connections are made every 5 seconds so as to operate two magnets to the armatures of which the time recording pens are attached. Offsets are thus made in the two time lines every 5 seconds.

Location Record.—An observer, by means of a telegrapher's key, battery circuit, and magnet to the armature of which a recording pen is attached, causes offsets to be made in the line giving a record of location as a given point on the test car passes numbered trolley line

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poles, stations, or other markers. This record enables one to correlate any position of the car with the profile.

Distance Record.—A pen operated electrically from a contacting device on one of the car wheels makes an offset in a line for every 50 feet of track passed over by the car. This record is used merely as a check on the distances obtained from a table giving the exact distance between any two trolley line poles.

Brake Cylinder Pressure.—A pressure gage connected to the brake cylinder has been modified so as to make a record upon the chart. This record is used merely to determine and to avoid those portions of tests during which the brakes were applied.

Auxiliary Apparatus.—The power consumed by the motors may be obtained from an integrating wattmeter. A separate wattmeter indicates the power consumed by the pump motors. For a part of the tests herein reported two water rheostats were placed in the trolley circuit for the purpose of cutting down the voltage and thereby obtaining lower speeds than it was otherwise possible to maintain.

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APPENDIX II.

Methods Employed in Calculating the Results.

The steps in the process by which the final results have been reached are here explained in some detail. The description of the general process which has been given in Section IV of the body of the report is to be considered as supplementary to this Appendix. The data taken and the methods by which they were obtained have been described there and in Appendix I. Fig. 9 has been reproduced from a portion of the chart made by the recording apparatus during test number 121.

Selection of the Sections.—From the chart produced during each test, sections have been chosen for calculation by comparison of the profile and chart. As has been stated, only those sections were considered where the car was running on straight track, and over which no brake applications were made. They were selected so that the average speeds over the sections chosen from any particular test should cover as wide a range as possible, and so that the number of sections from east bound runs would be balanced by the same or approximately the same number from west bound runs. In this way the number of resistance determinations made from data obtained during runs against the wind were balanced by an approximately equal number for the opposite condition. In addition, the following were considered sufficient reasons for the rejection of any section of chart:

a. Any considerable variation of the current or voltage from the average value.

b. Heavy grades between the section limits. Level track or light uniform grades are of course the preferable conditions.

c. A large difference in the speed of the car when entering and when leaving the track section or at any points between the section limits.

Aside from other considerations it is highly desirable to choose both track and chart sections so that the energy consumed by acceleration and grade shall be as small as possible, in order that errors in their calculation shall have slight effect on the accuracy of the values of average net car resistance. When the variation in speed is large the energy required to produce acceleration is alone frequently greater than that required to overcome all other resistances combined.

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Fig. 9 shows one of the chart sections selected from a run made over a track section lying between trolley line poles numbered 4 and 12. These limiting points are designated on Fig. 9 by numbered offsets in the location record. Parallel lines drawn through them and perpendicular to the location record line intercept all but four of the records at the points occupied by the pens at the moment when the car entered and left the track section. The four records in question, namely, current, voltage, brake cylinder pressure, and speed by the electrical recorder, are made by pens which move along arcs of circles and which are adjusted so that in their zero position their recording points lie in the same straight line as the other pens. For any but the zero reading, however, these pens take positions ahead of the others and consequently allowance must be made for this fact in determining their positions at the moments when the car passed the points of entrance and exit. This matter has been cared for by means of templates, as shown in Fig. 9.

The Calculations.—The result desired from each such section of chart is a value of average net car resistance. The following explanation of the derivation of each column of data in the tables should make clear all the processes involved in the production of the final results.

Table 1 gives a summary of test conditions. Tables 3 to 12 inclusive present original data, and intermediate and final results as follows:

Column 1. (Item No.) is self explanatory.

Column 2. (Test No.) is self explanatory.

Column 3. (Wind opposing or helping) is from original data.

Column 4. (Section limits) defines the track section as well as the chart section by stating the limiting points which have been designated by trolley line pole numbers. The location of each line pole or other marker is shown accurately upon the profiles.

Column 5. (Grade) is derived from column 4 and the profiles, and expresses the grade as the difference in elevation in feet between the entrance and exit points. A positive sign indicates that the car is going up grade.

Column 6. (Length of section) is derived from column 4 and a table giving the exact distance between any two trolley line poles.

Column 7. (Time) is derived from the time record on the chart by determining the number of whole and decimal parts of 5-second intervals included between the parallel lines drawn through the limiting points of the section.

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Column 8. (Motor data) is from the original data.

Column 9. (Efficiency) is derived from columns 8, 13 and 14, and a table of efficiencies.

Tables 3 to 12 inclusive, in Appendix III, show that the voltage (column 13) varied from about 100 to 600 volts. The low voltages were due largely to the method of using rheostats on the car for the purpose of obtaining low speeds. It was thought that the efficiency curve for the motors at 500 volts was not accurate enough under such conditions and therefore one of the motors was removed from the truck and a series of efficiency tests made upon it at five different voltages, namely, 120, 200, 300, 400, and 500. From the results of these tests a set of efficiency curves for the test car motors with gears was constructed, the Westinghouse Electric & Machine Company's standard gear losses being used. The 500-volt curve, thus determined, corresponds with the curve provided by the manufacturer. From these curves the above mentioned table was compiled.

Column 10. (Item No.) is self explanatory.

Column 11. (Speed at entrance to the section) The height of the speed curve above its base line at the point occupied by the recording pen at the moment the car entered the track section represents the speed of the car at that moment, to some scale determined by a calibration curve. Such a curve was made for each test by plotting average speeds calculated from columns 6 and 7 against the corresponding average heights of the speed curve in sufficient number to determine a curve. Straight lines were found to satisfy the conditions and for each one the equation was found and used in the calculations.

Column 12. (Speed at exit from the section) is determined from the height of the speed curve at the point of exit, and the equation of the calibration curve as explained above.

Column 13. (Average voltage) is determined by first obtaining the average height of the voltage curve between the section limits. From a calibration table the voltage corresponding to the average height may be found and it is recorded in column 13.

Column 14. (Average current) is determined from the current curve in the same manner as column 13 from the voltage curve.

Column 15. (Energy imparted to the car by the current) is calculated by means of the equation—

$$E_c = W \times e \times T \times 2655.$$

in which E_c = the energy in foot pounds imparted to the car by the current.

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W = Average watts, calculated from columns 13 and 14.

e = The efficiency of the motors and gears for the given current and voltage (Column 9).

T = The time in hours for the car to traverse the section = $\frac{\text{column 7}}{3600}$

2655 = foot pounds in one watt hour.

Column 16. (Energy imparted to the car by the change in kinetic energy between the entrance and exit points) is determined from the entrance and exit speeds (columns 11 and 12), and an equation expressing the relation between kinetic energy and the speed of the car. This equation was derived from a series of determinations of the force required to produce acceleration in the rotation of the revolving parts as well as of the force required to produce the acceleration in the motion of translation of the car as a whole. The equation follows:

$$E_k = (1995 + 5P)\ (S_1^2 - S_2^2)$$

in which E_k = The energy in foot pounds imparted to the car by the change in kinetic energy between the entrance and exit points.

S_1 = The speed of the car in miles per hour at the moment of entering the track section (column 11).

S_2 = The speed of the car in miles per hour at the moment of leaving the track section (column 12).

P = The number of additional 150 pounds above the weight of the car and its regular equipment, that is above 55 150 pounds. This item includes live load and special equipment.

1995 and 5 are constants derived from the inertia calculations referred to above.

A negative sign before the figures in column 16 indicates that the speed of the car had increased in passing over the track section and consequently that the kinetic energy possessed by the car at the moment it left the section was greater than when it entered it by the amount given. This energy had to be supplied by the current, by the grade, or by both current and grade.

Column 17. (Energy imparted to the car by the grade) is the product of the weight of the car (column 2, Table 1) and the difference in elevation in feet between the points of entrance and exit. A negative sign indicates that the car is going up grade and is consequently requiring additional energy for that purpose.

Column 18. (Net car resistance in pounds per ton)—The net

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energy used in overcoming car resistance alone while traversing the track section is first found from columns 15, 16 and 17 by correcting the energy delivered by the current, for changes in speed and grade. From the net energy the average value for the section is found by dividing by the length of that section in feet (column 6), and this in turn is reduced to net car resistance in pounds per ton of car weight by dividing by the weight of the car in tons. This is the result desired from each chart section.

Column 19. (Average speed) is calculated from the length of the track section (column 6) and the time required to traverse that distance (column 7). Columns 18 and 19 provide the co-ordinates of the points plotted in Figs. 10 to 19.

Method of Obtaining the Mean Curves of Car Resistance for Tests of Groups A and B.—As described on page 13, the ordinates of points on the mean curves for the individual tests of groups A and B have been calculated by means of an equation which is based on the assumption that wind resistance varies as the square of the speed. The derivation of the equation follows:

Let R_s = the resistance at speed s excluding air resistance

$$R = \text{the total resistance at speed } s = R_s + cs^2 \qquad (1)$$

in which c = some constant quantity

$$R_a = \text{the resistance for a run against the wind at speed } s$$
$$= R_s + c\,(s + v)^2 \qquad (2)$$

in which v = the component of the wind velocity parallel to the track.

$$R_w = \text{the resistance for a run at speed } s \text{ with the wind}$$
$$= R_s + c\,(s - v)^2 \qquad (3)$$

Subtracting equation (3) from (2) and solving for c

$$c = \frac{R_a - R_w}{4\,sv} \qquad (4)$$

Substituting (4) in equation (2) and solving for R_s

$$R_s = R_a - \frac{R_a - R_w}{4\,sv}\,(s + v)^2 \qquad (5)$$

Substituting equation (5) in equation (1)

$$R = R_a - \frac{R_a - R_w}{4\,s}\,(2s + v) \qquad (6)$$

Equation (6) is the one sought. The mean curves of groups A and B, calculated by means of this equation, and those of group C, determined as described on page 16, after being modified as described on page 14 to conform to the general shape, were accepted as the results desired from the individual tests.

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APPENDIX III.

The Results of the Individual Tests.

Note.—In the figures solid dots are for runs with the wind, and open circles are for runs against the wind.

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Table 3

Test No. 15-16.

1	2	3	4	5	6	7	8	9
Item No.	Test No.	Wind Opposing or Helping	Section Limits. — Trolley Line Pole Numbers.	Grade. — Rise or Fall Over Section. — + Up — Down	Length of Track Section	Time to Run Over Section	Motor Data: Number in Use and Connection*	Motor Data: Efficiency of Motors and Gears
		O or H		Feet	Feet	Sec's.		%
1	15	H	1 620–1 575	+13.20	4 009	77.21	4 M	85.1
2	"	"	1 470–1 440	+ 4.17	2 991	72.14	"	78.4
3	"	"	1 440–1 380	− 5.34	5 994	127.94	"	79.6
4	"	"	1 380–1 345	−10.15	3 503	63.14	"	77.5
5	"	"	1 335–1 305	− 3.45	2 986	53.30	"	81.0
6	"	"	1 295–1 270	+ 4.05	2 495	42.50	"	81.4
7	"	"	1 250–1 230	− 3.74	1 987	32.20	"	82.4
8	"	"	1 070– 970	− 9.92	9 987	173.99	"	81.2
9	16	O	970–1 000	+ 4.23	2 958	61.88	"	82.0
10	"	"	1 000–1 035	+ 3.53	3 480	69.21	"	82.2
11	"	"	1 035–1 070	+ 2.15	3 549	66.02	"	82.9
12	"	"	1 115–1 135	− 1.19	1 988	38.47	"	85.4
13	"	"	1 270–1 295	− 4.05	2 495	42.82	"	82.4
14	"	"	1 305–1 330	+ 3.41	2 488	43.66	"	81.8
15	"	"	1 400–1 430	+ 1.57	2 998	67.76	"	80.7
16	"	"	1 455–1 480	+ 0.42	2 492	52.40	"	81.0
17	"	"	1 480–1 535	+17.12	5 473	110.77	"	81.4
18	"	"	1 580–1 620	−14.74	3 576	67.54	"	81.4

*S=Series-multiple. M=Multiple. Sw=Switching. C=Coasting.

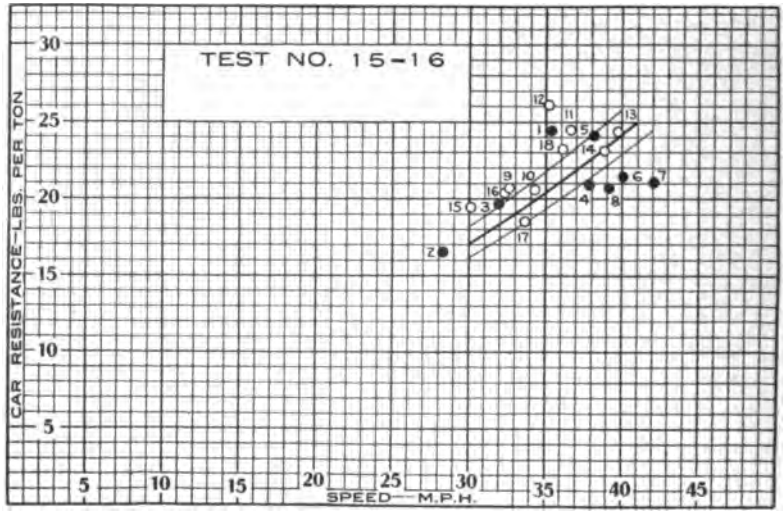

Fig. 10.

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Table 3 (Continued)

Test No. 15-16.

10	11	12	13	14	15	16	17	18	19
	Speed				Energy Imparted to Car				
Item No.	At Entrance to Section	At Exit From Section	Average Voltage	Average Current	By the Current	By the Change in Kinetic Energy	By the Grade	Net Car Resistance	Average Speed Over the Section
	M. P. H.	M. P. H.	Volts	Amp's.	Ft. Lbs.	Ft. Lbs.	Ft. Lbs.	Lbs. Per Ton	M. P. H.
1	33.46	28.32	510	171.7	4 243 500	— 720 400	—747 100	24.46	35.42
2	26.65	29.24	351	132.1	1 984 200	— 298 900	—236 000	16.51	28.27
3	29.24	35.08	394	128.4	3 800 000	— 775 700	+302 200	19.60	31.95
4	35.08	36.37	411	113.4	1 682 000	— 190 300	+574 500	20.84	37.83
5	37.99	38.64	469	129.9	1 940 000	— 102 860	+195 300	24.05	38.20
6	39.94	39.29	494	129.9	1 637 000	+ 106 350	—239 200	21.44	40.04
7	37.99	40.58	534	133.3	1 392 800	— 420 200	+211 700	21.06	42.07
8	34.10	41.23	467	131.5	6 398 500	—1 109 100	+561 500	20.69	39.15
9	29.57	32.16	436	141.1	2 302 000	— 330 150	—239 400	20.69	32.59
10	32.16	35.40	452	141.1	2 676 000	— 452 000	—199 800	20.55	34.28
11	35.40	37.02	492	142.0	2 320 000	— 242 300	—121 700	24.45	36.64
12	29.89	36.37	554	170.1	2 263 000	— 886 600	+ 67 350	26.01	35.24
13	36.70	38.96	523	135.1	1 839 000	— 353 100	+229 200	24.28	39.73
14	37.99	37.34	492	132.6	1 718 000	+ 101 100	—193 000	23.09	38.85
15	27.62	31.19	388	128.8	2 172 000	— 433 550	— 88 860	19.44	30.18
16	28.60	31.19	422	134.6	1 778 000	— 319 800	— 23 770	20.34	32.43
17	31.19	32.16	436	136.6	3 961 000	— 126 900	—969 000	18.49	33.68
18	29.24	36.37	456	134.4	2 485 100	— 966 000	+834 300	23.25	36.10

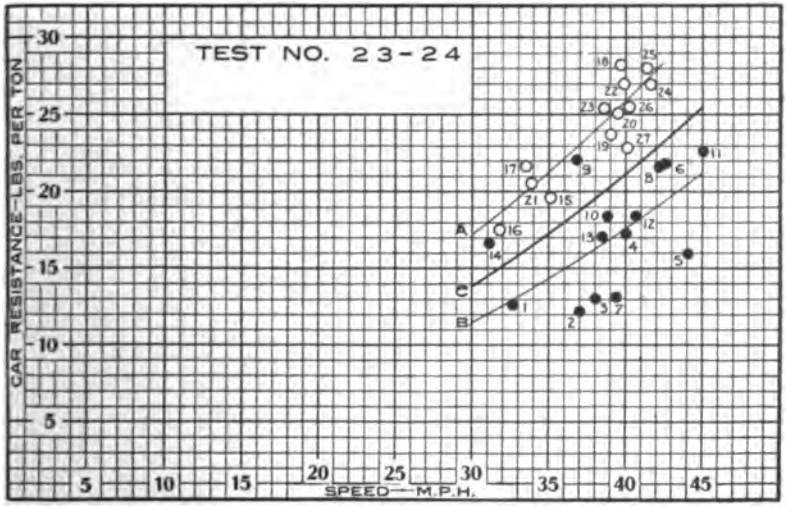

Fig. 11.

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Table 4

Test No. 23-24.

1	2	3	4	5	6	7	8	9
Item No.	Test No.	Wind Opposing or Helping	Section Limits. — Trolley Line Pole Numbers.	Grade — Rise or Fall Over Section — + Up — Down	Length of Track Section	Time to Run Over Section	Motor Data: Number in Use and Connection *	Motor Data: Efficiency of Motors and Gears
		O or H		Feet	Feet	Sec's		%
1	23	H	1 471 -1 462	+0.2	897	18.71	4 M	83.8
2	"	"	1 462 -1 453	+0.6	896	16.50	" "	82.6
3	"	"	1 402.7-1 396.7	—1.94	600	10.75	" "	76.6
4	"	"	1 394.2-1 388.2	—0.16	594	10.12	" "	80.8
5	"	"	1 361 -1 343	—1.49	1 804	27.91	" "	79.2
6	"	"	1 291 -1 282	—0.06	901	14.42	" "	80.0
7	"	"	1 061 -1 075	+0.78	582	10.07	" "	84.5
8	"	"	1 012 - 970	—4.76	4 144	66.90	" "	81.8
9	"	"	917.2- 901.75	+1.02	1 529	28.33	" "	83.3
10	"	"	871.5- 853.4	+2.15	1 803	31.65	" "	79.9
11	"	"	780.4- 720.5	—6.74	5 970	90.30	" "	83.4
12	"	"	567.4- 540.5	+0.43	2 689	45.07	" "	82.9
13	"	"	489.3- 468.5	+0.50	2 063	36.89	" "	81.2
14	"	"	190.5- 150.5	—3.96	3 948	86.45	" "	80.0
15	24	O	168.8- 190.5	+3.19	2 090	40.56	" "	83.2
16	"	"	403.1- 420.2	+1.33	1 664	35.60	" "	83.1
17	"	"	459.5- 492.3	+0.53	3 276	66.55	" "	80.5
18	"	"	550.5- 570.3	—0.62	1 977	33.94	" "	82.5
19	"	"	625.3- 687.5	+2.29	6 270	109.49	" "	84.3
20	"	"	726.3- 780.4	+5.57	5 392	92.98	" "	84.0
21	"	"	1 039 -1 075	+2.53	3 678	74.00	" "	83.3
22	"	"	1 111 -1 141	—0.98	2 982	50.94	" "	84.5
23	"	"	1 226.5-1 251.5	+5.48	2 483	43.86	" "	83.9
24	"	"	1 273 -1 297.5	—2.46	2 443	40.00	" "	83.8
25	"	"	1 306.8-1 334	+2.99	2 707	44.56	" "	83.7
26	"	"	1 343 -1 373	+4.97	3 004	50.88	" "	82.2
27	"	"	1 406 -1 465	—5.11	5 889	100.11	" "	83.0

*S=Series-multiple. M=Multiple. Sw=Switching. C=Coasting.

Table 5

Test No. 73-74.

1	2	3	4	5	6	7	8	9
Item No.	Test No.	Wind O or H	Section Limits	Grade	Length of Section	Time	Motor Data: No. and Connection *	Motor Data: Efficiency
1	73	H	1 470-1 455	+0.56	1 495	23.78	4 M	81.1
2	"	"	" "	"	"	24.60	4 M	81.9
3	"	"	" "	"	"	29.00	4 M	80.4
4	"	"	" "	"	"	28.58	4 S	84.7
5	"	"	" "	"	"	28.31	4 S	84.7
6	"	"	" "	"	"	37.80	C	——
7	74	O	1 455-1 470	—0.56	"	26.91	4 M	82.7
8	"	"	" "	"	"	28.40	4 M	83.3
9	"	"	" "	"	"	30.35	4 S	85.5
10	"	"	" "	"	"	31.94	4 S	85.8
11	"	"	" "	"	"	38.58	C	——

*S=Series-multiple. M=Multiple. Sw=Switching. C=Coasting.

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TABLE 4 (Continued)

TEST No. 23-24.

10	11	12	13	14	15	16	17	18	19
	Speed				Energy Imparted to Car				
Item No.	At Entrance to Section	At Exit from Section	Average Voltage	Average Current	By the Current	By the Change in Kinetic Energy	By the Grade	Net Car Resistance	Average Speed Over Section
	M. P. H.	M. P. H.	Volts	Amp's.	Ft. Lbs.	Ft. Lbs.	Ft. Lbs.	Lbs. Per Ton	M. P. H
1	31.33	34.97	464	153.8	822 230	— 493 520	— 11 250	12.58	32.69
2	34.97	37.16	455	145.3	664 520	— 323 040	— 33 750	12.21	37.02
3	38.61	39.71	435	108.5	286 630	— 176 180	+109 120	13.01	38.05
4	39.52	40.07	482	127.0	369 150	— 89 520	+ 9 000	17.28	40.02
5	43.53	44.62	477	118.5	922 650	— 196 490	+ 83 810	15.96	44.05
6	46.99	46.81	503	120.3	514 820	+ 34 530	+ 3 380	21.81	42.60
7	39.34	40.98	528	159.4	528 160	— 269 380	— 43 870	13.13	39.40
8	41.71	44.62	529	129.5	2 764 800	— 513 750	+267 750	21.61	42.23
9	37.34	39.16	515	144.0	1 290 700	— 284 730	— 57 380	22.06	36.80
10	39.16	39.16	459	123.0	1 052 900	0	—120 900	18.38	38.84
11	42.26	47.90	605	132.6	4 455 700	—1 039 900	+379 120	22.60	45.08
12	37.16	40.98	536	137.5	2 030 800	— 610 420	— 24 190	18.46	40.68
13	35.15	37.34	465	121.5	1 350 900	— 324 650	— 28 130	17.07	38.50
14	29.14	37.34	422	127.2	2 737 900	—1 114 800	+222 700	16.62	31.14
15	34.24	36.97	464	149.6	1 727 500	— 397 550	—179 400	19.57	35.13
16	29.14	33.70	448	151.4	1 479 800	— 586 000	— 74 810	17.50	31.85
17	32.42	33.70	426	130.8	2 201 500	— 173 080	— 29 810	21.69	33.57
18	38.61	38.25	526	136.0	1 477 200	+ 56 580	+ 34 880	28.21	39.71
19	33.15	41.89	548	151.6	5 655 200	—1 341 200	—128 800	23.73	39.04
20	37.34	40.62	548	146.9	4 636 900	— 522 920	—313 310	25.06	39.50
21	30.05	37.16	476	150.0	3 245 900	— 977 230	—142 300	20.55	33.89
22	37.89	40.98	571	149.4	2 708 100	— 498 380	+ 55 130	27.00	39.89
23	39.16	39.52	536	147.1	2 139 800	— 57 920	—308 300	25.40	38.61
24	41.53	42.99	560	142.0	1 965 800	— 252 350	+138 400	26.95	41.64
25	42.80	41.89	548	142.4	2 146 500	+ 157 600	—168 200	28.05	41.41
26	42.26	40.44	517	133.3	2 125 700	+ 307 800	—279 560	25.49	40.26
27	38.25	42.07	515	132.2	4 121 800	— 627 450	+287 440	22.83	40.11

TABLE 5 (Continued)

TEST No. 73-74.

10	11	12	13	14	15	16	17	18	19
	Speed				Energy Imparted to Car				
Item No.	Entrance	Exit	Average Voltage	Average Current	By Current	By Kinetic Energy	By Grade	Net Car Resistance	Average Speed
1	41.35	42.21	516	125.8	923 200	—146 950	—31 720	17.58	42.87
2	40.83	41.52	520	130.5	1 008 000	—116 200	"	20.31	41.45
3	33.56	35.29	443	127.7	972 800	—243 600	"	16.47	35.13
4	36.16	36.85	528	80.9	762 700	—103 000	"	14.83	35.72
5	35.12	35.99	530	81.2	761 100	—126 500	"	14.23	36.00
6	30.10	24.09	—	—	—	+666 000	"	14.98	26.96
7	36.68	37.20	507	138.6	1 153 000	— 78 560	+31 720	26.12	37.87
8	34.60	36.33	506	144.9	1 279 000	—250 950	"	25.02	35.89
9	32.52	33.39	530	92.1	934 100	—117 250	"	20.02	33.58
10	30.97	32.87	528	98.0	1 046 000	—248 050	"	19.59	31.91
11	30.10	24.05	—	—	—	+669 950	"	16.56	26.42

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Table 6

Test No. 77-78.

1	2	3	4	5	6	7	8	9
Item No.	Test No.	Wind Opposing or Helping	Section Limits. — Trolley Line Pole Numbers	Grade — Rise or Fall Over Section — + Up — Down	Length of Track Section	Time to Run Over Section	Motor Data: Number in Use and Connection *	Motor Data: Efficiency of Motors and Gears
		O or H		Feet	Feet	Sec's.		%
1	77	H	1 470–1 455	+0.56	1 495	23.03	4 M	78.5
2	78	O	1 455–1 470	—0.56	"	27.68	4 M	82.3
3	77	H	1 470–1 455	+0.56	"	32.87	2 M	85.0
4	78	O	1 455–1 470	—0.56	"	32.57	2 M	85.7
5	77	H	1 470–1 455	+0.56	"	35.48	2 M	81.6
6	78	O	1 455–1 470	—0.56	"	36.26	2 M	78.9
7	77	H	1 470–1 455	+0.56	"	38.01	2 M	79.3
8	77	H	1 470–1 455	+0.56	"	57.22	C	
9	78	O	1 455–1 470	—0.56	"	95.00	C	
10	77	H	1 470–1 455	+0.56	"	85.00	C	
11	78	O	1 455–1 470	—0.56	"	79.45	C	
12	77	H	1 470–1 455	+0.56	"	77.58	C	
13	78	O	1 455–1 470	—0.56	"	80.06	C	
14	78	O	1 455–1 470	—0.56	"	29.82	4 M	82.2

*S=Series-multiple. M=Multiple. Sw=Switching. C=Coasting.

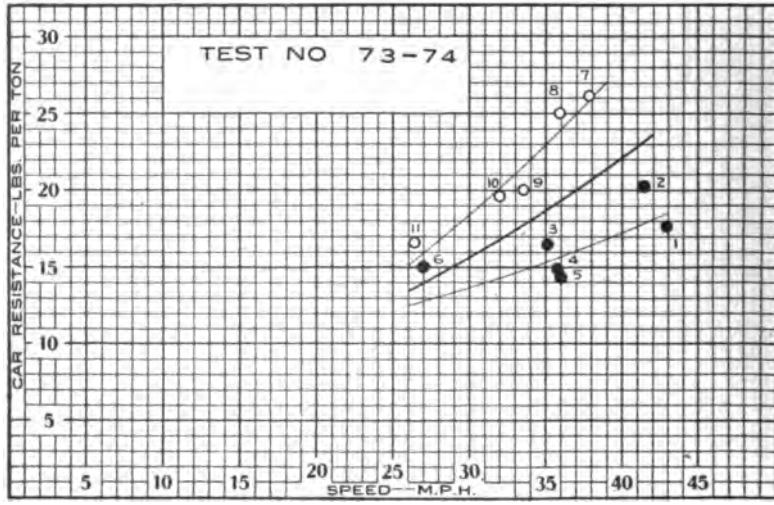

Fig. 12.

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TABLE 6 (Continued)

TEST NO. 77-78.

10	11	12	13	14	15	16	17	18	19
	Speed				Energy Imparted to the Car				
Item No.	At Entrance to Section	At Exit from Section	Average Voltage	Average Current	By the Current	By the Change in Kinetic Energy	By the Grade	Net Car Resistance	Average Speed Over the Section
	M. P. H.	M. P. H.	Volts	Amp's.	Ft. Lbs.	Ft. Lbs.	Ft. Lbs.	Lbs. Per Ton	M. P. H.
1	45.22	44.46	507	112.4	759 750	+139 720	—31 810	20.44	44.25
2	35.48	37.01	499	136.8	1 146 960	—227 360	+31 810	22.41	36.82
3	29.94	31.85	480	89.5	885 210	—241 940	—31 810	14.40	31.01
4	34.34	36.05	507	98.7	1 029 860	—246 750	+31 810	19.19	31.29
5	28.61	29.94	510	64.9	706 830	—159 640	—31 810	12.14	28.72
6	27.84	26.31	499	57.6	606 320	+169 840	+31 810	19.03	28.11
7	26.12	25.17	420	61.1	570 050	+ 99 890	—31 810	15.03	26.82
8	19.06	12.37				+431 050	—31 810	9.40	17.81
9	14.85	6.64				+361 690	+31 810	9.27	10.73
10	15.43	10.46				+263 780	—31 810	5.46	11.99
11	17.15	8.55				+453 090	+31 810	11.42	12.83
12	15.04	9.89				+263 200	—31 810	5.45	13.14
13	17.72	8.93				+480 220	+31 810	12.06	12.73
14	31.66	34.72	412	146.0	1 086 840	—416 400	+31 810	16.54	34.18

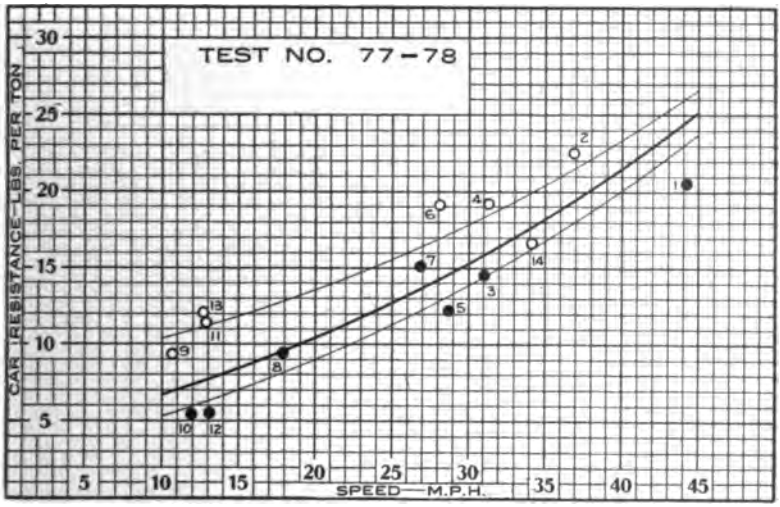

FIG. 13.

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Table 7

Test No. 91-92.

1	2	3	4	5	6	7	8	9
Item No.	Test No.	Wind Opposing or Helping	Section Limits. — Trolley Line Pole Numbers	Grade. — Rise or Fall Over Section. — + Up — Down	Length of Track Section	Time to Run Over Section	Motor Data: Number in Use and Connection *	Motor Data: Efficiency of Motors and Gears
		O or H		Feet	Feet	Sec's.		%
1	92	O	1 455–1 470	—0.56	1 495	31.50	4 M	81.5
2	91	H	1 470–1 455	+0.56	"	25.61	4 M	78.0
3	92	O	1 455–1 470	—0.56	"	29.80	4 M	82.0
4	91	H	1 470–1 455	+0.56	"	30.35	C	
5	92	O	1 455–1 470	—0.56	"	43.89	C	
6	91	H	1 470–1 455	+0.56	"	36.46	C	
7	92	O	1 455–1 470	—0.56	"	37.86	C	
8	91	H	1 470–1 455	+0.56	"	44.96	C	
9	92	O	1 455–1 470	—0.56	"	45.94	C	
10	91	H	1 470–1 455	+0.56	"	41.47	C	
11	92	O	1 455–1 470	—0.56	"	63.71	C	

*S=Series-multiple. M=Multiple. Sw=Switching. C=Coasting.

Table 8

Test No. 95-96.

1	2	3	4	5	6	7	8	9
Item No.	Test No.	Wind O or H	Section Limits	Grade	Length of Section	Time	Motor Data: No. and Connection *	Motor Data: Efficiency
1	95	H	1 470–1 455	+0.56	1 495	21.97	4 M	79.6
2	96	O	1 455–1 470	—0.56	"	27.55	"	82.6
3	95	H	1 470–1 455	+0.56	"	23.90	"	81.2
4	96	O	1 455–1 470	—0.56	"	27.42	"	82.0
5	95	H	1 470–1 455	+0.56	"	24.70	"	81.2
6	96	O	1 455–1 470	—0.56	"	28.40	"	82.6
7	95	H	1 470–1 455	+0.56	"	43.37	4 S	72.9
8	96	O	1 455–1 470	—0.56	"	58.10	4 S	73.6
9	95	H	1 470–1 455	+0.56	"	32.00	4 M	83.0
10	96	O	1 455–1 470	—0.56	"	33.10	"	83.1
11	95	H	1 470–1 455	+0.56	"	27.50	"	80.5
12	96	O	1 455–1 470	—0.56	"	30.20	"	82.7
13	95	H	1 470–1 455	+0.56	"	38.05	C	
14	96	O	1 455–1 470	—0.56	"	33.30	C	
15	95	H	1 470–1 455	+0.56	"	37.25	C	
16	96	O	1 455–1 470	—0.56	"	35.20	C	
17	95	H	1 470–1 455	+0.56	"	48.05	C	
18	96	O	1 455–1 470	—0.56	"	39.75	C	

*S=Series-multiple. M=Multiple. Sw=Switching. C=Coasting.

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TABLE 7 (Continued)

TEST No. 91-92.

10	11	12	13	14	15	16	17	18	19
Item No.	Speed		Average Voltage	Average Current	Energy Imparted to the Car			Net Car Resistance	Average Speed Over the Section
	At Entrance to Section	At Exit from Section			By the Current	By the Change in Kinetic Energy	By the Grade		
	M. P. H.	M. P. H.	Volts	Amp's.	Ft. Lbs.	Ft. Lbs.	Ft. Lbs.	Lbs. Per Ton	M. P. H.
1	32.71	33.24	454	134.7	1 157 860	— 71 130	—31 560	25.05	32.36
2	40.52	41.41	472	112.5	782 280	—148 390	+31 560	15.80	39.82
3	34.13	34.48	472	137.1	1 166 190	— 48 870	—31 560	25.78	34.21
4	35.01	32.00				+410 460	+31 560	10.50	33.59
5	27.56	20.10				+723 530	—31 560	16.43	23.22
6	28.80	25.78				+335 430	+31 560	8.71	27.96
7	30.93	22.76				+892 650	—31 560	20.44	26.92
8	22.76	18.85				+331 080	+31 560	8.61	22.67
9	27.38	17.43				+907 320	—31 560	20.79	22.19
10	26.67	22.23				+441 830	+31 560	11.24	24.58
11	20.63	12.10				+568 150	—31 560	12.74	16.00

TABLE 8 (Continued)

TEST No. 95-96.

10	11	12	13	14	15	16	17	18	19
Item No.	Speed		Average Voltage	Average Current	Energy Imparted to Car			Net Car Resistance	Average Speed
	Entrance	Exit			By Current	By Kinetic Energy	By Grade		
1	46.44	46.13	518	117.5	785 010	+ 58 398	—31 556	19.27	46.40
2	37.87	37.24	499	138.7	1 161 600	+ 96 295	+31 556	30.61	37.00
3	41.21	43.11	516	126.6	934 980	—326 020	—31 556	13.71	42.65
4	37.24	37.24	482	135.4	1 082 200	0	+31 556	26.44	37.17
5	39.94	42.00	510	127.0	958 050	—343 500	—31 556	13.84	41.27
6	35.34	36.76	488	140.0	1 182 000	—208 350	+31 556	23.87	35.89
7	22.96	25.18	539	45.6	573 090	—217 480	—31 556	7.69	23.50
8	17.40	18.20	522	46.8	770 440	— 57 957	+31 556	17.66	17.54
9	27.72	33.75	450	150.4	1 325 700	—754 300	—31 556	12.82	31.85
10	29.62	33.27	452	151.2	1 386 400	—467 130	+31 556	22.57	30.80
11	35.97	37.56	477	125.4	976 580	—237 920	—31 556	16.79	37.07
12	32.16	33.75	477	142.7	1 253 800	—213 260	+31 556	25.45	33.75
13	30.26	26.45				+439 690	—31 556	9.69	26.79
14	34.54	26.61				+986 810	+31 556	24.18	30.61
15	29.31	25.02				+474 310	—31 556	10.51	27.36
16	32.32	25.34				+819 020	+31 556	20.19	28.96
17	22.96	18.83				+351 230	—31 556	7.59	21.21
18	28.35	20.89				+747 520	+31 556	18.50	25.64

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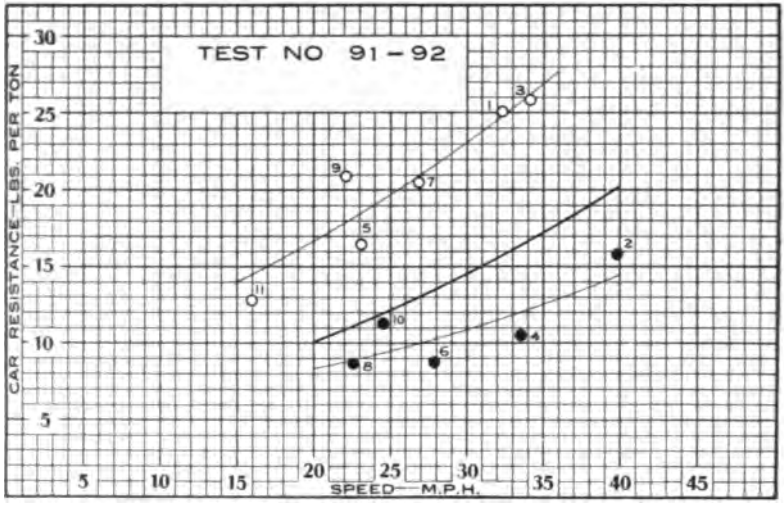

FIG. 14.

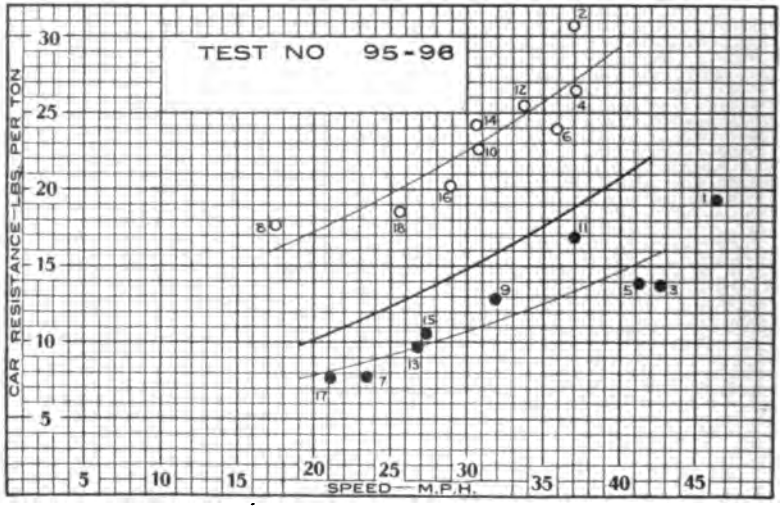

FIG. 15.

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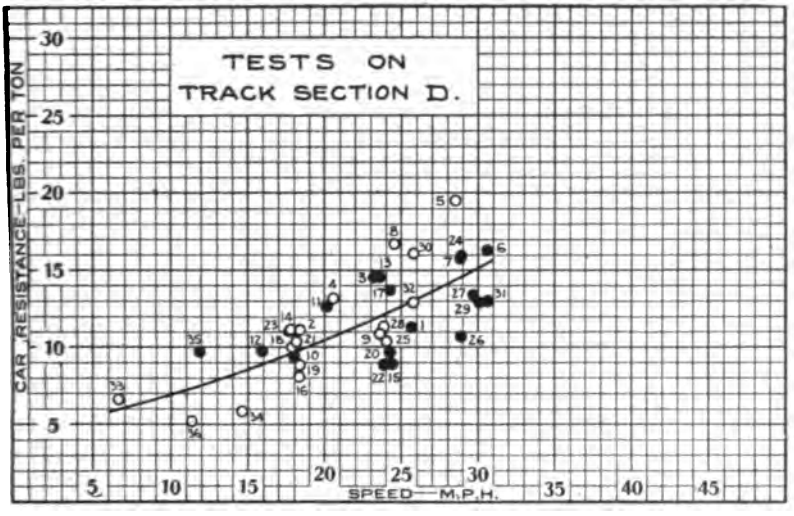

FIG. 16.

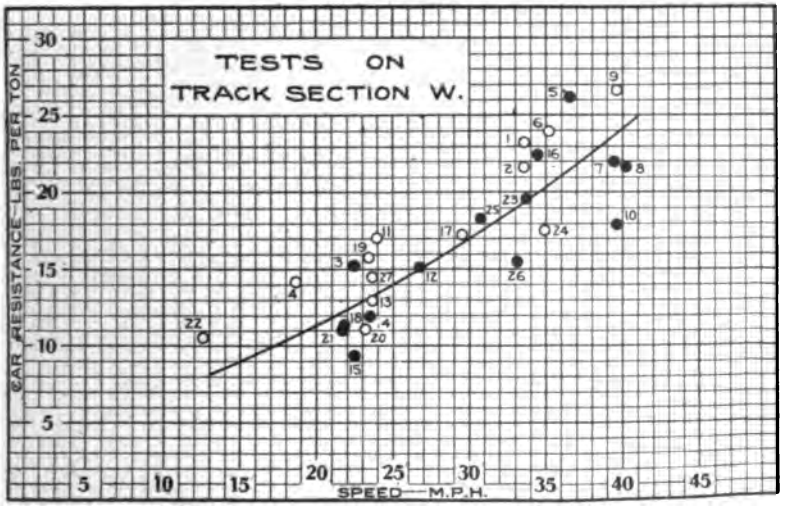

FIG. 17.

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TABLE 9

TESTS ON TRACK SECTION D.

1	2	3	4	5	6	7	8	9
Item No.	Test No.	Wind Opposing or Helping	Section Limits. — Trolley Line Pole Numbers	Grade — Rise or Fall Over Section — + Up — Down	Length of Track Section	Time to Run Over Section	Motor Data — Number in Use and Connection *	Motor Data — Efficiency of Motors and Gears
		O or H		Feet	Feet	Sec's.		%
1	109	H	1 571–1 566	+0.37	445	11.8	4 S	70.1
2	110	O	1 575–1 581	+2.54	522	19.3	4 S	81.4
3	109	H	1 571–1 563	+0.42	714	20.9	4 S	71.6
4	110	O	1 575–1 581	+2.54	522	17.3	4 S	82.5
5	110	O	1 575–1 581	+2.54	522	12.5	4 M	84.5
6	109	H	1 571–1 563	+0.42	714	15.9	4 M	79.9
7	109	H	1 571–1 563	+0.42	714	16.9	4 M	74.5
8	110	O	1 575–1 583	+4.60	700	19.4	4 M	80.9
9	110	O	1 575–1 583	+4.60	700	20.2	1	86.0
10	109	H	1 571–1 566	+0.37	445	16.8	C	——
11	111	H	1 581–1 575	—2.54	522	17.6	C	——
12	111	H	1 581–1 575	—2.54	522	22.3	C	——
13	111	H	1 571–1 563	+0.42	714	20.6	4 S	73.9
14	112	O	1 575–1 581	+2.54	522	20.0	4 S	79.4
15	111	H	1 571–1 566	+0.37	445	12.4	4 S	70.5
16	112	O	1 575–1 581	+2.54	522	19.4	4 S	76.6
17	111	H	1 571–1 566	+0.37	445	12.5	4 S	68.7
18	112	O	1 575–1 583	+4.60	700	26.7	4 S	80.8
19	112	O	1 575–1 583	+4.60	700	25.9	4 S	79.5
20	111	H	1 571–1 566	+0.37	445	12.5	4 S	61.5
21	112	O	1 575–1 583	+4.60	700	26.3	4 S	79.6
22	111	H	1 571–1 566	+0.37	445	12.7	4 S	66.0
23	112	O	1 575–1 583	+4.60	700	26.9	4 S	78.5
24	111	H	1 571–1 566	+0.37	445	10.5	4 M	72.6
25	112	O	1 575–1 581	+2.54	522	14.8	4 M	82.1
26	111	H	1 571–1 566	+0.37	445	10.5	4 M	67.8
27	111	H	1 571–1 566	+0.37	445	10.2	4 M	73.8
28	112	O	1 575–1 583	+4.60	700	20.0	4 M	74.5
29	111	H	1 571–1 566	+0.37	445	10.1	4 M	73.6
30	112	O	1 525–1 583	+4.60	700	18.5	4 M	80.8
31	111	H	1 571–1 566	+0.37	445	9.9	4 M	73.8
32	112	O	1 575–1 583	+4.60	700	18.5	4 M	81.5
33	114	O	1 575–1 578	+0.48	256	26.3	2 M	71.0
34	114	O	1 566–1 571	—0.37	445	20.7	2 M	65.0
35	113	H	1 571–1 566	+0.37	445	25.5	2 M	66.7
36	114	O	1 566–1 571	—0.37	445	26.7	2 M	70.5

*S = Series-multiple. M = Multiple. Sw = Switching. C = Coasting.

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Table 9 (Continued)

Tests on Track Section D.

10	11	12	13	14	15	16	17	18	19
	Speed				Energy Imparted to Car				
Item No.	At Entrance to the Section	At Exit from the Section	Average Voltage	Average Current	By the Current	By the Change in Kinetic Energy	By the Grade	Net Car Resistance	Average Speed Over the Section
	M. P. H.	M. P. H.	Volts	Amp's.	Ft. Lbs.	Ft. Lbs.	Ft. Lbs.	Lbs. Per Ton	M. P. H.
1	24.30	23.94	492	42.9	128 600	+ 35 700	— 21 070	11.30	25.72
2	16.74	17.64	496	65.0	373 400	— 63 580	—144 700	11.11	18.44
3	22.50	21.60	472	45.5	237 200	+ 81 560	— 23 920	14.50	23.23
4	19.80	20.34	565	64.8	385 300	— 44 530	—144 700	13.19	20.57
5	27.36	28.98	460	174.0	622 000	—187 600	—144 700	19.49	28.56
6	31.50	32.76	449	124.0	521 600	—166 400	— 23 920	16.29	30.60
7	27.90	27.90	344	107.5	343 200	0	— 23 920	15.71	28.80
8	23.94	24.30	368	148.0	630 300	— 35 700	—262 000	16.68	24.59
9	22.86	22.86	484	77.2	478 600	0	—262 000	10.86	23.61
10	19.80	18.00				+139 800	— 21 070	9.36	18.03
11	19.62	19.08				+ 42 510	+143 100	12.62	20.22
12	15.30	15.30				0	+143 100	9.72	15.95
13	23.04	22.50	496	47.8	266 100	+ 50 040	— 23 670	14.54	23.62
14	17.64	18.00	475	59.9	333 000	— 26 110	—143 100	11.13	17.78
15	24.12	24.12	467	43.9	132 100	0	— 20 850	8.88	24.46
16	18.00	18.00	443	54.0	262 000	0	—143 100	8.09	18.33
17	24.12	23.40	460	42.2	122 800	+ 69 620	— 20 850	13.68	24.26
18	17.64	18.00	477	63.6	482 400	— 26 110	—259 200	10.00	17.87
19	18.36	18.36	475	39.9	431 900	0	—259 200	8.76	18.42
20	23.22	22.68	417	38.8	91 680	+ 50 450	— 20 850	9.67	24.27
21	18.00	17.64	467	60.5	436 800	+ 26 110	—259 200	10.32	18.13
22	23.94	23.76	463	39.8	113 800	+ 17 460	— 20 850	8.81	23.88
23	18.36	17.46	454	58.1	410 500	+ 65 600	—259 200	11.00	17.74
24	28.44	28.26	337	105.5	200 000	+ 20 800	— 20 850	15.94	28.88
25	22.14	24.66	372	160.4	535 000	—240 000	—143 100	10.32	24.03
26	27.72	27.72	306	96.2	154 400	0	— 20 850	10.65	28.88
27	30.06	30.24	365	104.0	210 800	— 22 100	— 20 850	13.38	29.72
28	23.22	24.30	358	149.0	586 400	—104 400	—259 200	11.30	23.85
29	29.34	29.52	352	106.0	204 600	— 21 550	— 20 850	12.93	30.02
30	25.20	25.38	381	141.5	594 500	— 18 520	—259 200	16.06	25.79
31	30.78	30.96	365	105.0	206 500	— 22 600	— 20 850	13.00	30.63
32	24.66	25.74	387	145.0	624 000	—110 800	—259 200	12.88	25.80
33	5.76	6.66	113	62.3	97 020	— 22 700	— 26 980	6.58	6.64
34	13.32	13.68	160	45.1	71 640	— 19 730	+ 20 790	5.81	14.64
35	11.34	10.08	142	48.9	87 180	+ 54 810	— 20 790	9.69	11.89
36	8.64	10.44	149	55.4	114 300	— 69 710	+ 20 790	5.23	11.35

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Table 10

Tests on Track Section W.

1	2	3	4	5	6	7	8	9
							Motor Data	
Item No.	Test No.	Wind O or H	Section Limits	Grade	Length of Section	Time	Number and Connection*	Efficiency
1	118	O	7-16	+1.37	1 189	24.1	4 M	78.4
2	118	O	21-30	—0.40	1 154	23.4	4 M	81.0
3	117	H	30-21	+0.40	1 154	35.1	4 S	63.1
4	118	O	7-16	+1.37	1 189	43.4	4 S	70.0
5	117	H	16- 7	—1.37	1 189	22.1	4 M	83.5
6	118	O	21-30	—0.40	1 154	22.3	4 M	80.1
7	117	H	16- 7	—1.37	1 189	20.5	4 M	78.2
8	117	H	16- 7	—1.37	1 189	20.1	4 M	79.2
9	118	O	21-30	—0.40	1 154	19.8	4 M	83.0
10	117	H	16- 7	—1.37	1 189	20.4	4 M	78.3
11	118	O	21-30	—0.40	1 154	32.7	4 S	73.7
12	117	H	30-21	+0.40	1 154	29.4	4 S	70.0
13	123	O	30-21	+0.40	1 154	33.2	4 S	70.6
14	124	H	21-30	—0.40	1 154	33.4	4 S	65.4
15	124	H	7-16	+1.37	1 189	35.9	4 S	66.8
16	124	H	21-30	—0.40	1 154	22.8	4 M	81.9
17	123	O	30-21	+0.40	1 154	36.6	4 M	82.9
18	124	H	21-30	—0.40	1 154	36.0	4 S	67.4
19	123	O	16- 7	—1.37	1 189	34.5	4 S	72.0
20	123	O	30-21	+0.40	1 154	33.8	4 S	74.7
21	124	H	21-30	—0.40	1 154	36.2	4 S	66.9
22	123	O	16- 7	—1.37	1 189	64.1	4 Sw	61.3
23	124	H	21-30	—0.40	1 154	32.3	4 M	77.9
24	123	O	30-21	+0.40	1 154	22.5	4 M	78.5
25	124	H	21-30	—0.40	1 154	25.5	4 M	74.3
26	124	H	21-30	—0.40	1 154	23.7	4 M	74.9
27	123	O	16- 7	—1.37	1 189	34.2	4 S	70.7

*S=Series-multiple. M=Multiple. Sw=Switching. C=Coasting.

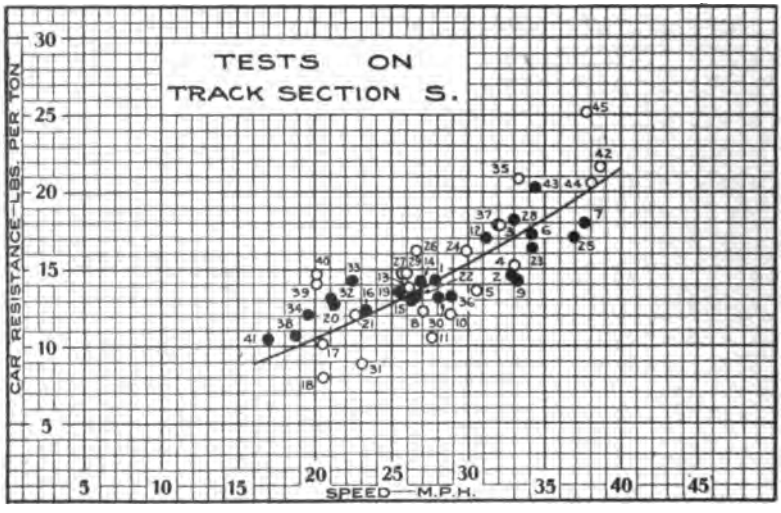

Fig. 18.

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Table 10 (Continued)

Tests on Track Section W.

10	11	12	13	14	15	16	17	18	19
Item No.	Speed		Average Voltage	Average Current	Energy Imparted to the Car			Net Car Resistance	Average Speed
	Entrance	Exit			By Current	By Kinetic Energy	By Grade		
1	35.28	34.02	407	119.0	674 850	+177 260	—76 990	23.20	33.61
2	33.48	34.56	449	131.7	826 550	—149 170	+22 480	21.58	33.60
3	23.76	21.24	415	42.5	288 140	+230 200	—22 480	15.22	22.41
4	19.44	18.54	399	52.0	464 910	+ 69 390	—76 990	14.10	18.66
5	36.18	38.34	537	153.5	1 121 800	—326 750	+76 990	26.10	36.65
6	36.00	36.00	454	125.8	752 340	0	+22 480	23.89	35.27
7	40.68	40.50	467	113.5	626 610	+ 29 660	+76 990	21.95	39.51
8	42.12	42.30	492	116.8	674 720	— 30 850	+76 990	21.58	40.31
9	40.32	41.04	555	142.5	958 580	—118 920	+22 480	26.59	39.70
10	40.32	41.04	473	114.5	638 020	—118 920	+76 990	17.84	39.72
11	23.76	24.30	555	59.0	582 090	— 52 680	+22 480	17.01	24.05
12	27.18	26.64	592	50.3	451 990	+ 59 000	—22 480	15.06	26.75
13	23.40	23.58	256	52.3	462 930	— 17 340	—22 700	12.92	23.69
14	23.04	22.50	225	44.7	324 130	+ 42 510	+22 700	11.89	23.54
15	20.70	20.34	220	46.6	362 570	+ 30 290	—77 750	9.34	22.56
16	34.20	35.28	460	136.7	865 940	—153 830	+22 700	22.44	34.47
17	25.74	29.88	427	152.5	1 059 000	—472 050	—22 700	17.23	29.55
18	20.70	21.06	222	47.5	377 400	— 30 820	+22 700	11.28	21.83
19	22.50	23.40	268	54.8	538 040	— 84 690	+77 750	15.74	23.47
20	20.34	23.40	290	60.9	657 690	—274 380	—22 700	11.01	23.25
21	20.52	20.88	220	46.8	367 750	— 30 550	+22 700	10.99	21.72
22	11.34	11.70	119	42.5	293 270	— 17 000	+77 750	10.49	12.63
23	32.76	32.76	397	116.0	616 430	0	+22 700	19.52	33.74
24	33.30	33.66	417	118.8	645 310	— 49 420	—22 700	17.50	34.95
25	30.96	30.60	346	109.6	529 860	+ 45 430	+22 700	18.26	30.83
26	32.04	32.40	372	108.6	528 900	— 47 560	+22 700	15.39	33.17
27	22.68	23.40	254	52.7	477 370	— 68 010	+77 750	14.43	23.69

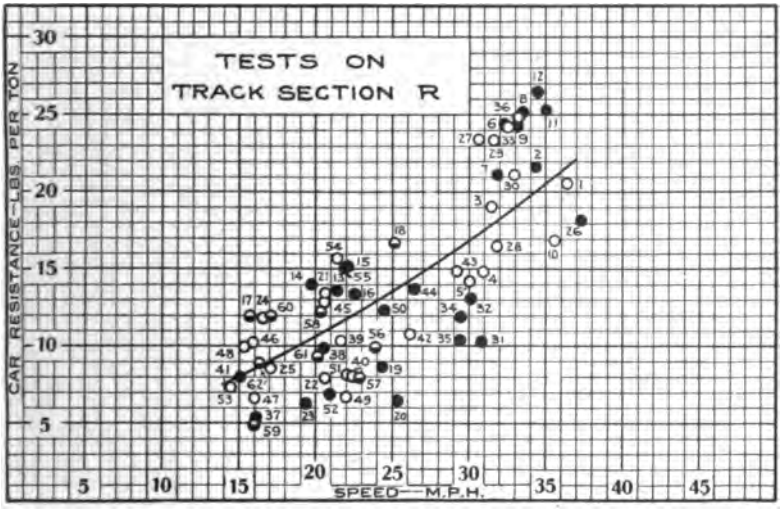

Fig. 19.

Note.—Solid dots are for runs with the wind, open circles are for runs against the wind, and circles with the lower half black are for runs in no wind.

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Table 11

Tests on Track Section S.

1	2	3	4	5	6	7	8	9
				Grade			Motor Data	
Item No.	Test No.	Wind Opposing or Helping	Section Limits. — Trolley Line Pole Numbers	Rise or Fall Over Section — + Up — Down	Length of Section	Time to Run Over Section	Number in Use and Connection *	Efficiency of Motors and Gears
		O or H		Feet	Feet	Sec's.		%
1	119	H	37-29	+0.76	1 057	25.9	4 M	68.3
2	119	H	12- 4	—3.40	1 054	21.9	"	60.4
3	120	O	29-37	—0.76	1 057	22.4	"	80.2
4	120	O	4-12	+3.40	1 054	21.8	"	60.1
5	120	O	29-36	—0.54	928	20.7	"	76.0
6	119	H	37-29	+0.76	1 057	21.1	"	85.1
7	119	H	12- 4	—3.40	1 054	19.1	"	67.2
8	120	O	29-37	—0.76	1 057	26.6	"	70.4
9	119	H	12- 4	—3.40	1 054	21.6	"	63.4
10	120	O	29-37	—0.76	1 057	25.0	"	71.0
11	120	O	4-12	+3.40	1 054	26.0	"	67.8
12	119	H	12- 4	—3.40	1 054	23.0	2 M	73.1
13	120	O	29-37	—0.76	1 057	27.5	"	84.7
14	119	H	12- 4	—3.40	1 054	26.7	"	74.3
15	119	H	12- 4	—3.40	1 054	27.3	4 S	70.5
16	119	H	37-29	+0.76	1 057	30.9	"	77.2
17	120	O	29-37	—0.76	1 057	35.1	"	74.9
18	120	O	4-12	+3.40	1 054	35.0	"	72.0
19	119	H	12- 4	—3.40	1 054	28.2	"	67.8
20	119	H	37-29	+0.76	1 057	33.9	"	79.3
21	120	O	4-12	+3.40	1 054	31.7	"	76.8
22	119	H	12- 4	—3.40	1 054	26.9	"	69.9
23	119	H	12- 4	—3.40	1 054	21.0	4 M	63.3
24	120	O	29-37	—0.76	1 057	24.1	"	78.1
25	119	H	12- 4	—3.40	1 054	19.5	"	68.7
26	120	O	29-37	—0.76	1 057	26.7	2 M	82.8
27	120	O	4-12	+3.40	1 054	28.0	"	84.8
28	119	H	12- 4	—3.40	1 054	21.8	"	80.8
29	120	O	4-12	+3.40	1 054	27.7	"	84.9
30	119	H	12- 4	—3.40	1 054	25.6	4 S	64.6
31	120	O	4-12	+3.40	1 054	31.3	"	71.9
32	122	H	29-37	—0.76	1 057	34.3	"	68.7
33	122	H	4-12	+3.40	1 054	32.0	"	70.7
34	122	H	29-37	—0.76	1 057	36.9	"	70.2
35	121	O	12- 4	—3.40	1 054	21.6	4 M	72.5
36	122	H	29-37	—0.76	1 057	25.0	"	73.6
37	121	O	12- 4	—3.40	1 054	22.5	"	62.2
38	122	H	29-37	—0.76	1 057	38.5	4 Sw	67.0
39	121	O	37-29	+0.76	1 057	35.8	"	72.6
40	121	O	12- 4	—3.40	1 054	35.7	"	56.0
41	122	H	29-37	—0.76	1 057	42.5	4 S	68.8
42	121	O	12- 4	—3.40	1 054	18.6	4 M	80.9
43	122	H	4-12	+3.40	1 054	20.9	"	82.7
44	121	O	12- 4	—3.40	1 054	18.9	"	66.6
45	121	O	37-29	+0.76	1 057	19.1	"	82.3

*S=Series-multiple. M=Multiple. Sw=Switching. C=Coasting.

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Table 11 (Continued)

Tests on Track Section S.

10	11	12	13	14	15	16	17	18	19
	Speed				Energy Imparted to the Car				
Item No.	At Entrance to the Section	At Exit from the Section	Average Voltage	Average Current	By the Current	By the Change in Kinetic Energy	By the Grade	Net Car Resistance	Average Speed Over the Section
	M. P. H.	M. P. H.	Volts	Amps.	Ft. Lbs.	Ft. Lbs.	Ft. Lbs.	Lbs. Per Ton	M. P. H.
1	28.05	26.98	280	95.7	349 380	+120 710	— 43 130	14.23	27.84
2	32.20	32.01	282	79.2	217 740	+ 25 000	+192 900	14.57	32.85
3	31.47	33.10	405	131.5	705 910	—215 760	+ 43 130	17.78	32.20
4	34.36	31.11	284	77.0	211 130	+436 200	—192 900	15.19	32.99
5	29.12	30.40	361	115.0	482 080	—156 180	+ 30 650	13.55	30.56
6	31.48	35.97	524	170.0	1 180 600	—620 840	— 43 130	17.23	34.15
7	37.40	37.04	354	86.6	289 940	+ 54 940	+192 900	17.98	37.62
8	26.08	26.98	300	102.0	422 600	— 97 900	+ 43 130	12.26	27.10
9	33.10	33.28	310	82.0	256 530	— 24 490	+192 900	14.21	33.27
10	27.88	28.77	316	102.0	422 170	—103 360	+ 43 130	12.07	28.81
11	28.22	26.80	284	94.0	346 860	+160 160	—192 900	10.50	27.67
12	31.11	30.40	350	52.2	226 290	+ 89 530	+192 900	17.01	31.22
13	24.62	27.32	407	94.1	657 240	—287 490	+ 43 130	13.77	26.20
14	25.89	26.25	319	57.9	270 080	— 38 480	+192 900	14.20	26.91
15	25.52	26.61	488	45.1	312 270	—116 480	+192 900	13.00	26.31
16	22.82	23.55	512	53.6	482 400	— 69 390	— 43 130	12.33	23.32
17	19.76	21.73	434	50.7	426 170	—167 560	+ 43 130	10.06	20.52
18	20.68	19.76	405	46.9	352 740	+ 76 270	—192 900	7.89	20.52
19	24.99	25.52	452	41.7	265 660	— 54 880	+192 900	13.50	25.50
20	20.30	22.11	501	58.5	580 900	—157 360	— 43 130	12.68	21.26
21	22.82	21.91	490	53.3	468 620	+ 83 440	—192 900	12.01	22.68
22	26.08	26.80	468	43.3	280 810	— 78 050	+192 900	13.23	26.72
23	33.81	33.45	308	81.6	246 270	+ 49 640	+192 900	16.35	34.24
24	29.11	30.75	370	125.0	642 430	—201 250	+ 43 130	16.15	29.92
25	36.33	36.33	356	90.0	316 350	0	+192 900	17.03	36.89
26	26.25	26.80	393	78.6	503 150	— 59 810	+ 43 130	16.22	26.72
27	24.44	24.99	417	94.4	689 070	— 55 730	—192 900	14.72	25.69
28	32.38	32.55	432	66.3	371 790	— 22 630	+192 900	18.13	33.00
29	24.62	25.18	419	95.0	689 940	— 57 170	—192 900	14.71	25.98
30	26.61	26.80	463	39.0	220 150	— 20 800	+192 900	13.12	28.10
31	22.63	21.39	454	45.8	344 890	+111 900	—192 900	8.82	23.08
32	20.48	20.48	202	49.7	348 960	0	+ 43 130	13.07	21.02
33	23.19	20.83	229	53.1	405 450	+212 970	—192 900	14.23	22.48
34	18.50	19.40	193	52.5	387 050	— 69 930	+ 43 130	12.01	19.54
35	33.09	32.91	338	104.0	406 300	+ 24 350	+192 900	20.85	33.30
36	26.98	28.05	301	116.0	473 860	—120 710	+ 43 130	13.21	28.86
37	32.55	31.66	266	81.2	222 730	+117 150	+192 900	17.81	31.98
38	17.78	18.50	174	50.0	331 000	— 53 550	+ 43 130	10.69	18.74
39	19.22	19.76	229	57.8	507 570	— 43 150	— 43 130	14.05	20.14
40	19.94	19.04	159	37.0	173 390	+ 71 920	+192 900	14.65	20.13
41	15.62	17.24	174	50.8	381 270	—109 130	+ 43 130	10.51	16.96
42	36.50	37.95	475	128.0	675 280	—221 300	+192 900	21.63	38.69
43	34.00	34.35	465	143.0	848 320	— 49 040	—192 900	20.27	34.42
44	38.68	37.78	355	85.4	281 280	+141 070	+192 900	20.57	38.08
45	38.31	38.31	505	136.0	796 440	0	— 43 130	25.12	37.75

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Table 12

Tests on Track Section R.

1	2	3	4	5	6	7	8	9
							Motor Data	
Item No.	Test No.	Wind O or H	Section Limits	Grade	Length of Section	Time	No. and Connection*	Efficiency
1	125	O	14 - 4	−2.62	1 320	24.7	4 M	80.3
2	126	H	4 - 14	+2.62	1 320	26.2	4 M	78.9
3	125	O	45 - 38	−2.93	920	19.9	4 M	84.3
4	125	O	14 - 4	−2.62	1 320	29.1	4 M	73.7
5	125	O	45 - 38	−2.93	920	20.9	4 M	80.5
6	126	H	38 - 45	+2.93	920	19.4	4 M	83.8
7	126	H	4 - 14	+2.62	1 320	28.3	4 M	75.9
8	126	H	38 - 45	+2.93	920	18.7	4 M	85.1
9	126	H	4 - 14	+2.62	1 320	27.1	4 M	77.4
10	125	O	45 - 38	−2.93	920	17.6	4 M	83.8
11	126	H	38 - 45	+2.93	920	17.9	4 M	85.0
12	126	H	4 - 14	+2.62	1 320	26.1	4 M	80.2
13	126	H	38 - 45	+2.93	920	29.3	4 S	77.3
14	126	H	4 - 14	+2.62	1 320	45.6	4 S	71.4
15	126	H	4 - 14	+2.62	1 320	40.9	4 S	75.3
16	126	H	38 - 45	+2.93	920	27.8	4 S	76.3
17	128	—	4 - 14	+2.62	1 320	57.1	4 S	72.4
18	127	—	14 - 4	−2.62	1 320	35.8	4 M	62.4
19	129	H	14 - 4	−2.62	1 320	36.9	4 S	66.6
20	129	H	45 - 38	−2.93	920	24.7	4 S	73.5
21	130	O	4 - 14	+2.62	1 320	43.6	4 S	72.0
22	130	O	38 - 45	+2.93	920	30.4	4 S	63.3
23	129	H	14 - 4	−2.62	1 320	46.5	4 S	38.0
24	130	O	4 - 14	+2.62	1 320	54.4	4 S	72.5
25	130	O	38 - 45	+2.93	920	36.8	4 S	70.7
26	129	H	14 - 4	−2.62	1 320	24.1	4 M	76.0
27	130	O	4 - 14	+2.62	1 320	29.4	4 M	82.5
28	130	O	38 - 45	+2.93	920	19.7	4 M	82.9
29	130	O	38 - 45	+2.93	920	19.8	4 M	83.2
30	130	O	4 - 14	+2.62	1 320	27.3	4 M	84.0
31	129	H	45 - 38	−2.93	920	20.3	4 M	72.7
32	129	H	14 - 4	−2.62	1 320	29.9	4 M	46.7
33	130	O	4 - 14	+2.62	1 320	27.7	4 M	82.7
34	129	H	45 - 38	−2.93	920	21.3	4 M	45.8
35	129	H	14 - 4	−2.62	1 320	30.6	4 M	65.4
36	130	O	4 - 14	+2.62	1 320	27.1	4 M	83.4
37	129	H	27 - 24	−0.26	372	16.7	4 S	49.5
38	142	H	1 725 -1 730	+1.63	519	17.2	4 S	74.0
39	141	O	1 741 -1 737.5	−1.90	338	10.6	4 S	55.0
40	141	O	1 730 -1 725	−1.63	519	15.8	4 S	65.4
41	142	H	1 737.5-1 741	+1.90	338	15.3	4 S	71.4
42	141	O	1 730 -1 725	−1.63	519	13.5	4 M	44.0
43	141	O	1 730 -1 725	−1.63	519	12.1	4 M	43.0
44	142	H	1 737.5-1 741	+1.90	338	8.7	4 M	69.4
45	141	O	1 741 -1 737.5	−1.90	338	11.2	4 S	69.3
46	141	O	1 730 -1 725	−1.63	519	22.2	4 S	53.6
47	141	O	1 741 -1 737.5	−1.90	338	14.4	4 S	66.5
48	141	O	1 730 -1 725	−1.63	519	23.0	4 S	43.0
49	141	O	1 730 -1 725	−1.63	519	16.1	4 S	60.3
50	142	H	1 737.5-1 741	+1.90	338	9.4	4 M	77.7
51	141	O	1 730 -1 725	−1.63	519	16.0	4 S	49.6
52	142	H	1 737.5-1 741	+1.90	338	11.0	4 M	63.1
53	141	O	1 730 -1 725	−1.63	519	24.4	4 S	55.1
54	141	O	1 730 -1 725	−1.63	519	16.5	4 S	58.4
55	142	H	1 737.5-1 741	+1.90	338	10.5	4 M	73.2
56	141	O	1 730 -1 725	−1.63	519	14.8	4 M	47.8
57	153	—	1 730 -1 725	−1.63	519	15.5	4 S	41.0
58	153	—	1 730 -1 725	−1.63	519	17.4	4 S	31.0
59	153	—	1 741 -1 737.5	−1.90	338	14.4	4 S	42.1
60	153	—	1 730 -1 725	−1.63	519	15.7	4 S	33.5
61	153	—	1 730 -1 725	−1.63	519	17.5	4 S	43.4
62	153	—	1 741 -1 737.5	−1.90	338	14.1	4 S	37.5

*S=Series-multiple. M=Multiple. Sw=Switching. C=Coasting.

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TABLE 12 (Continued)

TESTS ON TRACK SECTION R.

10	11	12	13	14	15	16	17	18
Item No.	Speed		Average Voltage	Average Current	Energy Imparted to the Car			Net C Resis ance
	Entrance	Exit			By the Current	By the Kinetic Energy	By the Grade	
1	25.29	36.52	431	128.4	809 490	—182 840	+150 260	20.53
2	34.73	33.11	380	127.7	739 860	+227 500	—150 260	21.59
3	29.16	33.67	441	168.7	920 480	—586 560	+168 040	19.03
4	29.68	30.96	321	112.2	569 730	—160 670	+150 260	14.78
5	26.80	30.60	365	145.2	657 630	—451 510	+168 040	14.18
6	32.02	30.78	387	139.7	648 150	+161 200	—168 040	24.31
7	32.58	30.22	340	119.2	642 040	+306 800	—150 260	21.10
8	32.58	34.01	490	178.8	1 028 200	—197 110	—168 040	25.13
9	35.47	32.75	361	122.6	684 670	+384 110	—150 260	24.27
10	32.76	36.52	480	156.2	815 570	—539 220	+168 040	16.84
11	34.38	35.28	496	173.2	964 010	—129 780	—168 040	25.25
12	36.19	33.82	397	132.1	809 540	+343 460	—150 260	26.49
13	20.32	21.41	264	70.3	620 020	— 94 160	—168 040	13.56
14	21.22	19.61	205	55.1	542 410	+136 070	—150 260	13.96
15	21.78	22.31	262	64.8	771 350	— 48 370	—150 260	15.13
16	20.88	21.41	266	68.1	566 770	— 46 400	—168 040	13.36
17	15.42	16.31	182	59.8	663 740	— 58 600	—150 650	11.98
18	27.48	25.51	193	84.0	267 060	+216 610	+150 650	16.71
19	23.78	25.58	214	46.4	359 960	—183 920	+150 260	8.62
20	22.13	26.29	265	59.0	418 670	—417 000	+168 040	6.43
21	21.22	19.97	214	55.6	551 010	+106 580	—150 260	13.40
22	20.52	18.54	175	43.4	216 130	+160 100	—168 040	7.89
23	18.54	18.54	112	29.4	85 830	0	+150 260	6.24
24	15.29	16.02	185	59.8	643 660	— 47 310	—150 260	11.78
25	18.17	18.00	180	55.1	380 700	+ 12 730	—168 040	8.54
26	37.98	38.18	376	112.2	569 890	— 31 740	+150 260	18.19
27	29.70	30.08	387	156.4	1 082 800	— 47 020	—150 260	23.39
28	30.79	32.07	413	154.5	768 540	—166 550	—168 040	16.45
29	31.51	31.51	413	156.2	783 740	0	—168 040	23.34
30	31.32	33.49	445	164.7	1 239 500	—291 120	—150 260	21.08
31	28.07	30.25	298	111.7	362 330	—263 180	+168 040	10.13
32	29.71	27.57	162	54.2	90 420	+253 740	+150 260	13.06
33	32.78	32.95	432	149.3	1 089 700	— 23 130	—150 260	24.21
34	29.52	28.81	158	152.3	59 450	+ 85 730	+168 040	11.87
35	27.72	28.28	232	89.3	305 820	— 64 920	+150 260	10.33
36	33.68	34.02	434	157.2	1 137 300	— 47 650	—150 260	24.81
37	14.76	14.76	99	34.4	41 520	0	+ 14 910	5.29
38	19.32	19.86	248	61.3	285 450	— 44 110	— 94 210	9.81
39	20.95	21.67	173	36.6	54 430	— 63 980	+109 820	10.27
40	20.58	21.85	202	45.1	138 850	—112 350	+ 94 210	8.05
41	15.36	14.64	149	59.4	142 600	+ 45 040	—109 820	7.97
42	25.62	25.28	139	50.2	30 570	+ 36 080	+ 94 210	10.72
43	29.05	28.32	176	59.1	39 910	+ 87 320	+ 94 210	14.77
44	26.52	25.28	246	100.0	109 540	+133 920	—109 820	13.68
45	18.60	19.87	202	50.6	117 300	—101 820	+109 820	12.83
46	15.18	15.36	109	36.5	69 830	— 11 460	+ 94 210	10.17
47	13.92	16.08	129	49.2	89 620	—135 110	+109 820	6.58
48	15.90	15.54	84	24.8	30 390	+ 23 890	+ 94 210	9.90
49	20.78	21.83	170	40.7	99 090	— 93 280	+ 94 210	6.66
50	26.35	26.35	298	143.0	229 520	0	—109 820	12.25
51	22.02	22.38	155	33.6	60 990	— 33 330	+ 94 210	8.12
52	21.12	20.04	191	85.6	83 700	+ 92 680	—109 820	6.81
53	13.02	14.28	117	37.5	87 010	— 71 720	+ 94 210	7.30
54	21.12	20.58	169	39.1	93 950	+ 46 950	+ 94 210	15.67
55	22.57	21.49	236	118.0	157 860	+ 99 210	—109 820	15.08
56	23.47	23.47	159	65.1	54 160	0	+ 94 210	9.89
57	21.57	21.92	223	27.4	57 270	— 31 740	+ 94 380	7.98
58	19.94	19.03	94	20.5	15 330	+ 73 940	+ 94 380	12.22
59	14.71	16.16	113	30.8	31 120	— 93 330	+110 010	4.88
60	22.11	21.39	115	22.5	20 070	+ 65 300	+ 94 380	11.96
61	19.94	19.94	131	31.0	45 490	0	+ 94 380	9.31
62	16.16	16.70	74	24.1	13 850	— 37 000	+110 010	8.88

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PUBLICATIONS OF THE ENGINEERING EXPERIMENT STATION

Bulletin No. 1. Tests of Reinforced Concrete Beams, by Arthur N. Talbot. 1904. *None available.*

Circular No. 1. High Speed Tool Steels, by L. P. Breckenridge. 1905. *None available.*

Bulletin No. 2. Tests of High-Speed Tool Steels on Cast Iron, by L. P. Breckenridge and Henry B. Dirks. 1905. *None available.*

Circular No. 2. Drainage of Earth Roads, by Ira O. Baker. 1906. *None available.*

Circular No. 3. Fuel Tests with Illinois Coal (Compiled from tests made by the Technologic Branch of the U. S. G. S., at the St. Louis, Mo., Fuel Testing Plant, 1904-1907), by L. P. Breckenridge and Paul Diserens. 1909. *Thirty cents.*

Bulletin No. 3. The Engineering Experiment Station of the University of Illinois, by L. P. Breckenridge. 1906. *None available.*

Bulletin No. 4. Tests of Reinforced Concrete Beams, Series of 1905, by Arthur N. Talbot. 1906. *Forty-five cents.*

Bulletin No. 5. Resistance of Tubes to Collapse, by Albert P. Carman and M. L. Carr. 1906. *None available.*

Bulletin No. 6. Holding Power of Railroad Spikes, by Roy I. Webber. 1906. *None available.*

Bulletin No. 7. Fuel Tests with Illinois Coals, by L. P. Breckenridge, S. W. Parr, and Henry B. Dirks. 1906. *None available.*

Bulletin No. 8. Tests of Concrete: I, Shear; II, Bond, by Arthur N. Talbot. 1906. *None available.*

Bulletin No. 9. An Extension of the Dewey Decimal System of Classification Applied to the Engineering Industries, by L. P. Breckenridge and G. A. Goodenough. 1906. Revised Edition 1912. *Fifty cents.*

Bulletin No. 10. Tests of Concrete and Reinforced Concrete Columns. Series of 1906, by Arthur N. Talbot. 1907. *None available.*

Bulletin No. 11. The Effect of Scale on the Transmission of Heat Through Locomotive Boiler Tubes, by Edward C. Schmidt and John M. Snodgrass. 1907. *None available.*

Bulletin No. 12. Tests of Reinforced Concrete T-Beams, Series of 1906, by Arthur N. Talbot. 1907. *None available.*

Bulletin No. 13. An Extension of the Dewey Decimal System of Classification Applied to Architecture and Building, by N. Clifford Ricker. 1907. *None available.*

Bulletin No. 14. Tests of Reinforced Concrete Beams, Series of 1906, by Arthur N. Talbot. 1907. *None available.*

Bulletin No. 15. How to Burn Illinois Coal Without Smoke, by L. P. Breckenridge. 1908. *Twenty-five cents.*

Bulletin No. 16. A Study of Roof Trusses, by N. Clifford Ricker. 1908. *Fifteen cents.*

Bulletin No. 17. The Weathering of Coal, by S. W. Parr, N. D. Hamilton, and W. F. Wheeler. 1908. *None available.*

Bulletin No. 18. The Strength of Chain Links, by G. A. Goodenough and L. E. Moore. 1908. *Forty cents.*

Bulletin No. 19. Comparative Tests of Carbon, Metallized Carbon and Tantalum Filament Lamps, by T. H. Amrine. 1908. *None available.*

Bulletin No. 20. Tests of Concrete and Reinforced Concrete Columns, Series of 1907, by Arthur N. Talbot. 1908. *None available.*

Bulletin No. 21. Tests of a Liquid Air Plant, by C. S. Hudson and C. M. Garland. 1908. *Fifteen cents.*

Bulletin No. 22. Tests of Cast-Iron and Reinforced Concrete Culvert Pipe, by Arthur N. Talbot. 1908. *None available.*

Bulletin No. 23. Voids, Settlement, and Weight of Crushed Stone, by Ira O. Baker. 1908. *Fifteen cents.*

**Bulletin No. 24.* The Modification of Illinois Coal by Low Temperature Distillation, by S. W. Parr and C. K. Francis. 1908. *Thirty cents.*

**Bulletin No. 25.* Lighting Country Home by Private Electric Plants, by T. H. Amrine. 1908. *Twenty cents.*

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THERMAL PROPERTIES OF STEAM

BY
G. A. GOODENOUGH

BUREAU OF MUNICIPAL RESEARCH

BULLETIN No. 75
ENGINEERING EXPERIMENT STATION
PUBLISHED BY THE UNIVERSITY OF ILLINOIS, URBANA

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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN No. 75. SEPTEMBER, 1914

THERMAL PROPERTIES OF STEAM

BY G. A. GOODENOUGH, PROFESSOR OF THERMODYNAMICS

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THERMAL PROPERTIES OF STEAM.

I. Introduction.

1. *Experimental Data on the Properties of Steam.*—The classic researches of Regnault gave the first reliable knowledge of the properties of steam. Although these experiments were made more than sixty-five years ago (most of Regnault's reports on the properties of steam bear the date 1847) with apparatus and methods incapable of the highest degree of accuracy, they are still deservedly held in high esteem, and until recently, Regnault's data formed the basis of all tabulations of the thermal properties of steam. Within the last ten years there has been a revival of interest in the subject and a number of important investigations, experimental and analytical, have been prosecuted. As a result, we have at present data of a high degree of accuracy covering nearly every phase of the work. The following is a summary of the more important of the experimental investigations:

a. The relation between the pressure and temperature of saturated steam has been established definitely by three series of experiments made respectively by Holborn and Henning, Holborn and Baumann, and Scheel and Heuse. The three series taken together cover the range 32° F. to the critical temperature. These experiments were conducted at the Reichsanstalt with all the resources afforded by modern apparatus and methods of precise measurement.

b. The relation between volume, pressure, and temperature of superheated steam has been determined by the experiments of Knoblauch, Linde, and Klebe at the Munich laboratory. These experiments afford satisfactory data for the range of pressure and superheat covered by them, but an extension over a much wider range should be made.

c. A number of experiments have been made to determine the specific heat of superheated steam. Of these, the experiments conducted in the Munich laboratory, first by Knoblauch and Jakob and afterward by Knoblauch and Mollier, are justly accepted as the most reliable. At the present time similar experiments covering a wider range of pressure are being made by Lanz and Schmidt.

d. The direct experiments of Griffiths, Joly, Smith, Henning, and Dieterici furnish data on the latent heat of saturated steam.

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e. The variation of the specific heat of water has been the subject of several investigations. For the range 32°-212° F. the experiments of Barnes have been verified by those of Callendar, and they are generally accepted. Above 212° F. precise measurements of this important property are lacking. The only available experiments are those of Regnault and Dieterici, and neither of these can be accepted as thoroughly reliable.

f. Four sets of experiments on the throttling of steam by Grindley, Griessmann, Peake, and Dodge, respectively, furnish valuable data that may be used for various purposes. Thus Davis has made effective use of them in establishing the curve of total heat of saturated steam; and in the present investigation they are used as a check on the heat content of superheated steam.

2. *Purpose and Scope of the Investigation.*—The various thermal properties of a vapor are not independent. On the contrary, the equations that express the variations of these properties are related through certain well-known thermodynamic laws. The development of a general theory applicable to a vapor, as water vapor, involves the following steps: 1. The establishment of a system of equations to represent the various thermal magnitudes. The forms of the equations chosen must be such that it is possible by proper selection of arbitrary functions to make the equations satisfy the thermodynamic relations. 2. The comparison of the equations with experimental data and the adjustment of constants to give the best possible agreement. 3. The satisfaction of the various thermodynamic relations. With sufficiently accurate and complete experimental data it should be possible to work out such a theory and thus deduce a set of consistent equations giving all the thermal properties as functions of the independent variables chosen. At the present time the character of the experimental evidence is such as to justify a fresh attack on the problem. It is the purpose of this paper to exhibit the development of a general theory which apparently gives with extreme accuracy the properties of saturated and superheated steam over a range of pressure and temperature far wider than the range covered in technical applications.

In order to make the presentation more useful, and perhaps more intelligible, it has been considered advisable to include (1) a section on thermodynamic relations, and (2) a section devoted to a brief historical review of some of the more important analytical investigations.

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3. *Acknowledgment.*—The very considerable labor of computation has been lightened by the assistance of a number of the writer's associates in the Department of Mechanical Engineering. In the earlier phases of the work Mr. A. Schaller and Mr. John A. Dent offered valuable suggestions and cheerfully assumed a considerable part of the work of computation. In the final revision, Mr. S. L. Simmering, fellow in the Engineering College, contributed largely to the progress of the investigation. Mr. G. Meyer, Jr., and Mr. T. E. Maury, senior students, assisted in some of the final computations, and Mr. Meyer made the drawings for the illustrations. The valuable assistance of these men is hereby gratefully acknowledged.

II. Thermodynamic Relations.

4. *Notation.*—Throughout the discussion the following notation is used:

J = mechanical equivalent of heat
$A = 1/J$, reciprocal of mechanical equivalent
t = temperature on F. or C. scale
T = absolute temperature
p = pressure
v = volume of unit weight (1 lb.) of fluid
γ = weight of unit volume
c_v = specific heat at constant volume
c_p = specific heat at constant pressure
u = intrinsic energy per unit weight
q = heat absorbed by fluid per unit weight
q' = heat of liquid
q'' = total heat of saturated steam
i = heat content $= u + Apv$
r = latent heat of vaporization
ρ = internal latent heat
$\psi = Ap\,(v'' - v')$ = external latent heat
s = entropy
μ = Joule-Thomson coefficient

The liquid state is characterized by a symbol with a prime, the state of saturated vapor by the same symbol with a double prime. Thus s', u', and i' denote respectively the entropy, energy, and heat content of the liquid, s'', u'', i'', the same properties of the saturated vapor.

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5. *Relations Between Certain Thermal Properties.*—The thermal properties of the liquid and saturated vapor are connected by relations that follow directly from the definition of the properties. Thus

$$r = \rho + \psi = \rho + Ap\,(v'' - v')$$

$$u'' = q' + \rho \qquad q'' = q' + r$$

$$s'' = s' + \frac{r}{T} \qquad i'' = i' + r$$

The heat content i is defined by the relation

$$i = u + Apv$$

u being expressed in thermal units; hence

$$i' = u' + Apv' \qquad i'' = u'' + Apv''.$$

The heat of the liquid is given by

$$q' = \int_{32}^{t} c'dt,$$

in which c' denotes the specific heat of the liquid. If the heat is absorbed by the water at constant atmospheric pressure from 32° to 212°, then for this range

$$q' = i' = \int_{32}^{t} c'dt.$$

The entropy of the liquid is given by

$$s' = \int \frac{dq'}{T} = \int \frac{c'dt}{T},$$

the integral being taken between proper limits.

Above 212° F. a distinction between q'' and i'' must be observed. By definition

$$i'' = u'' + Apv''$$

and $\qquad q'' = u'' + \psi = u'' + Apv'' - Apv'.$

Hence $\qquad i'' - q'' = Apv',$

similarly $\qquad i' - q' = Apv'.$

At low temperatures the difference Apv' is small and negligible. The term *total heat* is often used indiscriminately for q'' and i''; in this discussion, however, total heat refers to q'' and the name *heat content* is reserved for the property denoted by i.

6. *General Equations.*[1]—From the two laws of thermodynamics the following principal equations are deduced.

1. Goodenough's Principles of Thermodynamics, Chap. IV.

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$$dq = c_v dT + AT\left(\frac{\partial p}{\partial T}\right)_v dv, \tag{1}$$

$$dq = c_p dT - AT\left(\frac{\partial v}{\partial T}\right)_p dp. \tag{2}$$

The energy equation

$$du = dq - Apdv$$

combined with (1) leads to the fundamental equation

$$du = c_v dT + A\left[T\left(\frac{\partial p}{\partial T}\right)_v - p\right]dv; \tag{3}$$

and the equation of definition

$$di = dq + Avdp$$

combined with (2) gives an analogous equation

$$di = c_p dT - A\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right]dp. \tag{4}$$

The application of the criterion of an exact differential to (4) leads to the important Clausius relation

$$\left(\frac{\partial c_p}{\partial p}\right)_T = -AT\left(\frac{\partial^2 v}{\partial T^2}\right)_p. \tag{5}$$

In the case of an isothermal process $dT = 0$ and eq. (4) becomes

$$\left(\frac{di}{dp}\right)_T = -A\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right]; \tag{6}$$

while for a throttling process, in which i remains constant, (4) becomes

$$\left(\frac{dT}{dp}\right)_i = \frac{A}{c_p}\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right]. \tag{7}$$

The derivative in the first member of (7) is the Joule-Thomson coefficient μ.

At the saturation limit the thermal properties are connected by the Clapeyron-Clausius relation, namely

$$r = AT(v'' - v')\left(\frac{dp}{dt}\right)_{sat}. \tag{8}$$

The derivative $\frac{dp}{dt}$ is obtained from the relation $p = f(t)$ connecting the pressure and temperature of saturated steam.

The relations (5) and (8) are specially important in the development of any general theory of vapors. The equations that express the various thermal properties cannot be developed independently. They are tied together by these relations, which must be satisfied as well as the experimental data.

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7. *Development of a General Theory.*—The Clausius relation may be made the basis of a set of equations for a superheated vapor. Let the relation between p, v, and T be given by a characteristic equation

$$v = f(p, T), \tag{9}$$

and let the specific heat c_p at constant pressure, which varies with both pressure and temperature, be expressed by

$$c_p = \phi\,(p, T). \tag{10}$$

Two differentiations of the first equation give the derivative $\frac{\partial^2 v}{\partial T^2}$ and one differentiation of the second equation gives the derivative $\frac{\partial c_p}{\partial p}$. The form of the function in (9) must be so chosen that the experimental measurements of the volume are satisfied, the function ϕ in (10) must likewise satisfy the measurements of specific heat, and furthermore the two functions must satisfy the Clausius relation, $-AT\frac{\partial^2 v}{\partial T^2} = \frac{\partial c_p}{\partial p}$. The development of the theory may proceed along either one of two lines of attack. (1) A charcteristic equation is assumed—this amounts to fixing the function f in (9)—and the constants are determined so as to make the equation represent the volume measurements. Successive differentiation gives the second derivative $\frac{\partial^2 v}{\partial T^2}$, and this is substituted in the Clausius relation. The result is an equation of the form

$$\frac{\partial c_p}{\partial p} = \phi\,(p, T),$$

whence an expression for c_p is found by an integration with respect to p. This will involve an arbitrary function of T as a constant of integration. Finally the equation for c_p is tested by comparison with experimental results. (2) The preceding method may be reversed. Starting with a system of c_p curves laid down from the available experimental evidence, the derivative $\frac{\partial c_p}{\partial p}$ is evaluated, then by two integrations the volume v is determined as a function of p and T.

The first method, which is employed in this investigation, has an inherent difficulty and for that reason has been condemned by Davis and Jakob. It requires the evaluation of the second derivative of an observed magnitude v. Now when a function is empirically fitted to a series of observed values, the errors of the function are liable to be magnified in the first derivative and the second derivative is still more

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uncertain. This point may be made clearer by a geometrical illustration. If the experimental points are plotted on the v, T plane in groups, each representing a constant pressure, a series of curves may be made to pass through the groups of points with reasonable accuracy. The second derivative $\frac{\partial^2 v}{\partial T^2}$ measures the curvature of these constant-pressure lines, hence the curves must not only fit the observed values of v, but they must have precisely the proper curvature.

The second method has been used by Jakob in his investigation (see p. 14). Its chief disadvantage is the practical difficulty of performing the mathematical operations involved. Jakob avoided this difficulty by the use of graphical methods, and his results were thus obtained by the measurement of lines and areas. Herein lies the advantage of the first method if it can be successfully applied. The mathematical manipulation is simple and direct, the various thermal magnitudes are given by closed expressions and therefore they may be calculated without recourse to measurement or approximation.

With the expressions for v and c_p established by either method, an equation for the heat content i is found upon substituting these expressions in the general equation (4). A similar substitution in the general equation (2) leads to an equation for the entropy. The heat content equation when applied at the saturation limit must give values that satisfy the Clapeyron relation, eq. (8). Another check on the i equation is furnished by the various sets of throttling experiments. According to thermodynamic theory the heat content i remains constant in a throttling process; hence the constant-i lines deduced from the equation for i should lie in close coincidence with the experimental points.

III. Review of Earlier Investigations.

8. *Zeuner's Theory.*—In this section is given a brief review of the more important theoretical investigations on the thermal properties of steam. Aside from historical interest, such a review is valuable in showing the development of the analytical processes with increasing experimental evidence; and also in exhibiting the various methods of attacking the problem.

The starting point of Zeuner's investigation[1] is the assumption of a characteristic equation for superheated steam of the form

$$pv = BT - R,$$

1. Zeuner's Technical Thermodynamics, Klein's Tr., Vol. II, pp. 229-248.

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in which the correction term R is considered to be a function of the pressure p only. Following Regnault's experiments, the specific heat c_p is taken as a constant, viz. $c_p = 0.4805$. Then from the fundamental equations of thermodynamics, the law governing the adiabatic expansion of superheated steam is deduced in the form

$$\frac{T}{T_1} = \left(\frac{p}{p_1}\right)^m$$

which is the form found for adiabatic expansion of a gas, provided

$$m = \frac{k-1}{k}.$$

By a comparison of two expressions for the entropy of saturated steam the constant m is given the value 0.25. The correction term R is taken proportional to the fourth root of the pressure, and the characteristic equation thereupon becomes

$$pv = BT - Cp^{1/4}.$$

The energy and heat content of unit weight of saturated steam are given respectively by the simple expressions

$$u = u_0 + \frac{Apv}{k-1}$$

$$i = u_0 + \frac{k}{k-1}Apv,$$

and with $k = \frac{4}{3}$, these become, in English units,

$$u = 857 + 3\,Apv$$
$$i = 857 + 4\,Apv.$$

Zeuner compared the saturation volumes deduced from the characteristic equation with those given in Regnault's tables and found excellent agreement. The volumes of the superheated vapor deduced from the equation were compared with a few experimental values given by Hirn and Battelli and fair agreement was observed. The heat content of the saturated vapor calculated from the formula for i checked with the values derived from Regnault's formula.

Zeuner's theory represented excellently the experimental evidence available at the time it was presented. The Munich experiments, however, have shown conclusively (1) that the form of the characteristic equation assumed by Zeuner cannot possibly represent correctly the volume measurements; (2) that the specific heat c_p is not a constant but a function of both the temperature and pressure. Furthermore, the admirable investigation of Davis has shown that Regnault's linear

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formula for the heat content is clearly in error. The foundations of Zeuner's theory are therefore torn away.

9. *Callendar's Theory*[1].—The theory proposed by Callendar is assumed to apply to all vapors. The starting point of the investigation is a modification of the Joule-Thomson equation

$$v = \frac{BT}{p} - \frac{a}{BT^2}$$

Callendar observes (1) that the equation of a perfect gas at high temperatures is not $pv = BT$, but $p(v-b) = BT$, where b is the minimum volume or "co-volume" of Hirn and van der Waals. The value of b may probably be taken as equal to the volume of the liquid when the vapor pressure is small. (2) If it be assumed that the average total kinetic energy of the molecules of a gas is directly proportional to the energy of translation, then the limiting value of the specific heat of a gas in the ideal state ($p = 0$, $v = \infty$) either at constant pressure or at constant volume must be constant. It is further assumed that the kinetic energy of the vapor is proportional to $p(v-b)$ at all stages. Then the exponent of T in the small correction term $\frac{a}{BT^2}$ of the Joule-Thomson equation is not 2 but $n = \frac{(c_v)_o}{B}$, the ratio of the limiting value of the specific heat at constant volume to the limiting value of pv/T. Next adopting the hypothesis of Maxwell regarding the partition of energy, the limiting value of this ratio for a triatomic gas like steam or CO_2 should be 3.5. The equation deduced from these considerations is

$$v - b = \frac{BT}{p} - c_o\left(\frac{T_o}{T}\right)^{3.5},$$

in which T_o denotes the absolute temperature corresponding to 0° C. (273.1 C. or 459.6 F.). Taking $c_o T_o^{3.5}$ as a single constant m, the equation becomes

$$v - b = \frac{BT}{p} - \frac{m}{T^{3.5}}.$$

By the application of the Clausius relation, the specific heat at constant pressure is given by the expression

$$c_p = (c_p)_o + \frac{Amn\,(n+1)}{T^{4.5}}$$

in which $(c_p)_o$ denotes the assumed constant limiting value of c_p when

1. On the Thermodynamical Properties of Gases and Vapors. Proc. of the Royal Soc. of London, Vol. 67 (1900), pp. 266-286.

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$p = 0$. Expressions for heat content, energy, and entropy are readily derived. The value of the constant $(c_p)_0$ is given as 0.4966, and with this Callendar computed values of c_p at the saturation limit for the range 0°–200° C. Values of the total heat, latent heat, and entropy for the same range were also computed and tabulated.

Callendar's work was published in 1900, five years before the appearance of the Munich experiments and eight years before the publication of Davis' work on the total heat of steam. It is interesting to note that Callendar, on purely theoretical grounds, and in opposition to the then existing experimental evidence, forecast the more important of the recent developments of the subject. He showed the variation of the specific heat with pressure and temperature, though the values he calculated are not in good agreement with those now generally accepted. He challenged the accuracy of Regnault's linear equation for total heat and predicted the proper form of the total heat curve, as it was afterward established by Davis. He calculated from Joly's experiments the value of the latent heat at 100° C. (212° F.) as 540.2 Cal., almost exactly the value that is now considered most probable. Upon the appearance of the Munich volume measurements it was found that Callendar's characteristic equation represented them quite accurately up to 160° C. Above 160° the agreement was not so good for the reason that the equation gives the isotherms on the pv–p plane as straight lines, while the experimental points indicated isotherms with an appreciable curvature.

The fundamental defect in Callendar's theory is the assumption of the constancy of the specific heat $(c_p)_0$ at zero pressure. In the absence of experimental evidence the assumption was natural and the basis for it appeared plausible. The experiments of Knoblauch and Jakob and the later confirmatory experiments of Knoblauch and Mollier showed conclusively that $(c_p)_0$ cannot possibly be a constant. The same thing is shown by the specific heat measurements of Mallard and Le Chatelier and of Langen at very high temperatures.

Callendar's paper is properly regarded as one of the most important of the contributions to the literature on the properties of vapors. It contains suggestions of the highest value, and it shows what may be accomplished by the application of pure theory. However, in the light of the experimental evidence accumulated since the publication of the paper, it is clear that Callendar's equations without modification should not at present be used as a basis for a tabulation of steam properties.

Professor Mollier of Dresden, recognizing the importance of Callendar's investigation, made it the basis of a set of steam tables pub-

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lished in 1906.[1] The work of Mollier has been practically duplicated by Smith and Warren[2] in their new steam tables published in 1912. It is difficult to justify the appearance of the last set of tables. The authors make a claim for the consistency of the values, a claim which is readily granted since the values were calculated from a set of equations properly related through the general thermodynamic equations. Consistency is obtained, however, at the expense of accuracy. No account whatever is taken of the Munich experiments on specific heat and even Barnes' noteworthy experiments on the specific heat of water are neglected. The Mollier and the Smith and Warren tables are doubtless much better than the older tables based entirely on Regnault's data; but on the score of accuracy, they are not to be compared with the Marks and Davis tables or with Peabody's latest tables.

10. *Davis' Investigations.*—To Dr. H. N. Davis of Harvard University we are indebted for two of the most valuable and important contributions to the literature of the properties of steam.

In his first paper,[3] Dr. Davis investigated the Joule-Thomson effect in the case of steam, using for this purpose the throttling experiments of Grindley, Griessmann, Peake, and Dodge. The object of the investigation was the verification of the law of corresponding states. From the four sets of throttling experiments, values of the Joule-Thomson co-efficient μ were determined, and the "reduced" values of μ were compared with values of μ for carbon dioxide. It was found that the law was verified within the limit of error of the experiments. With respect to the properties of steam, the most useful result of the investigation was the establishment of a curve showing the variation of μ with the temperature.

The important feature of Davis' second paper[4] is the discussion of the new formula for the heat content of saturated steam. The throttling experiments were undertaken primarily for the purpose of gaining information on the specific heat of superheated steam. All attempts along this line were unsuccessful for the reason that the calculated specific heat depends upon the *rate* of variation of the heat content, and consequently errors in the formula for heat content are enormously magnified in the calculated values of the specific heat. Grindley and Griessmann used Regnault's linear formula for heat content and were

1. Neue Tabellen und Diagramme für Wasserdampf. Berlin. 1906.
2. The New Steam Tables, D. Van Nostrand Co., N. Y. 1912.
3. On the Applicability of the Law of Corresponding States to the Joule-Thomson Effect in Water and Carbon Dioxide. Proc. Am. Acad. Arts & Sciences, Vol. 45, pp. 243-264.
4. Notes on Thermal Properties of Steam. Proc. Am. Acad., Vol. 45, pp. 267-311, 1900. See also Proc. A. S. M. E., Vol. 30, p. 1419, 1908.

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unable to obtain any consistent results. When the calculation is reversed, that is, when the variation of i is determined from the assumed values of c_p, errors in c_p are reduced in the calculation of i. The experiments of Knoblauch and Jakob give reliable values of c_p. With these available, Davis conceived the possibility of reversing the method. Taking the two sets of experimental data (1) the throttling experiments, (2) the Knoblauch values of c_p, he deduced his well-known formula

$$i'' = i_{212} + 0.3745\ (t - 212) - 0.00055\ (t - 212)^2$$

which is universally accepted for the temperature range 212°–400° F. That the Regnault linear formula for heat content was inexact had been apparent for some time; in fact, Callendar in 1900 had called attention to the matter, and from his theory had obtained an i-curve differing materially from Regnault's straight line. The achievement of Davis in definitely settling this vexed question must be regarded as one of the most noteworthy in the history of the subject.

A section of the paper is devoted to a critical discussion of the specific heat of superheated steam, and the results of the researches of the Joule-Thomson effect are used to develop important relations between the specific heat and the coefficient μ. (See p. 36.)

Davis also discusses the test furnished by the Clausius relation (page 8) and points out that Linde's characteristic equation can not be reconciled with the Knoblauch specific heat measurements through this relation. He goes so far as to say that such reconciliation is impossible, taking the accepted volume and specific heat measurements.

The investigations of Davis are embodied in the Marks and Davis steam tables.

11. *Jakob's Investigation.*[1]—The Clausius relation is taken as the basis of the investigation, but the method used is the reverse of that used in the development of the present theory. Jakob first lays down a system of c_p-curves in accordance with the experiments of Knoblauch and Jakob and Knoblauch and Mollier. In the adjustment of the curves use is made of the thermodynamic relation suggested by Davis (see p. 36). The characteristic equation is given the form

$$v = \frac{BT}{p} - R,$$

in which the correction term R is a function of p and T. Since

$$-\frac{\partial^2 v}{\partial T^2} = \frac{\partial^2 R}{\partial T^2}$$

1. Zeit. des Verein. deutsch. Ing., Vol. 56, pp. 1980-1988. 1912.

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the Clausius relation gives

$$\left(\frac{\partial c_p}{\partial p}\right)_T = AT\frac{\partial^2 R}{\partial T^2},$$

whence

$$R = \frac{1}{A}\int\int\frac{1}{T}\frac{\partial c_p}{\partial p}dt^2.$$

To obtain a lower integration limit, Jakob attempts to set a value of T at which temperature the derivative $\frac{\partial c_p}{\partial p}$ reduces to zero. By a rather arbitrary process he arrives at the value of $T = 1200°$ C., approximately. Hence at this temperature the term R changes sign. No attempt is made to deduce an equation $c_p = \phi\ (p, T)$, and consequently the integrations required in the determination of R are performed graphically. The final result of the process is a set of values of the specific volume v, which are compared with the volumes calculated from Linde's equation.

Jakob also gives a tabulation of the heat content of superheated steam. From the assumed specific heat curves, the mean specific heat from saturation to a given temperature t is obtained by measurement; the product $c_{pm}\ (t - t_s)$ then gives the heat absorbed during superheat, and the sum $i_{sat} + c_{pm}\ (t - t_s)$ is the required value of i.

12. *Heck's Theory.*—Professor R. C. H. Heck has published several papers on the properties of steam, and in his latest paper[1] he has developed a complete theory.

The starting point of Heck's investigation is the Joule-Thomson effect. Taking the identity

$$\left(\frac{\partial i}{\partial p}\right)_t = -\left(\frac{\partial i}{\partial t}\right)_p\left(\frac{\partial t}{\partial p}\right)_i = -c_p\,\mu,$$

the variation of μ with the temperature is known at least approximately from the work of Davis. A relation between i and p with t constant is thus determined, and ultimately an equation for i is deduced. This has the form

$$i = i_o + y'p + z'p^{2.4}$$

in which i_o, y' and z' are rather complicated functions of the temperature. The function i_o is the heat content for $p = 0$. To get this the specific heat c_{po} for $p = 0$ has to be determined, and for this purpose

1. Journal A. S. M. E., Nov. 1913, pp. 1619-1630.

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the Knoblauch and Mollier experiments are used. The equation for c_{po} is

$$c_{po} = 0.302 + \frac{114.66}{t + 688} + 0.000144t.$$

The general equation (6), p. 7, now furnishes a relation from which the form of the characteristic equation may be inferred. That this relation may be satisfied it is sufficient to give the characteristic equation the form

$$pv = BT - yp - zp^{2.4}$$

in which y and z are functions of t that are related respectively to the functions y' *and* z'. The constants in the various functions are so adjusted that the i and v equations satisfy the Clausius relation, and at saturation the Clapeyron relation.

Heck's theory satisfies all the requirements imposed by thermodynamic laws. Unfortunately, the paper was published in abstract and the most valuable part—the comparison of the theory with experimental data—was omitted. It may safely be assumed, however, that the agreement is satisfactory.

13. *Other Investigations.*—Other investigations of less immediate importance may be noted.

Linde[1] in his discussion of the experiments of the Knoblauch, Linde, and Klebe experiments, after establishing his two characteristic equations, attempted to deduce from them the latent heat of steam and the specific heat of superheated steam. The results obtained were not confirmed by the later experiments on specific heats.

Dieterici's paper entitled "Energieisothermen des Wassers bei hohen Temperaturen"[2] describes an attack on the problem of the properties of steam by an original and ingenious method. In the light of our present knowledge, some of the basic assumptions are unsound, and the results therefore have no significance.

Schüle[3] has recently published a tabulation of steam properties. The values were obtained by purely empirical processes, and no attempt was made to correlate them through thermodynamic relations. Perhaps the most valuable part of Schüle's work is the attempt to determine the properties in the vicinity of the critical temperature.

1. Mitteil. uber Forschungsarbeiten. Vol. 21, pp. 57-92.
2. Annalen der Physik (4), Vol. 16, pp. 907-930.
3. Zeit. des Verein. deutsch. Ing., Vol. 55, pp. 1506-1512, 1561-1567.

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IV. Development of a General Theory of the Properties of Steam.

14. *Units and Constants Employed.*—In this section is given the development of the equations that represent the various properties of saturated and superheated steam. The order of topics is as follows:

A. Relation between pressure and temperature of saturated steam.
B. Specific volumes. Characteristic equation of superheated steam.
C. Specific heat of superheated steam.
D. Heat content, latent heat, heat of liquid, energy, and entropy.
E. The Joule-Thomson effect.
F. Thermal properties near the critical temperature.

As a preliminary step, the establishment of certain units and constants is necessary. These are

1. The thermal unit.
2. The mechanical equivalent of heat.
3. The temperature of melting ice on the absolute scale.

In this investigation the mean B.t.u. is taken as the thermal unit. This is defined as 1/180th of the heat required to raise the temperature of a pound of water from 32° to 212° F. The corresponding mean calorie is by Griffiths[1] identified with the 17½° calorie and by Barnes with the 16° calorie.

The various determinations of the mechanical equivalent seem to justify the value established by Griffiths[1] in 1893, namely,

1 mean calorie = 4.184 joules
1 mean B.t.u. = 777.64 standard ft. lb.

This value has been used.

Various determinations of the absolute temperature of the ice-point are discussed by Marks and Davis[2], and the value chosen by them is 491.°64. It does not seem that the evidence is sufficient to justify the degree of accuracy indicated by the fifth figure; hence in this investigation 491.°6 has been used. The relation between absolute and ordinary temperatures is thus given by

$$T = t + 459.6.$$

A. Relation between Temperature and Pressure of Saturated Steam.

15. *Experimental Data.*—The important relation $p = f(t)$ for saturated steam has been definitely determined by three sets of experi-

1. The Thermal Measurement of Energy.
2. Steam Tables, p. 88.

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ments conducted at the Reichsanstalt within the last five years. Each set covers a different range of temperature; Scheel and Heuse's[1] experiments cover the lower range 0–50° C. (32°–122° F.), Holborn and Henning's[2] the range 50°–200° C. (122°–392° F.), while Holborn and Baumann's[3] experiments extend from 200° to the critical temperature. The character of the work done at the Reichsanstalt is a sufficient guaranty that every precaution was taken and that the measurements are as precise as can be obtained with modern instruments and methods.

The values of the saturation pressure as deduced from the respective sets of experiments are given in the following tables. In the third table the values are not those given by Holborn and Baumann but values deduced therefrom by Prof. Marks.[4]

TABLE 1.

SCHEEL AND HEUSE.

Temp. C.	Pressure in mm. of mercury									
	0	1	2	3	4	5	6	7	8	9
0	4.579	4.926	5.254	5.685	6.101	6.543	7.014	7.514	8.046	8.610
10	9.210	9.845	10.519	11.233	11.989	12.790	13.637	14.533	15.840	16.481
20	17.539	18.655	19.832	21.074	22.383	23.763	25.217	26.747	28.558	30.052
30	31.834	33.706	35.674	37.741	39.911	42.188	44.577	47.082	49.708	52.459
40	55.341	58.36	61.52	64.82	68.28	71.90	75.67	79.62	83.74	88.05
50	92.54									

TABLE 2.

HOLBORN AND HENNING.

Temp. C.	Pressure in mm. of mercury				
	0	2	4	6	8
50	92.3	101.9	112.3	123.6	135.9
60	149.2	163.6	179.1	195.9	214.0
70	233.5	254.5	277.1	301.3	327.2
80	355.1	384.9	416.7	450.8	487.1
90	525.8	567.1	611.0	657.7	707.3
100	760.0	815.9	875.1	937.9	1004
110	1074.5	1149	1227	1310	1397
120	1489	1586	1687	1795	1907
130	2026	2150	2280	2416	2560
140	2709	2866	3030	3202	3381
150	3569	3764	3968	4181	4402
160	4633	4874	5124	5384	5655
170	5937	6229	6533	6848	7175
180	7514	7866	8230	8608	8999
190	9404	9823	10256	10705	11168
200	11647	12142	12653		

1. Annalen der Physik (4), Vol. 31, pp. 715-735, 1910.
2. Annalen der Physik (4), Vol. 25, pp. 833-883, 1908.
3. Annalen der Physik (4), Vol. 31, pp. 945-970, 1910. See also articles by Risteen; The Locomotive, Vol. 26, pp. 85, 183, 246; Vol. 27, p. 54; Vol. 28, pp. 88, 118.
4. Proc. A. S. M. E., Vol. 33, p. 572.

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TABLE 3.

HOLBORN AND BAUMANN.

Temp. F.	Pressure in lb. per sq. in.									
	0	10	20	30	40	50	60	70	80	90
400	246.99	276.24	306.23	342.18	380.92	421.85	465.96	512.65	565.06	620.18
500	679.26	742.55	810.31	882.58	959.85	1042.2	1130.2	1223.7	1322.0	1428.3
600	1539.9	1667.8	1782.9	1915.3	2055.1	2208.1	2359.2	2523.4	2697.1	2883.3
700	3063.4									

16. *Pressure-Temperature Formulas.*—A large number of formulas have been proposed to represent the relation between pressure and temperature of saturated steam. The greater number of these are purely empirical, but a few have a semi-rational basis.

Of the earlier formulas suggested, two have been quite generally used, namely, Roche's and Biot's.[1] From certain theoretical considerations Roche deduced the form

$$p = a\,\alpha^{\frac{\theta}{m+n\theta}}$$

in which a, α, m, and n are constants and $\theta = t + \text{const.}$, that is, θ denotes the temperature measured from some zero arbitrarily chosen. Biot suggested the more general form.

$$\log p = \alpha + b\alpha^{\theta} + c\beta^{\theta}$$

which has five constants. The Biot formula has been generally used to represent Regnault's experiments.

A group of formulas may be deduced from the following considerations. The Clapeyron relation

$$r = AT\,(v'' - v')\,\frac{dp}{dt}$$

may be written in the form

$$\frac{dp}{p} = \frac{1}{T}\;\frac{r}{Ap\,(v'' - v')}\,dT.$$

In the fraction of the second term the numerator is the latent heat r and the denominator is the external latent heat,

$$\psi = Ap\,(v'' - v')$$

that is, the part of r that is used in overcoming external resistance. If values of r and of ψ be plotted as ordinates against temperatures as abscissas, the resulting curves are of such character that they can be represented very closely by equations of the type

1. Preston's Theory of Heat, Art. 188.

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$$r = a_1 + a_2T + a_3T^2 + a_4T^3 + \ldots\ldots$$
$$\psi = T\,(b_1 + b_2T + b_3T^2 + b_4T^3 + \ldots\ldots)$$

The second curve should properly pass through the origin, since the product pv'' should approach zero as T approaches zero. By division

$$\frac{r}{\psi} = \frac{1}{T}\,(c_1 + c_2T + c_3T^2 + c_4T^3 + \ldots\ldots)$$

and the substitution of this expression in the original Clapeyron relation leads to the differential equation

$$\frac{dp}{p} = \frac{1}{T^2}\,(c_1 + c_2T + c_3T^2 + c_4T^3 + \ldots\ldots)\,dT.$$

The integration of this equation gives the following general form for the relation $p = f(t)$

$$\log p = A + \frac{B}{T} + C \log T + DT + ET^2 + FT^3 + \ldots\ldots \qquad (11)$$

The number of constants may be increased indefinitely by taking more terms involving the higher powers of T; hence the equation may be fitted if desired to a large number of experimental points. The signs of the coefficients B, C, D, E, etc., may, of course, be either positive or negative.

Several formulas that have been proposed are simply modifications of this general equation.

a. The *Dupré-Hertz formula,* which is usually written in the form

$$\log p = k - m \log T - \frac{n}{T},$$

includes the first three terms of the general equation.

b. *Callendar's formula.*[1]—Callendar deduced from his equations for total heat and entropy a pressure-temperature relation of complicated form, but remarked that the equation could be written in the form

$$\log p = A + \frac{B}{T} + C \log T + \text{small terms.}$$

c. *Bertrand's formulas.*[2]—If the expressions for r and $Ap\,(v'' - v')$ are taken as linear in T, the integrand assumes a form that permits integration in finite terms. Bertrand assumed that for water vapor the following relations are sufficiently well satisfied.

$$p\,(v'' - v') = B\,(T + a), \quad r = M - nT$$

The Clapeyron relation then becomes

$$\frac{dp}{p} = \frac{m - nT}{ABT\,(T + a)}\,dT$$

1. Proc. Royal Soc. of London, Vol. 67, p. 285, 1900.
2. Chwolson, Lehrbuch der Physik, Vol. 3, p. 736.

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from which $$p = k \frac{T^{\alpha}}{(T+a)^{\beta}}$$

or $$\log p = \log k + \alpha \log T - \beta \log (T+a)$$

In deducing Bertrand's second formula, it is assumed that the relation

$$\frac{Ap\,(v'' - v')}{r} = aT - b$$

holds good for steam. From this relation the equation

$$p = k \left(\frac{T-m}{T}\right)^{n}$$

or $$\log p = \log k - n \log \frac{T}{T-m}$$

is easily obtained. While Bertrand's equations are convenient for the purposes of calculation, they cannot be extended over any considerable temperature range without change of constants. The same statement applies to the Dupré-Hertz formula.

d. Marks' formula.—If in the general equation (11) the constant C is made equal to zero and terms containing powers of T above the second are suppressed, the resulting equation has the form deduced from quite different considerations by Prof. Marks.[1]

One other formula should be mentioned. Thiesen's formula involves the critical temperature t_k and atmospheric pressure. In the metric units (p in mm. of mercury and t in deg. C.) it may be written in the form

$$T \log \frac{p}{760} = A\,(t-100) - B\,(t_k - t)^4 - (t_k - 100)^4$$

Thiesen's formula is used as a standard of reference by Henning[2] in an elaborate discussion of various pressure-temperature measurements.

It is an easy task to fit any of the proposed equations to the experimental values through a limited temperature range; but to obtain a single equation that will satisfactorily represent the experiments over the entire range from 32° to the critical temperature is a problem of some difficulty.

Marks' equation,

$$\log p = A - \frac{B}{T} - CT + DT^2,$$

represents the experimental values above 400° F. with remarkable accu-

1. Proc. A. S. M. E., Vol. 33, p. 573.
2. Annalen der Physik (4), Vol. 22, pp. 609-630, 1907.

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racy and may be extended to 300° F. with satisfactory results. Below 300° the Marks curve begins to run below the experimental points, as shown in Fig. 1, and the Scheel and Heuse points are missed entirely. Heck[1] has slightly changed the constants of the Marks equation with the avowed purpose of getting the proper value of p at 212° F. At this temperature the Marks equation gives $p = 14.672$, while the exact value is 14.697. The two sets of constants are as follows:

	Marks	Heck
$A =$	10.515345	10.606400
$\log B =$	3.6878597	3.6897500
$\log C =$	$\bar{3}.6075880$	$\bar{3}.6205462$
$\log D =$	$\bar{6}.1439400$	$\bar{6}.1601803$

$$T = t + 459.64$$

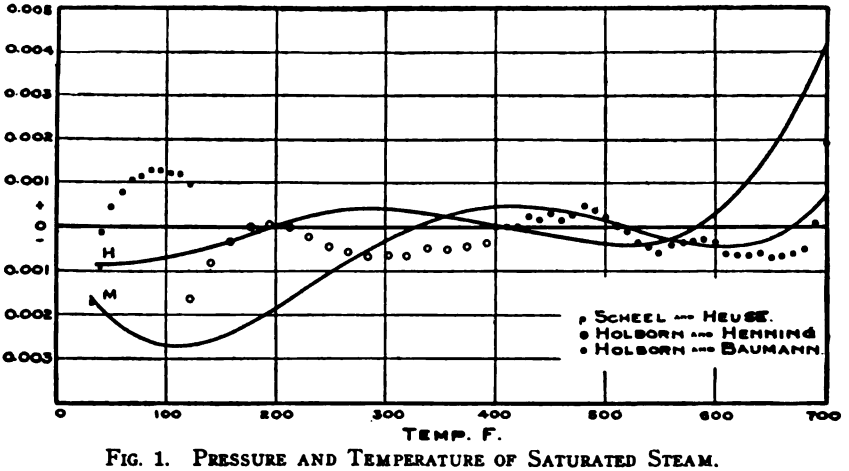

FIG. 1. PRESSURE AND TEMPERATURE OF SATURATED STEAM.

The curve H, Fig. 1, shows the agreement between Heck's equation and the experimental points. It is apparent that Heck's modification improves the agreement throughout the lower range of temperature, which is the most important. Above 600°, however, Heck's curve begins to deviate widely from the Holborn and Baumann points.

The equation worked out in the present investigation may in a sense be regarded as a modification of Marks' equation. In the first place a term involving log T was added as is suggested by the general form (11). The equation thus took the form

1. Journal A. S. M. E., Vol. 35, p. 1627, 1913.

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$$\log p = A - \frac{B}{T} - C \log T - DT + ET^2,$$

and the following constants were determined:

$$A = 10.5688080 \qquad \log D = \bar{3}.6088020$$
$$\log B = 3.6881209 \qquad \log E = \bar{6}.1463000$$
$$C = 0.0155 \qquad T = t + 459.6$$

With these constants, the equation represents the Holborn and Baumann points above 400° with substantially the same accuracy as the Marks equation, and it represents the experiments throughout the range 32°–300° very much better than the Marks equation. It is, however, open to two objections: (1) At 212° it gives $p = 14.694$, which is still too low; (2) at 32° it gives a value of p considerably higher than is indicated by the Scheel and Heuse experiments.

It was found that these objections could be removed and the agreement improved throughout by the inclusion of a small correction term. The final equation is, therefore,

$$\log p = A - \frac{B}{T} - C \log T - DT + ET^2 - \Delta \qquad \text{(A)}$$

where $$\Delta = 0.0002\,(10 - 10\,\theta^2 + \theta^4),$$

and $$\theta = \frac{t - 370}{100}.$$

The addition of the term Δ amounts to the inclusion of terms involving T^3 and T^4 in the general formula (11). The values of $C \log T$ and Δ are easily calculated, and since Δ is an even function of t the values below 370° are duplicated above 370°. The labor of calculation is therefore not materially increased by the inclusion of these terms.

17. *Comparison of Formulas.*—In Fig. 1, equation (A) is used as a standard of reference. The points plotted are taken from the preceding tables, and the curves M and H represent respectively the equations of Marks and Heck. Ordinates represent the relative deviation from the value of p calculated from the formula. It is seen that the proposed equation (A) represents the experimental values with a high degree of accuracy. The deviations except at two or three isolated points are well within 1 part of 1000, which is probably within the limit of accuracy of the experiments. For the lower range 32° to 400°, Heck's formula is superior to Marks', and the proposed formula (A) is superior to both. Above 450° Marks' formula gives slightly better results than the new formula, and Heck's equation shows considerable deviation from the experimental points.

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It will be seen that none of the three curves follows the general course of the Scheel and Heuse points. Both the M and H curves tend to favor the low Holborn and Henning point at 122° F. rather than the high Scheel and Heuse point at the same temperature. The curve of equation (A) on the other hand, lies nearer the Scheel and Heuse points throughout except at the extreme end of the range near 32°. The discrepancy between the two points at 122° is greatly magnified in the figure; the actual pressure difference is 0.24 mm. of mercury. Scheel and Heuse have noted this discrepancy and they defend the higher point. Equation (A) is probably more accurate throughout the range 40°–200° than either of the others. At 32° the value calculated from the equation is 4.587 mm. while the accepted value is 4.579 mm. of mercury. The discrepancy is unimportant so far as pressures are concerned. The significant fact is that, accepting the Scheel and Heuse points as authentic, the derivative $\frac{dp}{dT}$ calculated from any one of the three of the equations must be too small in the range 32°–80°. The effect of this error will be shown in another section.

B. VOLUME OF SUPERHEATED AND SATURATED STEAM. CHARACTERISTIC EQUATIONS.

18. *Experimental Data.*—Direct experiments on the specific volume of saturated and superheated steam have been made by Ramsay and Young[1], by Battelli[2], and by Knoblauch, Linde, and Klebe[3]. The experiments in the Munich laboratory conducted by Knoblauch, Linde, and Klebe were so superior in all respects to those of the other investigators, that the results are generally accepted as decisive.

The apparatus used in the Munich experiments was so arranged that three sets of observations were made. 1. The pressure of the saturated vapor corresponding to the temperature. 2. Simultaneous values of pressure and temperature corresponding to a predetermined constant volume of superheated steam. 3. The corresponding saturation values of p and t for the given volume. In conducting the experiment the volume of a predetermined weight of steam was kept constant and corresponding temperatures and pressures were observed. These observed values of p and t when plotted give a constant volume curve, or "isochor" on the pt-plane. It was found that the curves, within the limits of accuracy of the experiments, were straight lines. These lines were

1. Phil. Trans. Roy. Soc. of London, Vol. 183-A, p. 107 (1892).
2. Annales de Chimie et de Physique (7), Vol. 3, p. 408 (1894).
3. Mitteilungen über Forschungsarbeit., Vol. 21, pp. 33-72 (1905).

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prolonged to intersect the saturation curve $p=f(t)$, and the points of intersection gave, therefore, simultaneous values of p, v, and t, at the saturation limit.

For convenience in establishing a characteristic equation, Linde made use of the scheme of representation devised by Amagat. Values of the product pv were plotted as ordinates against values of p as abscissas. The experimental points were not taken for this purpose but rather the points determined by the intersection of the successive isochors by lines of constant temperature. In this way the points on the pv-p plane are separated into groups, each of which is associated with a particular temperature. In other words, curves through the successive sets of points are lines of constant temperature, or isotherms. Fig. 2 shows the points as thus determined. These were not copied from Linde's chart, but were calculated independently from the experimental data by Mr. Simmering.

19. *Characteristic Equations.*—A large number of equations have been proposed to represent the relation between the p, v, and t of superheated vapors. In general these equations have the form

$$pv = BT - R$$

or the form

$$p(v-b) = BT - R,$$

in which R is the so-called *correction term.* In the second form account is taken of the co-volume (see p. 11). The term R is taken as a function of one or more of the variables, p, v, t. Zeuner makes $R = Cp^{1/4}$, Tumlirz assumes simply $R = Cp$. In Callendar's equation, namely

$$p\,(v-b) = BT - pc_0\left(\frac{T_0}{T}\right)^{-2.5}$$

R is proportional to the product $pT^{-2.5}$. It may be noted that if R is taken as a function of p and T, or as a function of p alone, and p appears in the first power only, then the isothermals on the pv-p plane are straight lines. The Tumlirz equation $pv = BT - cp$ gives a group of parallel straight lines, while the Callendar formula gives straight lines inclined at different angles. In one group of equations the term R is made a function of v alone or a function of v and t. In the well known van der Waals equation

$$R = C\,\frac{v-b}{v^2},$$

and in the Clausius equation

$$R = \frac{C}{T}\,\frac{v-a}{(v+B)^2}.$$

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In the first empirical formula proposed by Linde

$$R = \frac{1}{v}\left[C\left(\frac{373}{T^2}\right) - D\right].$$

Since p and T are always taken as the independent variables and v is the magnitude calculated, it is extremely inconvenient to have an equation with a power of v higher than the first; hence equations having R a function of p and T have a practical advantage. With this point in view, Linde constructed his second equation, namely

$$pv = BT - p\,(1 + ap)\left[C\left(\frac{373}{T}\right)^3 - D\right].$$

This equation has been generally accepted, and from it have been calculated the values of v that appear in the Marks and Davis and the Peabody steam tables.[1]

While Linde's second equation represents the experiments within the limits of accuracy it is open to two serious objections. (1) At 402° C. the correction term R vanishes and for higher temperatures it changes sign. In the language of Linde, the vapor bcomes a more than perfect (übervollkomenes) gas at temperatures above 402°. (2) Taking the specific heat measurements of Knoblauch and Mollier as decisive, it is impossible to satisfy the Clausius thermodynamic relation with the values of the derivative $\frac{\partial^2 v}{\partial T^2}$ obtained from Linde's equation. This means that while the constant pressure curves deduced from Linde's equation pass through the experimental points with sufficient accuracy, they have not the proper curvature. Dr. Davis in his paper on the properties of steam[2] has pointed out this defect, and has expressed the opinion that no reliable c_p values can be obtained through the Clausius equation from any volume measurements as yet available. That this view is not justified will be shown in the following section.

In the course of the present investigation a number of equations have been developed. These differ slightly in form, and each may be considered a modification of Linde's second equation. The first equation[3] was given the form

$$v + c = \frac{BT}{p} - (1 + ap)\,\frac{m}{T^n}.$$

1. Marks and Davis, Steam Tables and Diagrams, p. 98.
2. Am. Acad. of Arts and Sciences, Vol. 45, p. 288 and p. 303.
3. Goodenough's Principles of Thermodynamics, p. 203.

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The equation resembles Linde's equation in retaining the expression $(1 + ap)$ which serves to give the parabolic form to the isothermals when drawn in the pv-p plane. See Fig. 2. The constant D in Linde's equation is dropped, but another constant is added to v. The values assigned to the constants were as follows:

Metric units (p in kg. per sq. m.)	English units (p in lb. per sq. in.)
$B = 47.113$	$B = 0.5963$
$\log m = 11.19839$	$\log m = 13.67938$
$n = 5$	$n = 5$
$c = 0.0055$	$c = 0.088$
$a = 0.00000085$	$a = 0.0006$

With these constants the equation represents the experimental results with substantially the same accuracy as Linde's equation; and by means of the Clausius relation the specific heat measurements of Knoblauch and Mollier are fairly well verified.

A careful study of conditions to be satisfied led to further modifications. It was found that more consistent results could be obtained by taking a fractional power of p in the correction term. Further, it was found that by taking 4 instead of 5 for the exponent n the constant c could be omitted. Hence the second equation was given the form

$$v = \frac{BT}{p} - (1 + 3ap^{1/2}) \frac{m}{T^n}$$

The constants for this equation are

Metric (p in kg. per sq. m.)	English (p in lb. per sq. in.)
$\log B = 1.67274$	$\log B = \bar{1}.77508$
$\log m = 8.65429$	$\log m = 10.88000$
$3a = 0.001131$	$3a = 0.03$
$n = 4$	$n = 4$

In all respects this equation is an improvement over the first equation.

Further consideration of the question led to another slight modification with a corresponding change of constants. Accepting Callendar's suggestion that the first member should contain a term to represent the co-volume, the equation was given the form

$$v - c = \frac{BT}{p} - (1 + 3ap^{1/2}) \frac{m}{T^n} \qquad \text{(B)}$$

and the value of c was taken as the specific volume of water. Hence, when the equation is used to determine the specific volume of saturated steam, the first member becomes simply $v'' - v'$. Since this difference, rather than the steam volume v'', occurs in the Clapeyron relation and

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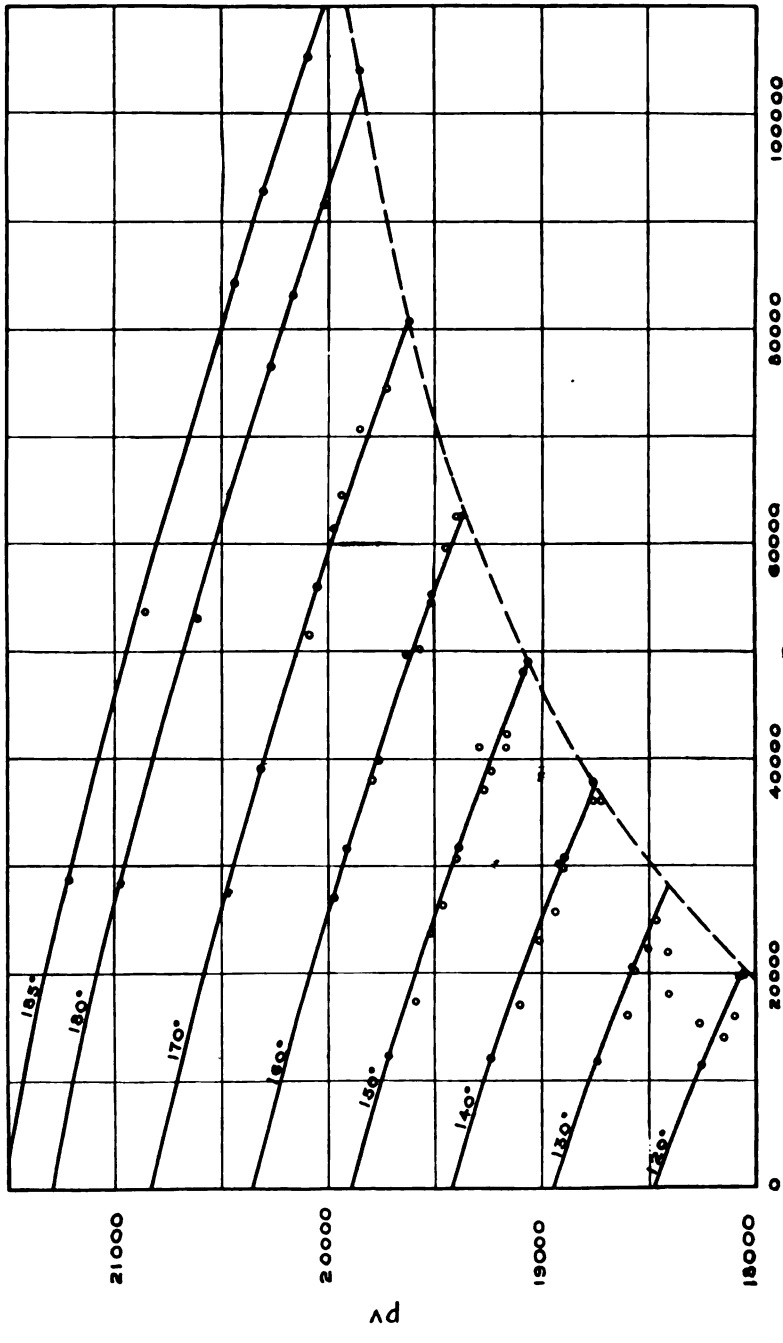

FIG. 2. ISOTHERMAL CURVES FROM EQ. (B). THE POINTS REPRESENT THE EXPERIMENTS OF KNOBLAUCH, LINDE, AND KLEBE.

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in other thermodynamic formulas, the inclusion of the constant c really simplifies the calculation of the thermal magnitudes that involve specific volumes. The constants of the final equation are as follows:

Metric	English
$\log B = 1.67206$	$\log B = \bar{1}.77441$
$\log m = 8.59929$	$\log m = 10.82500$
$\log 3a = \bar{3}.28644$	$\log 3a = \bar{2}.71000$
$n = 4$	$n = 4$

With the satisfaction of the Clausius relation in view, Heck[1] has developed an equation which is in form considerably different from any heretofore suggested. It is

$$v = \frac{BT}{p} - \frac{E}{(t+130)^2} - \frac{F}{(t+40)^3} p^{1.4}$$

The constants are: $B = 0.5956$, $\log E = 4.66365$, $\log F = 11.02244$. Pressures are to be taken in lb. per sq. *inch*.

It may safely be assumed that Heck's equation represents the volume measurements sufficiently well and that it satisfies the Clausius relation.

20. *Tests of the Characteristic Equation.*—The following comparisons are made to establish the validity of equation (B) as far as specific volumes are concerned. The question of the satisfaction of the Clausius relation is discussed in the following section.

1. Isothermal curves are calculated from (B) and drawn on the pv-p plane along with the points plotted from the measurements of Knoblauch, Linde and Klebe. The result is shown in Fig. 2, which may be compared with Fig. 13 in Linde's paper.[2] The agreement between the curves and points is thoroughly satisfactory.

2. As has been stated, the constant volume lines of Knoblauch, Linde, and Klebe when drawn on the pt-plane appeared to be straight lines. Linde[3] has compared the slopes of these lines as observed with the slopes calculated from his first equation. The slopes *at the saturation limit* have been calculated from (B), and the following table gives a comparison

TABLE 4. VALUES OF $\left(\frac{\partial p}{\partial T}\right)_v$

$t =$	101.4	112.3	126.5	139.2	150.3	163.2	170	180.6	183
From (B)	30.7	44.0	68.4	99.0	134.7	189.2	224.8	291.5	309.7
Linde	30.4	43.9	67.4	97.9	132.9	188.9	221.7	287.5	308.7
Observed	30.4	44.3	68.0	99.3	134.0	188.6	225.0	293.0	313.0
Heck	30.5			96.3		178.7			285.2

1. Journal Am. Soc. Mech. Eng'nrs, Nov., 1913, p. 1619.
2. Mitteil. über Forschungsarbeit., Vol. 21, p. 58.
3. Loc. cit., p. 67.

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The comparison is somewhat misleading, because both Linde's second equation and equation (B) give values of the slope that vary slightly with the temperature. This variation is shown for four of the experiments.

TABLE 5.

VARIATION OF $\left(\frac{\partial p}{\partial T}\right)_v$ WITH TEMPERATURE.

Exper. No. 1	$t = 101.4$	120.0	140.0	160.0
$\left(\frac{\partial p}{\partial T}\right)_v$	$= 30.7$	30.2	30.1	29.9
No. 12	$t = 126.5$	140.0	160.0	180.0
$\left(\frac{\partial p}{\partial T}\right)_v$	$= 68.4$	67.7	67.0	66.9
No. 22	$t = 150.3$	160.0	180.0	
$\left(\frac{\partial p}{\partial T}\right)_v$	$= 134.7$	133.2	131.2	
No. 29	$t = 170.0$	180.0		
$\left(\frac{\partial p}{\partial T}\right)_v$	$= 224.8$	219.9		

The change in slope introduces a curvature in the constant volume lines, so slight however, that it can scarcely be detected in the figure. It is clear that the slopes deduced from (B) agree sufficiently well with the observed slopes.

3. Values of v'', the specific volume of the saturated vapor, as calculated from (B) are compared with values calculated from Linde's equation. The following table shows the comparison:

TABLE 6.

COMPARISON OF SPECIFIC VOLUMES.

Temp. C.	100	110	120	130	140	150	160	170	180
v'' from (B).....	1.6738	1.211	0.8926	0.6692	0.5095	0.3928	0.3075	0.2431	0.1943
v'', Linde.....	1.6740	1.211	0.8922	0.6690	0.5091	0.3921	0.3073	0.2430	0.1943

At the saturation limit the two formulas give practically identical values throughout the range of temperature covered by the experiments.

4. For the range 0°–100° C. and for temperatures above 180° C. there are no reliable experimental values to serve as a check of the formula. However, the specific volumes of the saturated vapor given in the Marks and Davis tables may be accepted as fairly accurate as they

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were calculated by means of the Clapeyron relation from values of the latent heat that are at least approximately correct. The following table shows a comparison of volumes obtained from (B) with the Marks and Davis values.

TABLE 7.

COMPARISON OF SPECIFIC VOLUMES.

Temp. F.	32	40	80	120	160	200	212	240	280	320	360	400	440	480	520	560	600
v'' from (B)	3296	2440	632.8	203.2	77.28	33.65	26.81	16.33	8.66	4.92	2.964	1.868	1.220	0.830	0.562	0.391	0.274
v'', M. & D.	3294	2438	632.8	203.1	77.20	33.60	26.79	16.32	8.64	4.91	2.957	1.872	1.229	0.81	0.55	0.39	0.27

From the comparison it appears that formula (B) may be accepted, so far as saturation volumes are concerned, for the entire range 32°–600° F.

5. For the volumes in the region of high superheat there is at present no check. The measurements of Knoblauch, Linde, and Klebe reach only to about 50° C. (120° F.) of superheat. The extension of any formula to 400° or 500° superheat is therefore an extrapolation, the validity of which is uncertain. On account of the character of the correction term in Linde's formula (see p. 26), it is practically certain that at high superheat Linde's values are too high. Jakob[1] in the course of this investigation has deduced values that run consistently lower than those of Linde at the higher superheats. As a matter of interest a comparison has been made between three sets of values for three different pressures, 1, 9 and 19 kg. per sq. cm. This is shown in the following table:

TABLE 8.

SPECIFIC VOLUMES OF SUPERHEATED STEAM.

p = 10000 kg. per sq. m.							
t =	99.1	130	160	190	220	250	300
v from (B)	1.7254	1.8777	2.0233	2.1675	2.3109	2.4536	2.6904
v Linde	1.7260	1.8783	2.0245	2.1695	2.3137	2.4571	2.6954
v Jakob	1.7281	1.8789	2.0237	2.1674	2.3107	2.4535	2.6909
p = 90000							
t =			174.6	190	220	250	300
v from (B)			0.2193	0.2293	0.2480	0.2659	0.2946
v Linde			0.2190	0.2293	0.2483	0.2665	0.2959
v Jakob			0.2195	0.2296	0.2479	0.2655	0.2939
p = 190000							
t =				208.9	220	250	300
v from (B)				0.1068	0.1107	0.1208	0.1362
v Linde				0.1068	0.1108	0.1213	0.1374
v Jakob				0.1071	0.1113	0.1210	0.1359

The comparison is shown graphically in Fig. 3. The ordinates of curve L represent the difference between Linde's values and the values given

1. Zeit. des Verein. deutsch. Ing., Vol. 56, p. 1987.

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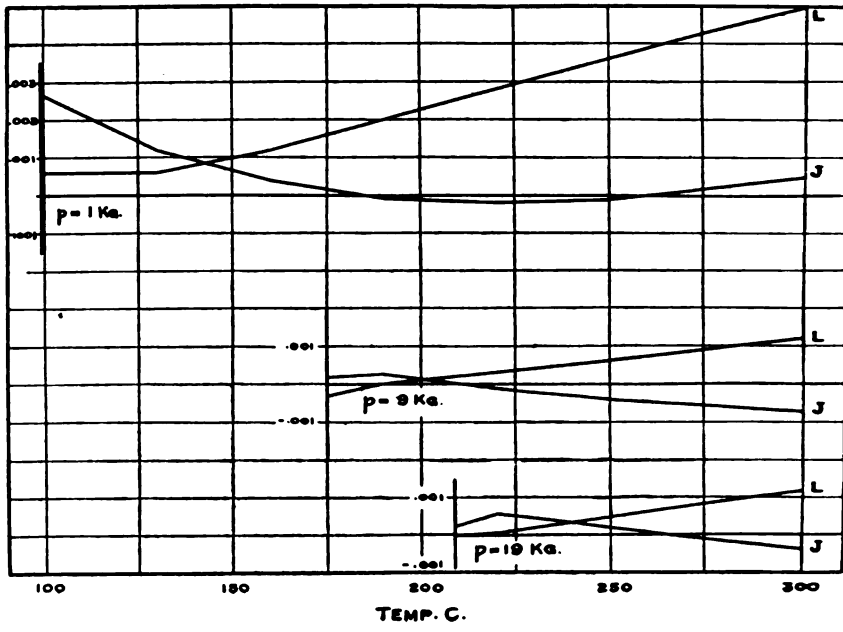

FIG. 3. COMPARISON OF SPECIFIC VOLUMES.

by eq. (B). Similarly, curve J applies to Jakob's values. It will be seen that, in general, the volumes calculated from (B) lie near Linde's volumes at the saturation limit, but approach more nearly Jakob's values with increasing superheat. It will be observed that the curve J cuts the base line (which represents formula (B)) in two points for the pressure of 1 kg. If the superheat is carried sufficiently high at the other pressures, a second intersection will likewise be obtained. The correction term R in Linde's equation changes sign at 402° C., and in Jakob's system the change of sign occurs at 920° C. Hence before reaching 920° Jakob's values must rise above those calculated from (B).

The correction term in eq. (B) does not change sign at any temperature. It approaches zero as T is indefinitely increased, and it becomes negligible compared with the term $\frac{BT}{p}$ when the pressure becomes very small. In these two limiting cases, therefore, equation (B) merges into the equation

$$p\,(v - c) = BT.$$

Evidently the *form* of equation (B) is such as to justify extrapola-

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tion beyond the region of the experiments; and it is probable that the values of v calculated from this equation are worthy of confidence both at the saturation limit and in the region of superheat.

C. SPECIFIC HEAT OF SUPERHEATED STEAM.

21. *Experimental Data.*—The experiments on specific heat may be divided into groups as follows:

1. The early experiments of Regnault with steam at atmospheric pressure and at temperatures relatively close to saturation.

2. The experiments of Maliard and Le Chatelier, Langen, and Pier at very high temperatures.

3. The experiments of Holborn and Henning with steam at atmospheric pressure and a temperature range of 110°–1400° C.

4. The experiments of Grindley and Griessmann using the throttling method.

5. Recent direct experiments with steam at various pressures. Of these, the experiments of Knoblauch and Jakob and of Knoblauch and Mollier performed in the Munich laboratory are specially noteworthy. Similar experiments have been made by Thomas.

Regnault's experiments made in 1862[1] indicated a constant value of $c_p = 0.4805$. Davis[2] has recomputed Regnault's values and has deduced a somewhat smaller value, namely, $c_p = 0.4762$. For the pressure and range of temperature covered in the experiment, Regnault's value agrees well with the results of recent experiments.

1. Mem. Inst. de France, Vol. 26, p. 167 (1862).
2. Proc. Am. Acad., Vol. 45, p. 286 (1910).

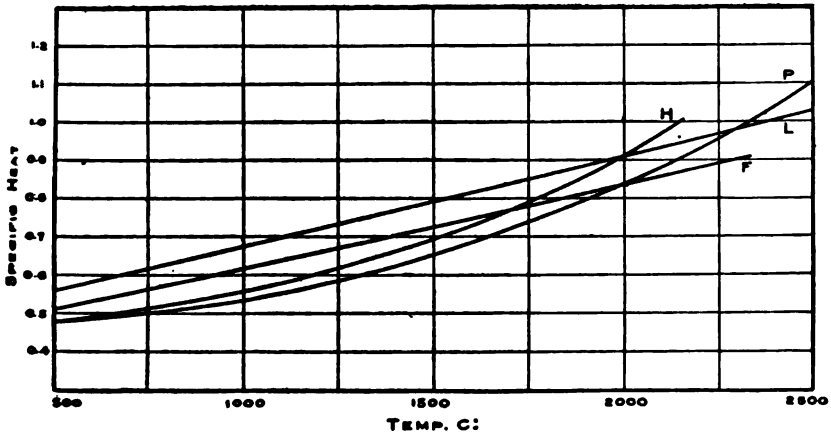

Fig. 4. Curves of Specific Heat.

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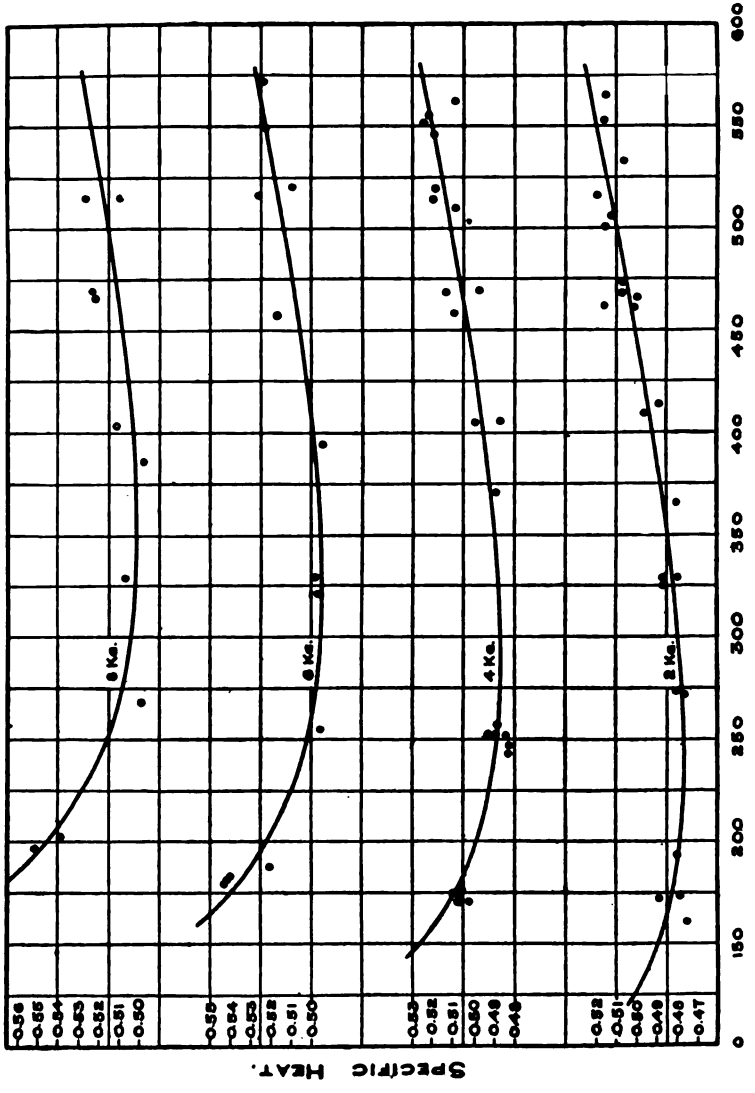

Fig. 5. Curves of Specific Heat Deduced from Eq. (C). The Points Represent the Experiments of Knoblauch and Mollier.

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The high temperature experiments noted in group 2 have only an indirect bearing on the present investigation. The results obtained by the different investigators are discordant, but they all agree in showing a marked increase of specific heat with rising temperature. In Fig. 4 the straight line L represents the linear relation

$$c_p = 0.439 + 0.000239t$$

established by Langen, curve P represents Pier's equation, and curve H the equation given by Holborn and Henning.

The experiments of Holborn and Henning[1] form a link between the high temperature experiments of group 2 and the experiments of group 5. These measurements indicate values of c_p consistently lower than those obtained in the Munich experiments. While considerable weight must be attached to the Holborn and Henning experiments, it seems probable that preference must be given the Knoblauch and Mollier measurements. Callendar[2] has expressed the opinion that the Holborn and Henning values are too low by as much as 10 per cent.

The efforts of Grindley and Griessmann to determine c_p by the method of throttling were futile, and the results obtained by them are without value.

Of the direct experiments, preference is justly given to those of Knoblauch and Jakob[3] and Knoblauch and Mollier.[4] The latter experiments supplement and extend the range of the former. A third set of experiments is now being conducted, and preliminary reports indicate that the earlier results will be sustained. The results reported by Thomas[5] are of value indirectly as in some degree corroborating the Munich experiments. As conclusively shown by Davis,[6] the Thomas experiments are not to be compared with the Knoblauch experiments in point of accuracy.

After reviewing all the experimental evidence one must be convinced that for the range of temperature covered, the Knoblauch and Mollier measurements should be accepted without modification. They are therefore used in the present investigation. The points plotted in Fig. 5 are those determined by Knoblauch and Mollier. For convenience in the identification of the measurements associated with the four pressures employed, the points have been separated into four groups.

1. Annalen der Physik, Vol. 18, p. 739 (1905); Vol. 23, p. 809 (1907).
2. Report of British Assoc. Committee on Gaseous Explosions, pp. 31, 32 (1908).
3. Mitteil. über Forschungsarbeit, Vol. 35, p. 109.
4. Zeit. des Ver. deutsch. Ing., Vol. 55, p. 665 (1911).
5. Proc. Am. Soc. Mech. Engrs., Vol. 29, p. 633 (1907).
6. Proc. Am. Acad. of Arts and Sciences, Vol. 45, pp. 269-272.

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22. *Systems of Specific Heat Curves.*—The Knoblauch and Jakob experiments showed unmistakably the variation of specific heat with both pressure and temperature. At constant temperature, an increase of pressure resulted in an increase of specific heat; and at constant pressure the value of c_p first decreased from the saturation limit, attained a minimum and then increased. Knoblauch and Jakob exhibited the variation of c_p by means of constant pressure curves drawn on a plane with c_p as ordinate and t as abscissa. The curves were so drawn as to represent as closely as possible the experimental points and by a doubtful process of extrapolation the pressures were carried up to 20 kg. per sq. cm.

Taking the Knoblauch and Jakob curves as a basis, Davis developed a system of specific heat curves (Marks and Davis, Steam Tables and Diagrams, p. 97) from which the properties of superheated steam were deduced. Davis modified the Knoblauch curves in two respects. 1. Accepting the Holborn and Henning measurements, he lowered the c_p curve for atmospheric pressure so that it would join the Holborn and Henning curve. In the light of the Knoblauch and Mollier results, this modification was doubtless a mistake. 2. The Knoblauch values at low pressures near the saturation limit were changed so as to bring them more nearly in accord with Regnault's value. Davis also developed a thermodynamic relation by which the spacing of the curves could be tested. Denoting by μ the Joule-Thomson coefficient, the relation is

$$\frac{c_p}{c_{po}} = e^{-\int_{p_o}^{p} \left(\frac{\partial \mu}{\partial T}\right)_p dp} \tag{12}$$

Davis had already investigated the Joule-Thomson effect for steam[1] and had obtained an approximate relation between μ and t. By means of the relations $\mu = f(t)$ he was able to evaluate the integral in (12) and thus to adjust the spacing of the c_p-curves.

Another system of c_p-curves has been worked out by Jakob.[2] The curves were adjusted to the Knoblauch and Mollier points, from Davis' relation the curve for c_{po} (*i. e.*, p = 0) was determined, and then by the same relation c_p-curves for 10, 12, 14, 16, 18, and 20 kg. per sq. cm. were established. Near the saturation limit the Jakob curves agree closely with the Davis curves; at the higher superheats the increase of c_p with the temperature is more marked in the Jakob curves, as it doubtless should be.

1 Proc. Amer. Acad. of Arts and Sciences, Vol. 45, p. 261.
2. Zeit. des Verein. deutsch. Ing., Vol. 56, pp. 1981-3 (1912).

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23. *Equation for Specific Heat.*—From the characteristic equation

$$v - c = \frac{BT}{p} - (1 + 3ap^{1/2}) \frac{m}{T^n}$$

the second derivative required in the Clausius relation is obtained. It is

$$\left(\frac{\partial^2 v}{\partial T^2}\right)_p = -(1 + 3ap^{1/2}) \frac{mn(n+1)}{T^{n+2}}.$$

Hence

$$\left(\frac{\partial c_p}{\partial p}\right)_T = -AT \frac{\partial^2 v}{\partial T^2} = \frac{Amn(n+1)}{T^{n+1}} (1 + 3ap^{1/2}).$$

An integration with T constant gives an expression for c_p, namely

$$c_p = F(T) + \frac{Amn(n+1)}{T^{n+1}} p(1 + 2ap^{1/2}).$$

The arbitrary function $F(T)$ is evidently c_{po}, that is, the specific heat at zero pressure. This was taken as a constant by Callendar. The experiments of Knoblauch and Mollier show that c_{po} cannot be constant, and this conclusion is confirmed by the high-temperature experiments of Langen and others. It has been suggested that a simple linear relation

$$c_{po} = \alpha + \beta T$$

may be assumed, and this assumption was made in the writer's earlier paper. It is found, however, that better agreement is obtained by a relation of the form

$$c_{po} = \alpha + \beta T + \frac{\gamma}{T^2}.$$

Writing the equation for c_p in the form

$$c_p = F(T) + f(p, T)$$

values of the term $f(p, T)$ may be calculated for each of the Knoblauch and Mollier experiments and by subtraction the corresponding values of $c_{po} = F(T)$ are found. From the curve through these points the constants α, β, and γ are obtained. The equation for c_p finally takes the form

$$c_p = \alpha + \beta T + \frac{\gamma}{T^2} + \frac{Amn(n+1)}{T^{n+1}} p(1 + 2ap^{1/2}) \qquad \text{(C)}$$

and the constants are

	Metric	English
α	$= 0.320$	0.320
β	$= 0.0002268$	0.000126
γ	$= 7371$	23583

The constants a, m, and n are those of the characteristic equation. (See p. 29.)

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In Fig. 5 the curves calculated from equation (C) are shown. The agreement between the curves and experimental points is sufficiently evident. It would be difficult to obtain a family of curves connected by a relation that would represent more accurately the groups of points.

The perfect correlation of the Munich experiments is shown by a comparison of Figs. 2 and 5. Equation (B) gives the curves of Fig. 2, equation (C) the curves of Fig. 5. The equations certainly represent the experimental data within the limits of accuracy of the experiments, and they are properly connected by the Clausius relation. The difficulty of making the correlation lies not in the measurements themselves,—they are sufficiently accurate—but in the choice of a proper form for the characteristic equation. The results here shown amply justify the method used in attacking the problem, and they strongly confirm the validity of equation (B).

In the following table values for c_p for various temperatures and pressures are given. The numbers in parentheses are the corresponding values deduced by Jakob from his c_p-curves:

TABLE 9.

SPECIFIC HEAT OF SUPERHEATED STEAM.

Pressure kg. per sq. cm.	Temperature C								
	150	200	250	300	350	400	450	500	550
0	0.457 (0.462)	0.460 (0.465)	0.466 (0.469)	0.473 (0.474)	0.480 (0.481)	0.489 (0.489)	0.498 (0.497)	0.508 (0.505)	0.518 (0.514)
2	0.490 (0.486)	0.479 (0.477)	0.477 (0.477)	0.480 (0.480)	0.485 (0.485)	0.492 (0.492)	0.500 (0.500)	0.509 (0.508)	0.519 (0.516)
4	0.527 (0.524)	0.498 (0.494)	0.489 (0.486)	0.487 (0.486)	0.490 (0.490)	0.496 (0.496)	0.503 (0.503)	0.511 (0.510)	0.520 (0.517)
6		0.522 (0.514)	0.503 (0.495)	0.496 (0.492)	0.496 (0.494)	0.500 (0.500)	0.505 (0.506)	0.513 (0.512)	0.521 (0.518)
8		0.545 (0.538)	0.517 (0.505)	0.504 (0.498)	0.502 (0.499)	0.504 (0.503)	0.508 (0.508)	0.515 (0.514)	0.523 (0.520)

The agreement between the two sets of values is remarkably close; except for three or four points the difference is well within one per cent. From 350° to 500° the values are practically identical, but above 500° the specific heats calculated from the equation begin to rise above the values found by Jakob, while these in turn are higher than the Davis values. The question of the proper course of the c_p-curves at 550° is important in connection with the c_p values at high superheat. The only guide in settling this question is the experimental evidence furnished by the high-temperature experiments. Referring to Fig. 4, curve F represents the function $F(T) = \alpha + \beta T + \frac{\gamma}{T^2}$ and at high tempera-

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tures this curve is practically identical with the specific heat curve, since the term $f(p, T)$ in the specific heat equation becomes vanishingly small. It will be seen that curve F is in good agreement with the other curves between 500° and 2000° C. If Jakob's curves were prolonged in the same way, they would merge into a single curve lying somewhat below curve F. This is more clearly shown in Fig. 6, in which various determinations of the function $c_{po} = F(T)$ are plotted. Curve H represents the variation of c_{po} assumed by Heck, curve A represents the preceding equation, and the points are plotted from the values of c_{po} given

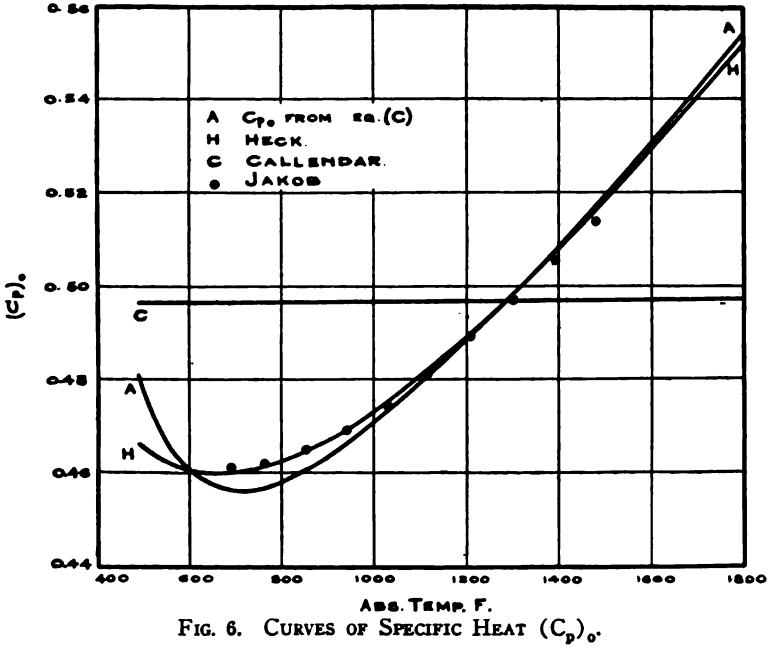

FIG. 6. CURVES OF SPECIFIC HEAT $(C_p)_0$.

by Jakob. As a matter of interest Callendar's constant value is shown by the straight line C. It appears that the three curves agree quite well. At the higher temperatures curves A and H are almost coincident, but the Jakob points show a tendency to run low. It is probable that equation (C) gives fairly good values of c_p up to about 2000° C., that Heck's equation may safely be used through the same range, but that Jakob's curves if prolonged lie somewhat low and are not valid above 1000° C.

At the lower temperatures Heck and Jakob are in close agreement, while curve A shows quite a different course. Experimental evidence in

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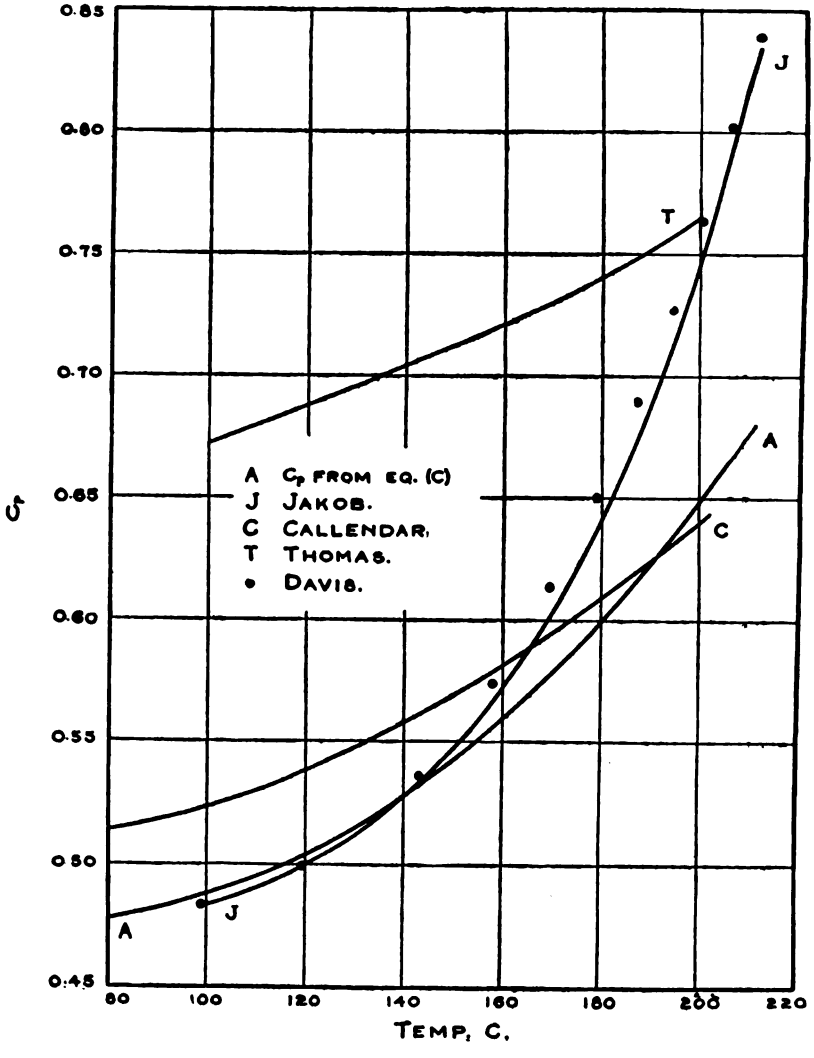

FIG. 7. CURVES OF SPECIFIC HEAT AT THE SATURATION LIMIT.

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this region is entirely lacking, and consequently no statement can be made as to the probable relative accuracy of the two curves.

In all discussions of specific heat much emphasis has been laid on the values of c_p at the saturation limit. These give a "saturation curve." From the specific heat equation (C) values of $(c_p)_{sat}$ are readily calculated by taking for p and t corresponding saturation values. Curve *A*, Fig. 7, is the curve thus derived. Davis[1] has given a very full discussion of this subject. The empirical curves of Knoblauch and Jakob were tested by Planck's thermodynamic relation

$$(c_p)_{sat} = \frac{dH}{dt} - \frac{r}{T} + AT\left(\frac{\partial v}{\partial t}\right)\left(\frac{dp}{dt}\right)_{sat}$$

and fair agreement was shown. Davis therefore accepted the Knoblauch values of $(c_p)_{sat}$ with a reservation. Regarding the tremendous rise of Knoblauch's saturation curve at even moderately high temperatures he says: "It is probable that this feature of Knoblauch's curves, although near enough the truth to satisfy the present needs of engineering practice, will have to be revised later." The points in Fig. 7 represent the values of $(c_p)_{sat}$ given by Davis.

Jakob arbitrarily established the values of $(c_p)_{sat}$ for four pressures: 2, 4, 6, and 8 kg. per sq. cm., and from these determined the constants in the assumed formula.

$$(c_p)_{sat} = 0.455 + 2 \cdot 10^{-20} \frac{T_s^8}{T_k - T_s}$$

in which T_s denotes the saturation temperature and T_k the critical temperature. The curve thus found is curve *J*, Fig. 7. It agrees very closely with the Davis curve but runs slightly lower. According to Jakob's formula $(c_p)_{sat}$ becomes infinite at the critical temperature.

Curve *T*, Fig. 7, represents the experiments of Thomas, and curve *C*, the saturation values of c_p calculated by Callendar from purely theoretical considerations.

It is a safe conclusion that curve *A* is nearer the truth than curve *J* or the Davis curve. In the first place, the values represented by curve *A* are obtained from an equation, and as shown in Fig. 5, this equation represents very accurately the best experimental data. The equation automatically extends the curves from the region of the experiments to the saturation curve, thus obviating a doubtful extrapolation that was necessary in laying down the empirical curves. Again, the relation of the curves to the experimental points exhibited in Fig. 5 indicates accurate spacing. The curves show no tendency to run high or low

1. Proc. Am. Acad., Vol. 45, pp. 295-303.

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with increasing pressure. Hence it appears probable that equation (C) may be used with confidence for pressures considerably higher than those used in the experiments. At any rate, there is no reasonable doubt that the c_p-curve for a pressure of 20 at. calculated from eq. (C) is likely to be nearer the truth than the corresponding curve obtained by the system of extrapolation employed by Knoblauch and Jakob.

It should be observed that the Davis equation for spacing the curves (see p. 36) and Planck's thermodynamic relation are both automatically satisfied, as both reduce to identities when applied to the equations developed in this investigation.

While Regnault's measurements of c_p at atmospheric pressure are not to be considered as possessing any great degree of precision, some importance may be attached to a comparison of Regnault's results with the corresponding values of c_p calculated from equation (C). The four series of experiments covered the temperature range 122.8°–231.1° C. The mean value of c_p given by Regnault was 0.4805, but this value is lowered to 0.4762 by Davis. All experiments were conducted at atmospheric pressure. The following table gives values of c_p at atmospheric pressure calculated from the equation, also the values assigned by Jakob for the slightly lower pressure, 1 kg. per sq. cm.

TABLE 10.

SPECIFIC HEAT AT ATMOSPHERIC PRESSURE.

Temp. C.	100	150	200	250	300	350	400
From Eq. (C).....	0.489	0.474	0.470	0.472	0.476	0.483	0.491
Jakob.....	0.482	0.473	0.471	0.473	0.477	0.483	0.490

The mean c_p deduced from the equation agrees very well with the recomputed value 0.4762, and the Regnault experiments therefore strengthen the evidence in favor of the specific heat equations.

D. TOTAL HEAT, LATENT HEAT, HEAT OF LIQUID.

24. *Equation for Heat Content.*—From the characteristic equation and the equation for specific heat an expression for the heat content is readily deduced. In the general equation

$$di = c_p\, dT - A\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right] dp$$

we introduce the expression for c_p given by eq. (C) and the expression for the derivative $\left(\frac{\partial v}{\partial T}\right)_p$ obtained from the characteristic equation, namely

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$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{B}{p} + \frac{mn}{T^{n+1}}(1 + 3ap^{1/2}).$$

The result of the substitutions is the equation

$$di = \left[a + \beta T + \frac{\gamma}{T^2} + \frac{Amn\,(n+1)}{T^{n+1}}\,p\,(1 + 2ap^{1/2})\right]dT$$

$$-A\left[\frac{m\,(n+1)}{T^n}\,(1 + 3ap^{1/2}) - c\right]dp,$$

which upon integration gives the following equation for the heat content

$$i = aT + \tfrac{1}{2}\beta T^2 - \frac{\gamma}{T} - \frac{Am\,(n+1)}{T^n}\,p\,(1 + 2ap^{1/2}) + Acp + i_0. \quad \text{(D)}$$

The constant i_0 is determined as follows. Corresponding saturation values of p and t at some definite temperature, say 212°, are substituted in the equation, which for this purpose may be written

$$i_{sat} = \phi\,(p, T) + i_0.$$

The function $\phi\,(p, T)$ is thus calculated, and i_{sat} being known, i_0 is found by subtraction.

Since the constant c is taken as the liquid volume v', the term Acp is Apv', which (see p. 6) is the small difference between the heat content i'' and the total heat q''. Hence, when applied at the saturation limit, equation (D) gives i'' and the same equation with the term Acp omitted gives q''.

25. *Tests of the i-equation.*—Three tests may be applied to the equation (D) for heat content. Two of these apply at the saturation limit, the third in the region of superheat.

1. Values of i_{sat} calculated from equation (D) are compared with values obtained independently by other means. The following table shows a comparison with the Marks and Davis values for i:

TABLE 11.

HEAT CONTENT OF SATURATED STEAM.

Temp. F.	i''		Temp. F.	i''		Temp. F.	i''	
	Eq. (D)	M. & D.		Eq. (D)	M. & D.		Eq. (D)	M. & D.
32	1072.83	1073.4	212	1151.58	1150.4	440	1202.33	1207.1
40	1076.64	1076.9	240	1161.90	1160.4	480	1200.43	1210
80	1095.30	1094.8	280	1175.15	1173.3	520	1194.02	1207
120	1113.34	1112.3	320	1186.30	1184.4	560	1182.76	1196
160	1130.64	1129.5	360	1194.86	1193.7	600	1166.	1176
200	1146.94	1145.8	400	1200.37	1201.3			

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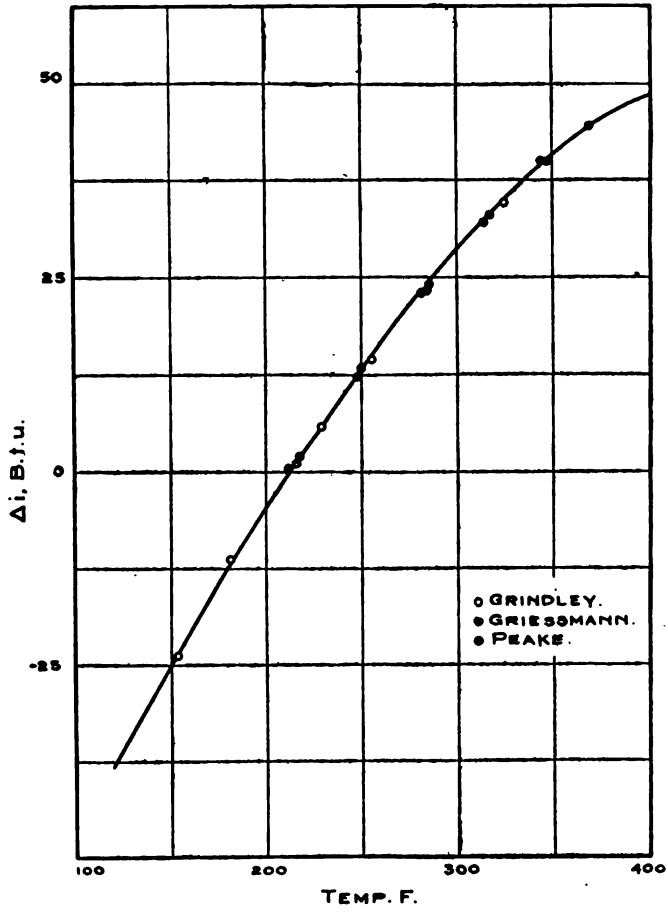

FIG. 8. COMPARISON OF i″-CURVE FROM EQ. (D) WITH POINTS DEDUCED FROM THE THROTTLING EXPERIMENTS.

The considerable discrepancy above 400° has no significance, for in this range Davis makes no claim for the accuracy of the M. and D. values. The effective test is furnished by a comparison of the two sets of values within the range 212° — 400°, where the Davis formula for heat content is surely valid. The comparison is shown graphically in Fig. 8. The points are those determined by Davis from the throttling experiments of Grindley, Greissmann and Peake, and they are plotted from the data given in Table 1 of Davis' paper.[1] The ordinates represent

1. Proc. Am. Acad., Vol. 45, p. 276.

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the difference between the i at the given temperature t and the i at 212°. The curve therefore represents the equation

$$i'' - i''_{212} = f(t),$$

where i'' is calculated from the formula and $i''_{212} = 1151.58$. The curve does not fit the points quite as well as the Davis second-degree curve, but the agreement is satisfactory, and is probably well within the limits of accuracy of the throttling experiments. Beyond the last point the curve begins to bend downward rather sharply and thus diverge from the prolonged Davis curve. The maximum value of i'' is reached at about 440°, the Davis equation gives the maximum at about 550°, and in the Marks and Davis tables the maximum occurs at 480°.

2. The Clapeyron-Clausius relation furnishes a valuable test within the range 32° — 212° (see p. 7). The discussion of this point is given in another section.

3. In the region of superheat the heat content formula may be checked by the throttling experiments of Grindley, Griessmann and Peake. According to the principles of thermodynamics a throttling process is also a constant-i process. That is, the points obtained in any particular throttling experiment when plotted on the p, t plane should lie on a curve $i =$ const.

In Fig. 9 the points are plotted from the experiments of Peake and Grindley, and constant-i curves calculated from eq. (D) are superposed. The general agreement is satisfactory, especially at the lower pressures. At the highest pressures Peake's highest curve has a smaller slope than the curve for $i = 1200$ and intersects it. It will be observed, however, that the curve has the slope indicated by Peake's points near saturation and the slope indicated by Grindley's points at some distance from the saturation limit.

The divergence of the curves from Peake's points naturally suggests the possibility that the curves run too low at the higher superheats. If this were the case, values of i calculated from equation (D) would be somewhat greater than the true values. Some light is thrown on this point by the series of throttling experiments made by Dodge.[1] In these experiments the steam was initially highly superheated, and the initial pressure was about 300 lbs. per sq. in. gauge. The points of 14 tests are shown in Fig. 10; these were taken directly from Davis'[2] discussion of the throttling experiments. Through each group of high-side points

1. Jour. A. S. M. E., Vol. 28, p. 1265 (1907); Vol. 30, p. 1227 (1908).
2. Proc. Am. Acad., Vol. 45, p. 253-257.

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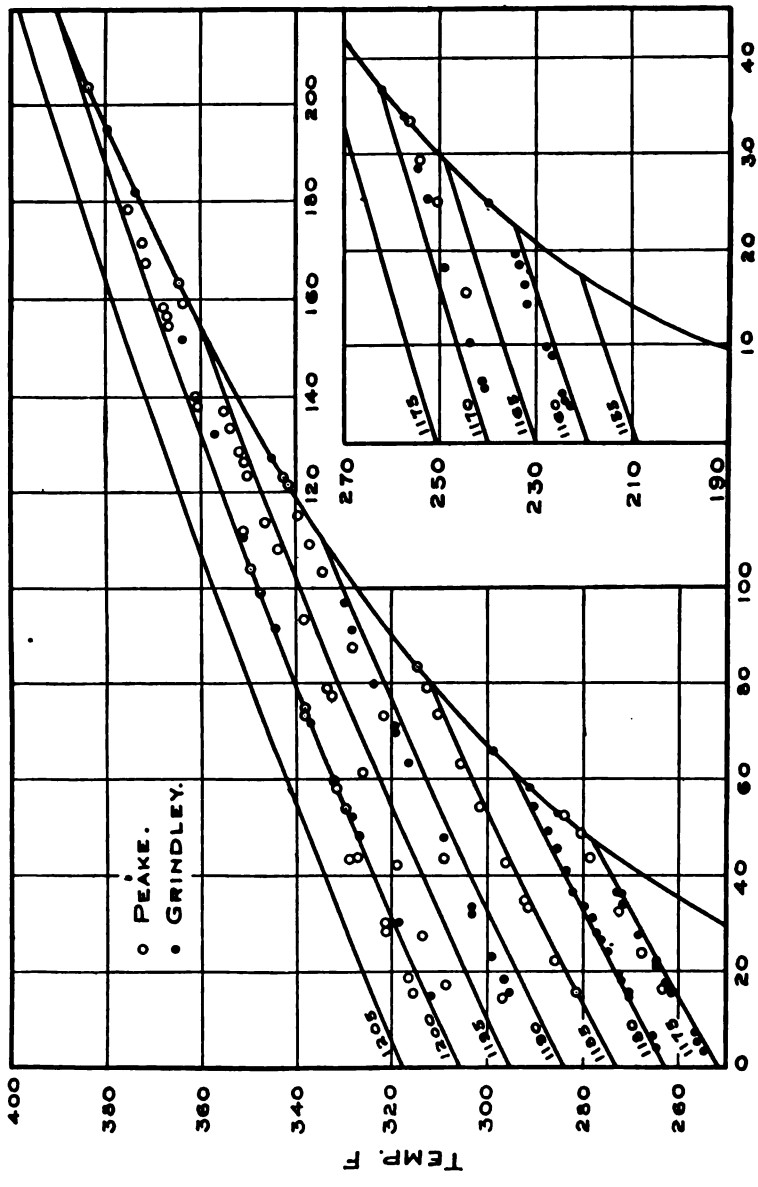

FIG. 9. CURVES OF CONSTANT HEAT CONTENT.

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a constant-*i* curve has been drawn. It will be seen that the curves agree fairly well with the points and at the higher superheats have the slopes indicated by general trend of the points. The marked drop from the initial point to the first of the low-side points is a phenomenon that is difficult to explain. In this connection Davis says: "It will be noticed that in every case a smooth curve through the low side points runs considerably below the corresponding high side points, just as did Grindley's

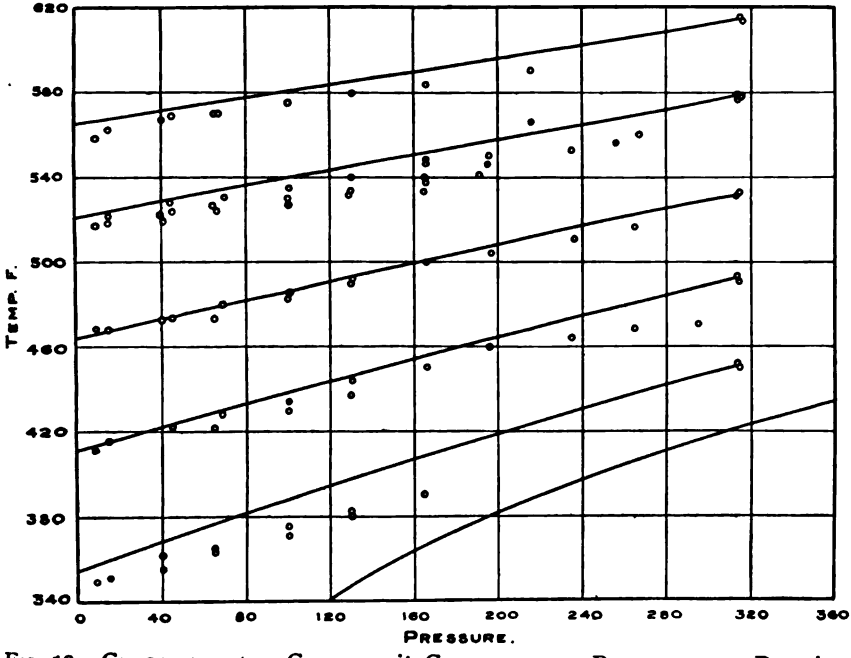

FIG. 10. COMPARISON OF CONSTANT-i' CURVES WITH POINTS FROM DODGE'S THROTTLING EXPERIMENTS.

curves. In Grindley's case this was because the entering steam carried water in suspension, the presence of which made the true total heat of the incoming mixture less than its apparent total heat regarded as homogeneous saturated steam, and dropped all the low side points onto throttling curves lower than those on which they apparently belonged. A similar phenomenon may be in evidence in Dodge's case, for although the incoming steam was superheated, it may still have been carrying in suspension a part of the water which had been sprayed into it for temperature regulation just before it reached the high-side chamber. It must, however, be admitted that if this explanation is to account for the whole of the discrepancy in Dodge's results, an extraordinarily large

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amount of water in suspension must have reached the high-side chamber —from one to one and a half per cent of the whole weight present. It is therefore probable that there is another source of error not yet discovered."

At the high superheats employed by Dodge it appears unlikely that such an amount of water could have been present, and it is probable that all the throttling experiments were vitiated to some extent by systematic errors. A probable source of error was the use of mercury thermometers. It has been shown quite conclusively that a mercury thermometer is not a good instrument for measuring temperatures in a current of superheated steam. If the throttling experiments could be repeated with the same precautions as have been observed in the other experiments on thermal properties, the results would have great value.

Bearing in mind the probable lack of precision in the sets of throttling experiments, it may be concluded that these experiments, on the whole, sustain the validity of the proposed heat content equation in the region of superheat.

A comparison of various values of the heat content i in the region of superheat is shown in Fig. 11, which was suggested by Fig. 21 of Heck's paper. The full lines represent equation (D), the dash lines represent Heck's new equation, and the points represent values given in the Marks and Davis tables. It is significant that the two equations deduced from entirely different bases should agree so closely.

26. *Specific Heat of Water. Heat of Liquid.*—For the temperature range 32°–212° F. (0°–100° C.) there are available five sets of experiments on the variation of the specific heat of water with the temperature. The curves that represent the results of these experiments are separable into two groups having quite different characteristics. Ludin[1] working with the method of mixtures obtained a curve which shows a minimum value of c' at about 20° C., then a rapid rise to a maximum at 87° C. (See Marks and Davis Steam Tables, Fig. 1, p. 88). The curves obtained by Dieterici[2] and Barnes[3] are similar in character; each shows a decrease of c' to a well defined minimum then a steady rise without any suggestion of a maximum. The experiments of Regnault and Dieterici above 100° C. show a steady rise of the specific heat with the temperature; hence if Ludin's curve be accepted, the specific heat after reaching its maximum at 87° must diminish and then increase

1. Inaug. Diss. Zurich, 1895.
2. Annalen der Physik (4), Vol. 16, pp. 593-620 (1905).
3. Phil. Trans., Vol. 199-A, pp. 55-148, 149-263 (1902).

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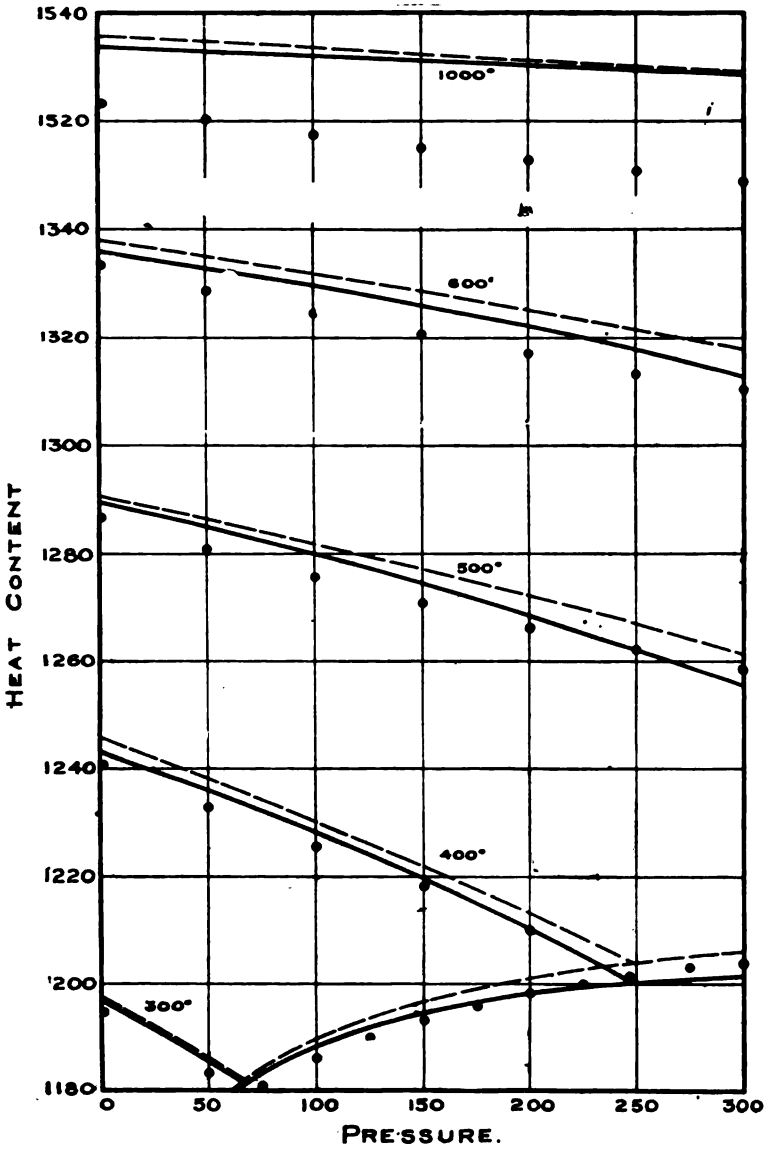

FIG. 11. ISOTHERMAL CURVES ON THE IP-PLANE.

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again. It is difficult to account for such a variation on any rational basis, and on the score of intrinsic probability, the curves of Barnes and Dieterici should be preferred to Ludin's curve. Davis[1] attached no weight whatever to Ludin's values and adopted a curve lying between those of Barnes and Dieterici, with Barnes' values given double weight. However, the question is again complicated by the experiments of W. R. and W. E. Bousfield[2] which reproduce Ludin's results, although the method employed (electric heating with a vacuum-jacket calorimeter) was entirely different from Ludin's method of mixtures. Callendar[3] has undertaken to throw light on the subject by a set of experiments in which a new and very accurate method is employed. Callendar's paper contains an exhaustive and valuable discussion of the whole subject.

The methods used by Barnes and Callendar, respectively, have the marked advantage of being continuous. In the Barnes experiments a steady current of water was heated through a small range of temperature by an electric current, and the result obtained was therefore the actual specific heat at a pre-determined temperature rather than the mean specific heat over a considerable range. Callendar used a continuous-mixture method in which two steady currents of water at different temperatures are passed through a system of concentric tubes which constitute a heat exchanger. The continuous flow methods have obvious advantages over other methods. The water equivalent of the calorimeter is not required, and various corrections that involve uncertain measurements are eliminated.

The results of Callendar's experiments by the continuous-mixture method completely verify the earlier experiments of Barnes by the continuous-electric method. As these two independent methods are much superior to the other methods used and give identical results, there can be no question that these results should be accepted.

Taking the specific heat of water at 20° C. as unity, Callendar gives the following equation for the variation of the specific heat with temperature

$$c' = 0.98536 + \frac{0.504}{t + 20} + 0.0084\frac{t}{100} + 0.0090\left(\frac{t}{100}\right)^2.$$

From the specific heat c' the heat content i' of the liquid is derived by the relation

$$i' = \int c'dt.$$

1. Steam Tables and Diagrams, p. 89.
2. Phil. Trans., Vol. 211-A, pp. 199-251 (1911).
3. Phil. Trans., Vol. 212-A, pp. 1-32 (1913).

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After changing from C. to F. temperatures and applying a factor to reduce from the 20°-calorie to the mean calorie, the equation for i' becomes

$$i' = 0.9838t + 2.0856 \log (t + 4) + 0.233 \left(\frac{t - 32}{100}\right)^2 + 0.09245\left(\frac{t - 32}{100}\right)^3 - 34.73.$$

Between 32° and 212° this equation gives values that agree very closely with the Marks and Davis values, as shown by the following table.

TABLE 12.

HEAT CONTENT OF LIQUID, 32°–212° F.

Temp. F.	40	80	120	160	200	212	240
Callendar	8.05	48.05	87.94	127.87	167.94	180	208.2
M. & D.	8.05	48.03	87.91	127.86	167.94	180	208.3

For temperatures above 212°, two sets of experiments are available, Regnault's and Dieterici's, neither of which can be accepted as thoroughly reliable. Regnault's results have been recomputed by various investigators. In Fig. 12, six mean values deduced from Callendar's computation are shown. The ordinates in this figure represent values of

$$\Delta i' = i' - (t - 32),$$

that is, the excess of the heat content over $t - 32$, the temperature range.

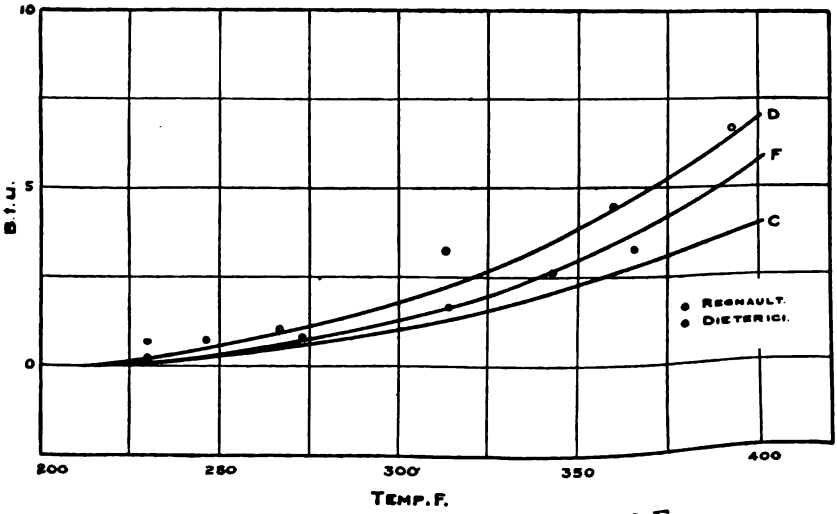

FIG. 12. HEAT CONTENT OF WATER, 212°-400° F.

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Abscissas are temperatures F. In the same figure are shown five points obtained by Dieterici, and in Fig. 13, the temperature range is extended and all of Dieterici's points are plotted. The curve *D*, Fig. 12, represents the equation adopted by Dieterici and curve *C* represents Callendar's equation extended beyond 212°.

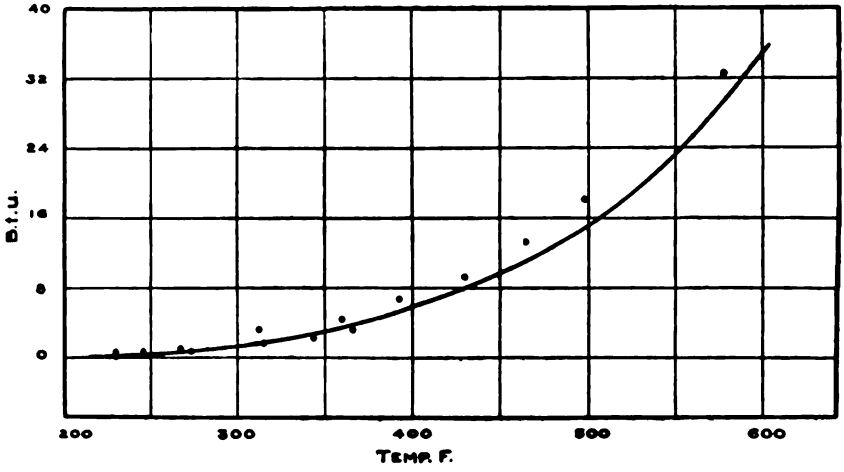

FIG. 13. HEAT CONTENT OF WATER, 212°-600° F.

Callendar questions the accuracy of Dieterici's experiments and gives preference to his equation extrapolated through the range 100° — 200° C. It is probable that Dieterici's points are considerably in error, as the method of the experiments involved large corrections, and it is also probable that Regnault's points are no more reliable. However, there seems to be no valid reason for choosing a curve, like curve *C*, lying below both sets of points.

27. *Latent Heat of Saturated Steam.*—The Clapeyron relation

$$r = A\,(v'' - v')\,T\,\frac{dp}{dT}$$

gives a means of calculating the latent heat. It is convenient to write the equation in the form

$$r = Ap\,(v'' - v')\,\frac{T}{p}\,\frac{dp}{dT}$$

in which the second member is made up of two factors. From the characteristic equation, the first is expressed by

$$Ap\,(v'' - v') = ABT - Ap\,(1 + 3ap^{1/2})\,\frac{m}{T^n}\,.$$

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Upon differentiating equation (A) connecting the pressure and temperature of saturated steam, namely

$$\log p = A - \frac{B}{T} - C \log T - DT + ET^2 - \Delta$$

the second factor is obtained in the form

$$\frac{T\,dp}{p\,dT} = 2.3026 \left[\frac{B}{T} - DT + 2ET^2 - T\frac{d\Delta}{dt} \right] - C$$

For the range 32° — 212°, within which the heat of the liquid is given accurately by the experiments of Barnes and Callendar, a second independent method of calculating the latent heat is available. Saturation values of i are calculated from the formula for heat content and from these are subtracted the corresponding known values of the heat of the liquid. The difference gives, of course, the latent heat. The following table gives values of r obtained by the two methods, and for comparison the Marks and Davis values are included.

TABLE 13.

LATENT HEAT, 32°–212° F.

Temp. F.	32	40	80	120	160	200	212
i'' from Eq. (D)	1072.83	1076.64	1095.30	1113.34	1130.64	1146.94	1151.58
i' Barnes & Callendar	0	8.05	48.05	87.94	127.87	167.94	180.00
r by subtraction	1072.83	1068.59	1047.25	1025.40	1002.77	979.00	971.58
r by Clapeyron relation	1072.19	1067.96	1046.81	1025.11	1002.62	978.97	971.58
r Marks & Davis	1073.4	1068.9	1046.7	1024.4	1001.6	977.8	970.4

Above 212° the heat of the liquid is so uncertain that the method of determining r by subtraction is hardly justified. Hence values of r are calculated from the Clapeyron relation, and subtracted from corresponding values of i''. The result is a set of values of i' that may be compared with the Regnault and Dieterici experimental values. The following table exhibits the details of the calculation.

TABLE 14.

LATENT HEAT AND HEAT OF LIQUID, 212°–600° F.

Temp. F.	212	240	280	320	360	400	440	480	520	560	600
i'' from Eq. (D)	1151.58	1161.90	1175.15	1186.30	1194.86	1200.37	1202.33	1200.43	1194.02	1182.76	1166.00
r from Clap. rel.	971.58	953.73	926.42	896.51	863.45	826.66	785.60	739.64	687.96	629.55	563.06
i' by subtraction	180	208.17	248.73	289.79	331.41	373.71	416.73	460.79	506.06	553.21	602.93
r Marks & Davis	970.4	952.1	924.3	894.2	861.8	827.2	790.1	748	700	642	572
i' " " "	180	208.3	249.0	290.2	331.9	374.1	417	462	507	554	604

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Referring to the first of the preceding tables, the close agreement of the two sets of values of r may be noted. The greatest difference, which occurs at 32°–40° is about 6 in 10 000. This agreement is a decisive test of the validity of the analysis. The two sets of numbers are obtained independently, one from the characteristic equation, the other from the heat-content equation, and the agreement between the two shows the satisfaction of the Clapeyron relation. Of the two sets the one obtained from the heat-content equation should be chosen, rather than the set derived by means of the Clapeyron relation. The reason for this lies in the slight uncertainty in the exact value of the derivative $\frac{dp}{dt}$ at low temperatures. It was shown in connection with Fig. 1 that the course of the Scheel and Heuse points indicates that the true value of this derivative at 32° is probably slightly greater than the value obtained from either of the three formulas. The slightly lower values of r calculated from the Clapeyron relation in the range of 32° — 80° may be ascribed, therefore, to a small error in the derivative.

For the range 212°–600° the important result is the set of values of i', heat of the liquid. The fairly close agreement of these values with the Marks and Davis values will be observed. The latter numbers were calculated from the Dieterici formula. In Fig. 12 curve F represents the new set of values for the range 212°–400° F., and in Fig. 13 the curve is extended to 600° F. The curve lies between Dieterici's curve and Callendar's extrapolated curve and represents very well the Regnault experiments as interpreted by Callendar. Above 400° the curve runs from 1 to 3 B. t. u. lower than the Dieterici points, a deviation of 0.2 to 0.6 per cent. Dieterici admits a possible error of 0.3 to 0.5 per cent in the experiments to determine the mean specific heat c_m and a further error in the reduction of c_m to the actual specific heat. It is likely that a possible error of at least 1 per cent may be attached to Dieterici's points; hence if the points are too high, as is indicated by Regnault's experiments and Callendar's extrapolated formula, the curve probably represents the true values fairly well.

28. *Experiments on Latent Heat.*—The values of the latent heat r given in Table 13 may be compared with direct experiments within the range 32°–212°. For this purpose four sets of experiments are available, those of Dieterici,[1] Griffiths,[2] Smith,[3] and Henning.[4] The

1. Annalen der Physik, Vol. 37, pp. 494-508 (1889).
2. Phil. Trans., Vol 186-A, pp. 261-341 (1895).
3. Phys. Review, Vol. 25, pp. 145-170 (1907).
4. Annalen der Physik (4), Vol. 21, pp. 849-878 (1906).

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following table gives the results of these experiments expressed in a common unit, the mean B. t. u.

TABLE 15.

EXPERIMENTAL DETERMINATIONS OF LATENT HEAT.

	Temp. F.	Latent Heat B. t. u.
Dieterici	32	1072.9
Griffiths	86	1045.1
	104.3	1034.1
Smith	57.1	1061.6
	70.1	1054.5
	82.5	1047.6
	103.6	1035.0
Henning	86.2	1043.2
	120.5	1026.2
	148.7	1008.0
	171.2	995.4
	192.7	983.3
	213.1	969.8

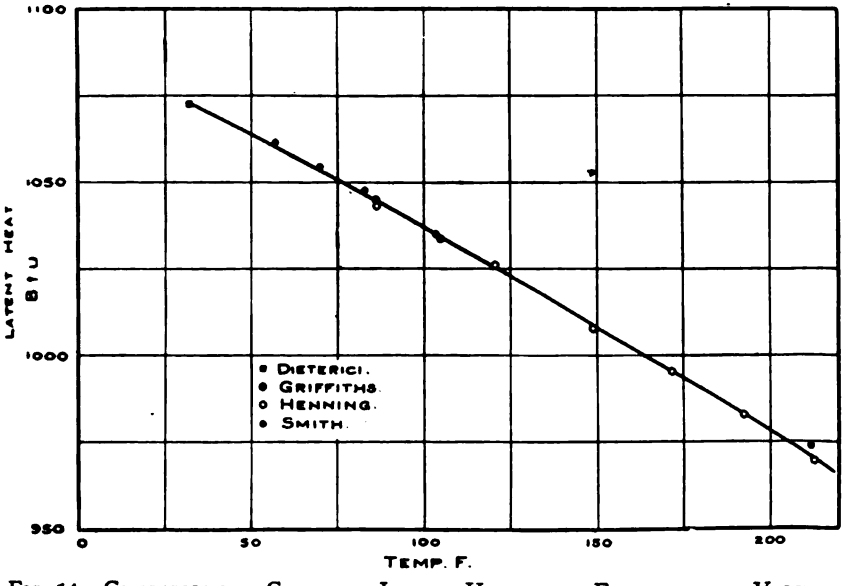

FIG. 14. COMPARISON OF CURVE FOR LATENT HEAT WITH EXPERIMENTAL VALUES.

In Fig. 14 these results are shown by the plotted points and the curve represents the variation of r according to the heat-content equation.

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The agreement is satisfactory, though Smith's points would indicate that the calculated values may be slightly low.

Special interest attaches to the value of r at 212° F. For years Regnault's number 966 B. t. u. was universally accepted. Callendar in his 1900 paper had the courage to raise this value to 972, which is almost precisely the value that is now considered most probable. Davis made use of the experiments of Henning and Joly at 212° and set the value of r at 970.4 B. t. u. Smith's recent experiments[1] on slow vaporization of water under atmospheric pressure indicate a value higher than any yet assumed, and Dr. Davis in a communication to the writer expresses the opinion that the number should be at least 972. Heck uses the value 971.2, Mollier uses 971.4. The present investigation leads to the value 971.6, which is probably quite close to the truth, though if anything slightly low.

29. *Entropy.*—An expression for the entropy of superheated steam is readily obtained from the fundamental equation,

$$dq = c_p dT - AT\left(\frac{\partial v}{\partial T}\right)_p dp.$$

Dividing by T,

$$ds = \frac{dq}{T} = c_p \frac{dT}{T} - A\left(\frac{\partial v}{\partial T}\right)_p dp.$$

From the characteristic equation

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{B}{p} + \frac{mn}{T^{n+1}}(1 + 3ap^{1/2}).$$

Introducing this and the expression for c_p in the preceding equation, the result is

$$ds = \left[\frac{\alpha}{T} + \beta + \frac{\gamma}{T^2} + \frac{Amn\,(n+1)}{T^{n+2}}\,p\,(1 + 2ap^{1/2})\right]dT - \frac{AB}{p}dp$$
$$- \frac{Amn}{T^{n+1}}(1 + 3ap^{1/2})\,dp.$$

The integration of this exact differential equation gives the following equation for the entropy

$$s = \alpha \log_e T + \beta T - \tfrac{1}{2}\frac{\gamma}{T^2} - AB \log_e p - \frac{Amn}{T^{n+1}}\,p\,(1 + 2ap^{1/2}) + s_0. \quad \text{(E)}$$

The constant s_0 is found by applying the equation at the saturation limit. The value thus determined is $s_0 = 0.08085$.

1. Physical Review, Vol. 33, p. 188 (1911).

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For the range 32°–212°, within which Callendar's formula for the heat of the liquid is surely applicable, there are available two independent methods of calculating the entropy of saturated steam. 1. The entropy of the liquid s' is determined by the integration of Callendar's equation for i' and the entropy of vaporization $\frac{r}{T}$ is added. 2. Corresponding saturation values of p and T are substituted directly in the preceding formula for s. The two methods should give substantially identical results.

Above 212° the entropy s'' of saturated steam is calculated from formula (E) and the entropy of the liquid s' is obtained by the relation

$$s' = s'' - \frac{r}{T}.$$

Integration of Callendar's i'-equation gives the following formula for s':

$$s' = 2.3623 \log T + 0.0045775 \log (t + 4) - 0.00022609\, T + 0.000\,000\,13867\, T^2 - 6.28787.$$

The following table shows the results of the calculation, the Marks and Davis values being included for comparison.

TABLE 16.

ENTROPY OF LIQUID AND SATURATED VAPOR, 32°–212° F.

Temp. F.	32	40	80	120	160	200	212
s' from Callendar Eq.	0.	0.0163	0.0933	0.1646	0.2312	0.2939	0.3120
$\frac{r}{T}$	2.1823	2.1389	1.9408	1.7692	1.6184	1.4842	1.4467
$s''=s'+\frac{r}{T}$	2.1823	2.1552	2.0341	1.9338	1.8496	1.7781	1.7587
s'' from Eq. (E)	2.1822	2.1550	2.0340	1.9338	1.8497	1.7782	1.7587
Marks & Davis							
s'	0.	0.0162	0.0932	0.1645	0.2311	0.2937	0.3118
$\frac{r}{T}$	2.1832	2.1394	1.9398	1.7674	1.6165	1.4824	1.4447
s''	2.1832	2.1556	2.0330	1.9319	1.8476	1.7761	1.7565

30. *Intrinsic Energy.*—From the defining equation

$$i = A\,(u + pv)$$

the energy u in thermal units is readily obtained by subtraction; thus

$$u = i - Apv$$

Combination of equations (B) and (D) gives therefore the following explicit expression

$$u = (a - AB)\,T + \tfrac{1}{2}\beta T^2 - \frac{\gamma}{T} - \frac{Amnp}{T^n}\left(1 + \frac{2n-1}{n} a p^{1/2}\right) + i_0. \quad \text{(F)}$$

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E. JOULE-THOMSON EFFECT.

31. *The Joule-Thomson Coefficient.*—The ratio of the drop in temperature to the decrease of pressure when a fluid is subjected to a throttling process is the Joule-Thomson coefficient μ. Since in such a process the heat content i is constant, the value of μ is given by the derivative $\frac{dT}{dp}$ evaluated under the condition that i is constant; that is

$$\mu = \left(\frac{dT}{dp}\right)_{i = \text{const.}}$$

It follows, therefore, that the slope of a throttling curve (See Fig. 9) at any point is the value of μ at that point.

An expression for μ is readily derived from the equation for heat content,

$$i = aT + \frac{1}{2}\beta T^2 - \frac{\gamma}{T} - \frac{Am(n+1)}{T^n}\, p\,(1 + 2ap^{1/2}) + Acp + i_0.$$

With i constant

$$\left(\frac{dT}{dp}\right)_i = -\left(\frac{\partial i}{\partial p}\right)_T \div \left(\frac{\partial i}{\partial T}\right)_p,$$

But

$$\left(\frac{\partial i}{\partial T}\right)_p = c_p,$$

hence

$$\mu = \frac{A}{c_p}\left[\frac{m\,(n+1)}{T^n}(1 + 3ap^{1/2}) - c\right] \qquad \text{(G)}$$

32. *Comparison with the Throttling Experiments.*—From the four available sets of throttling experiments, Davis[1] has computed values of μ covering a wide range of pressure and temperature. As might be expected, the results when plotted showed a broad band of points (See Fig. 6 of Davis' paper) and individual measurements showed considerabl discrepancy. By passing a smooth curve through the band of points Davis obtained the relation between μ and t that was afterwards used by himself in spacing the specific heat curves, and by Heck in the development of his theory. Davis therefore assumes that μ is a function of the temperature only, while according to the preceding equation (G) μ must vary with the pressure also.

A comparison of equation (G) with the values of μ computed by Davis is shown in Fig. 15. Four different pressures are used, 36, 120, 200, and 300 lb. per sq. in., and the points are plotted from the data

1. Proc. Am. Acad., Vol. 45, pp. 243-264.

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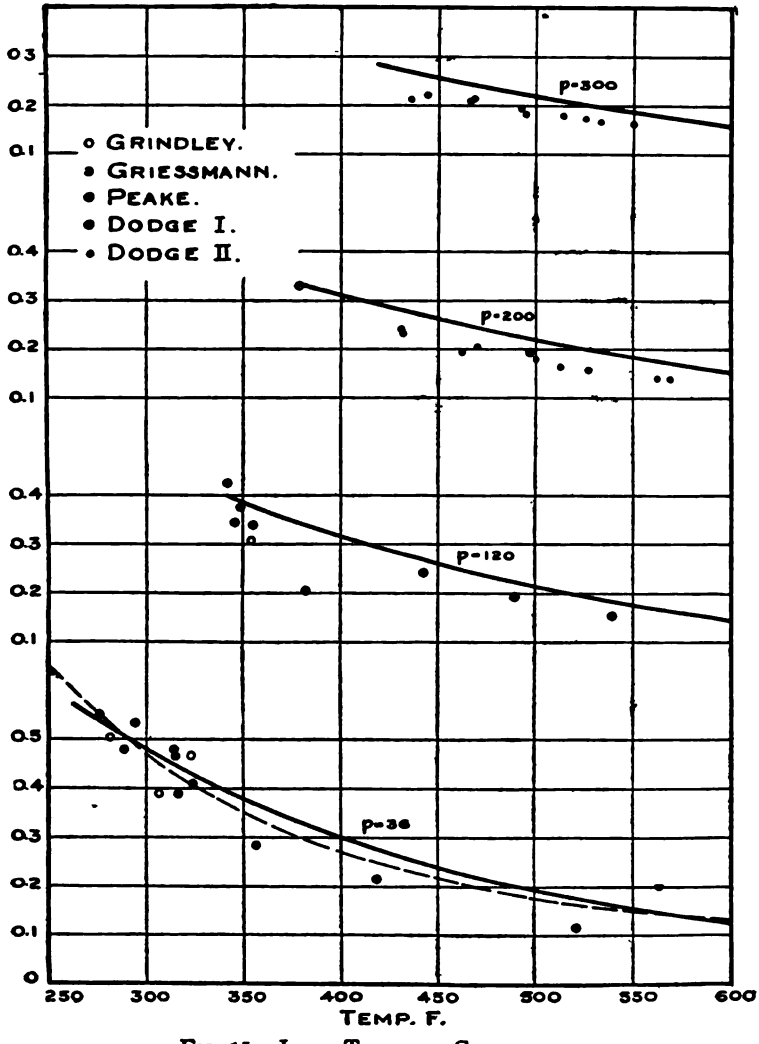

FIG. 15. JOULE-THOMSON COEFFICIENT.

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given in the tables of Davis' paper. The points in each group are so chosen that the pressure in the experiments represented by them is very nearly the pressure attached to the corresponding curve. The dash curve through the lowest group is the Davis curve for μ.

It will be seen that the curves agree sufficiently well with the points obtained from the first three sets of throttling experiments, but that they lie somewhat above the points deduced from Dodge's experiments. Reference to Fig. 10 will afford an explanation of the discrepancy in the case of Dodge's first series. The peculiar discontinuity between the high-side point and the first low-side point has already received comment (See p. 47). Davis rejected the high-side point entirely and obtained values of μ by a comparison of low-side points only. This was equivalent to taking the slopes of smooth curves running through the low-side points, and it is evident from the figure that the values of μ thus obtained are smaller than the values given by the slopes of the theoretical curves. Dodge's second series of 33 experiments was of such a character that the high-side observations had to be used. The values of μ obtained directly from the observations were, according to Davis, abnormally high, and certain corrections were made to reduce them to the level required by the law of corresponding states. The chief correction was a reduction of the high-side temperature by an amount ranging from 11° to 14°. If the high-side points are incorrect, as assumed by Davis, because of water carried in suspension in the highly superheated steam, the plotted points representing Dodge's experiments must be accepted, and the curves obtained from eq. (G) run somewhat high. If, on the other hand, the error lies in the low-side points, the values of μ calculated by Davis are too low, and the curves probably are nearer the truth than the points. The question can only be settled by a new series of experiments with suitable precautions to insure the dryness of the steam and with improved methods of temperature measurement.

The other question that presents itself relates to the variation of μ with the pressure. According to equation (G) this variation must exist, though it is small. Davis states that he was unable to detect such variation from an examination of the observations. Heck assumed at the start of his investigation that μ depended on t alone, but the further development of the theory led him to the conclusion that the assumption could not be rigorously correct. It is apparently impossible to construct a general theory that will give satisfactory agreement in other parts of the field and at the same time leave the coefficient μ unaffected by the

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pressure. We conclude, therefore, that μ *is* a function of the pressure; and so far as our present knowledge extends, the value of μ is given satisfactorily by equation (G).

F. THERMAL PROPERTIES NEAR THE CRITICAL TEMPERATURE.

33. *Previous Investigations.*—That the equation (B) to (F) (at the saturation limit) should hold good up to the critical temperature is not to be expected. In the vicinity of the critical point the variation of some of the thermal properties is so rapid that to represent these properties, equations of a different form must be used. As there are no trustworthy experiments in the range 600° F. to the critical temperature, any formulation covering this region must be regarded as a guess. However, even a conjecture may have a qualitative value, and in the present case the investigation at least serves to establish quite closely the upper limit of temperature for the equations developed in the preceding sections.

Several attacks have been made on this problem. Davis[1] by the use of Thiesen's exponential formula determined the latent heat r, and by "the law of the straight diameter" established tentatively the specific volume v''. Schüle[2] has likewise estimated certain of the thermal properties near the critical temperature, and his work appears to have a reasonable basis. Another tabulation was published in *Engineering* (Jan. 4, 1907); this is palpably incorrect and has no value whatever.

34. *Thiesen's Exponential Formula.*—At the critical temperature, the latent heat r becomes zero, and furthermore $\frac{dr}{dt} = -\infty$. An equation connecting r and t may be given a form such that these conditions will be satisfied. The simplest form is that suggested by Thiesen, namely.

$$r = C\,(t_k - t)^n,$$

in which t_k denotes the critical temperature, and $n < 1$. If now the constants can be so determined as to give correct values of r at ordinary temperatures, then it may be inferred that the formula will give reasonably good values for higher temperatures. The formula may be written

$$\log r = \log C + n \log (t_k - t)$$

a form more convenient for purposes of calculation.

Davis[3] found that with $\log C = 2.14302$, $n = 0.315$, and $t_k = 689$, the formula represented the available experimental values of r with remarkable accuracy. The number 689 is however the older value of

1. Marks and Davis Steam Tables, pp. 102-105.
2. Zeit. des Verein. deutsch. Ing., Vol. 55, pp. 1561-1567 (1911).
3. Proc. Am. Acad., Vol. 45, p. 284.

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t_k due to Cailletet and Colardeau. The recent experiments of Holborn and Baumann[1] raise the value of t_k to 706.3° F., and with this value it appears to be impossible to make Thiesen's formula fit any series of probable values of r through a wide range of temperature. It is found, however, that the equation

$$\log r = 2.00117 + 0.3685\ (706.3 - t)$$

fits reasonably well the values of r from 360° to 560°. The following table shows the agreement.

TABLE 17.

COMPARISON OF VALUES OF LATENT HEAT.

Temp. F.	360	400	440	480	520	560	600
r deduced from Eq. (D)	863.5	826.7	785.5	739.6	688.0	629.6	563.1
r from Thiesen's Eq.	864.9	826.6	785.1	739.4	688.2	629.6	559.7

Below 360° and above 560° the two sets of values diverge rapidly. It is assumed that the formula will give approximate values of r between 560° and 706.3°, but too much confidence should not be placed in the accuracy of the results thus obtained.

35. *Heat Content of Liquid and Vapor.*—The law of the mean diameter has frequently been applied in determining the specific volume of the vapor. A somewhat similar application is here made in connection with the heat contents i' and i''. Taking the sum $i' + i''$ for the temperature range 400°–600° F., it is found that this sum is given very accurately by the equation

$$i' + i'' = 956.94 + 1.923t - 0.00095t^2.$$

It is assumed that this relation also holds good from 600° to the critical temperature. Hence with the sum $i' + i''$ determined by this equation and the difference $i'' - i' = r$ determined from the Thiesen exponential equation, values of i'' and i' are readily calculated. The variation of these thermal magnitudes near the critical temperature is shown in Fig. 16. Curve CD is the mean diameter, its ordinate being $\frac{1}{2}(i' + i'')$, curve AD represents the heat content i'' of the vapor, and curve BD the heat content i' of the water. These are two branches of a continuous curve, and the common value of i' and i'' at the critical temperature is apparently about 921 B. t. u.

36. *Specific Volume of Steam.*—The method of Davis[2] has been followed, except that 706.3 is used for the critical temperature. Values of v'' the specific volume of steam were calculated from the characteristic

1. Annalen der Physik, Vol. 31, pp. 945-970 (1910).
2. Steam Tables and Diagrams, pp. 103-105.

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equation (B). For the values of v', the specific volume of water, the formula of Avernarius

$$v' = a - b \log (t_k - t)$$

was found to give exact results for the temperature range 280° — 600° F., with the following constants: $a = 0.044825$, $b = 0.0105$. Denoting by $2s$ the sum of the densities of the liquid and vapor, that is

$$2s = \frac{1}{v'} + \frac{1}{v''}$$

the variation of this sum with the temperature is given by the equation

$$2s = 67.1 - 0.029131\, t - 0.000009342\, t^2.$$

The Clapeyron relation gives another relation connecting v' and v'', namely

$$r = A\,(v'' - v')\,T\frac{dp}{dT}$$

in which the derivative $\frac{dp}{dT}$ is found from eq. (A) and r from the Thiesen formula. The values of v'' and v' are therefore readily obtained by the solution of a quadratic equation.

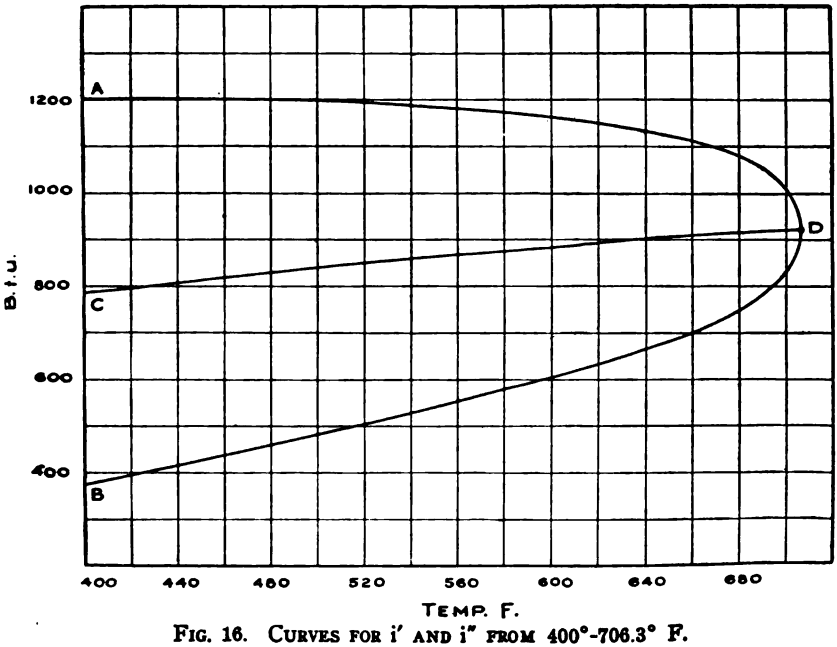

FIG. 16. CURVES FOR i′ AND i″ FROM 400°-706.3° F.

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37. *Table of Properties at High Temperatures.*—The following table gives the thermal properties of saturated steam between 600° and the critical temperature, as calculated by the methods just described. It is to be understood that the tabulation is merely a conjecture and that no claim of extreme accuracy is asserted.

TABLE 18.

PROPERTIES OF SATURATED STEAM FROM 600° TO 706.3° F.

Temp. F.	Pressure: lb. per sq. in.	Volume of 1 lb.	Weight of 1 cu. ft.	Heat Content		Latent heat
				of liquid	of vapor	
600	1540.4	0.272	3.68	604.5	1164	560
620	1658.7	0.241	4.15	633	1151	518
640	2056.6	0.187	5.35	663	1136	473
660	2360.8	0.151	6.63	700	1112	412
680	2699.1	0.118	9.86	745	1080	335
700	3074.5	0.080	12.46	823	1016	193
706.3	3200	0.048	20.92	921	921	0

Comparing the equations used in deducing the values in this table with the original equations (B) and (D), it is found that the former merge into the latter at a temperature lying between 560° and 600° F. The discrepancy at 600° is small. It may be reasonably assumed, therefore, that the general equations developed in this investigation hold good up to 600° F. when applied to the saturated vapor. The corresponding pressure limit is above 1500 lb. per sq. in.

V. RESUMÉ OF FORMULAS.

For convenience of reference the principal formulas developed in the preceding section are here collected. The constants are for English units with pressures expressed in pounds per square *inch*. The logarithms involved are common logarithms.

1. Relation between pressure and temperature:

$$\log p = 10.5688080 - \frac{4876.643}{T} - 0.0155 \log T - 0.00406258\, T$$

$$+ 0.00000400555T^2 - 0.00002\left[10 - 10\left(\frac{t-370}{100}\right)^2 + \left(\frac{t-370}{100}\right)^4\right] \quad \text{(A)}$$

$$T = t + 459.6$$

2. For the specific volume:

$$v - c = 0.59465\frac{T}{p} - (1 + 0.05129p^{1/2})\frac{C_1}{T^4} \qquad \text{(B)}$$

in which $\log C_1 = 10.82500$, and c may be given a mean value 0.017.

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3. For the specific heat:

$$c_p = 0.320 + 0.000126T + \frac{23583}{T^2} + \frac{C_2 p\,(1 + 0.0342p^{1/2})}{T^3} \qquad \text{(C)}$$

$$\log C_2 = 11.39361.$$

4. For the heat content:

$$i = 0.320T + 0.000063T^2 - \frac{23583}{T} - \frac{C_3 p\,(1 + 0.0342p^{1/2})}{T^4}$$

$$+ 0.00333p + 948.7 \qquad \text{(D)}$$

$$\log C_3 = 10.79155.$$

5. For the entropy of superheated steam

$$s = 0.73683 \log T + 0.000126T - \frac{11791.5}{T^2} - 0.25355 \log p$$

$$- \frac{C_4 p\,(1 + 0.0342p)}{T^5} - 0.08085 \qquad \text{(E)}$$

$$\log C_4 = 10.69464$$

6. For the intrinsic energy of superheated steam, in B. t. u.:

$$u = 0.2099T + 0.000063T^2 - \frac{23583}{T} - \frac{C_5 p\,(1 + 0.02992p^{1/2})}{T^4} + 948.7 \quad \text{(F)}$$

$$\log C_5 = 10.69464$$

7. Heat of the liquid between 32° and 212° from Callendar's formula:

$$i' = 0.9838t + 2.0856 \log (t + 4) + 0.233 \left(\frac{t - 32}{100}\right)^2 +$$

$$0.09245 \left(\frac{t - 32}{100}\right)^3 - 34.73.$$

8. Entropy of the liquid between 32° and 212°, from Callendar's formula

$$s' = 2.3623 \log T + 0.0045775 \log (t + 4) - 0.00022609\,T$$

$$+ 0.00000012867T^2 - 6.2879.$$

9. External latent heat of saturated steam:

$$\psi = 0.1101\,T - \frac{C_6 p\,(1 + 0.05129p)}{T^4}$$

$$\log C_6 = 10.09258.$$

10. Latent heat of saturated steam:

$$r = \psi \frac{T}{p} \frac{dp}{dT}$$

in which $\frac{T}{p} \frac{dp}{dT}$ is derived from formula (A).

The preceding equations are sufficient for the calculation of all the thermal properties of saturated and superheated steam that are usually found in steam tables.

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VI. Conclusion.

38. *Comparison of Tables and Formulations.*—A table of the properties of steam, or a formulation from which a table may be calculated, should possess two characteristics, *consistency* and *accuracy*. A table or formulation is consistent when the values are obtained from equations that are properly connected by thermodynamic relations, such as the Clausius and Clapeyron relations. It may be considered accurate if the calculated values show close agreement with the most reliable experimental data. In this sense, accuracy is only relative. A table that would have been considered accurate ten years ago would be regarded today as extremely inaccurate in the light of our present knowledge of the subject.

Taking these two characteristics as a basis of classification, the existing tables and formulations may be divided into four groups:

1. Those that possess neither consistency nor accuracy.
2. Those that are consistent but inaccurate.
3. Those that are reasonably accurate but lack consistency.
4. Those that are both consistent and accurate.

In the first group may be placed all the older tables, such as Rankine's, Buel's, Porter's, and Dery's. The tabular values were calculated from empirical formulas based chiefly on Regnault's data, and the necessary connection through thermodynamic relations was not recognized.

In the second group are the tables based on Callendar's theory. As regards consistency these tables are entirely satisfactory, as the thermodynamic relations are satisfied. Mollier's tables appeared before the publication of the decisive Munich experiments, which invalidated some of the fundamental assumptions of Callendar's theory. Smith and Warren, however, have apparently ignored completely the mass of experimental evidence accumulated since 1900, and have deliberately sacrificed accuracy for the sake of consistency. While these two sets of tables are probably more accurate than those in the first group, they are nevertheless grossly inaccurate according to present standards.

The noteworthy researches of Davis have been embodied in the Marks and Davis steam tables and in the last edition of Peabody's tables. The establishment of the new formula for the heat content of saturated steam was the principal achievement of Dr. Davis, and the incorporation of this formula in the tables marks an important advance in accuracy. The Marks and Davis tables are not rigorously consistent, and in some particulars they also exhibit a lack of extreme accuracy. The system of

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specific heat curves used as the basis of the values in the region of superheat is probably somewhat in error, as noted in a preceding section. The values of the specific volume of superheated steam are probably too high, and the values of the heat content in the region of superheat are almost certainly too low (See Fig. 11). The value 970.4 B. t. u. assumed for the latent heat at 212° F. is doubtless slightly low. Notwithstanding minor defects in accuracy and consistency, the Marks and Davis tables are justly held in high regard, and they have been very generally accepted as standard in engineering practice.

In the fourth group may be included the formulations of Jakob and Heck, and the formulation developed in the preceding sections. In each of these, consistency is secured by a strict observance of thermodynamic relations; at the same time the available experimental data are used to check the equations, and thus a high degree of accuracy is insured. It is probable that a table derived from any one of these formulations would surpass in accuracy the best of existing tables.

With respect to the relative advantages of the three formulations, the following statements may be made. Jakob's scheme is open to the objection that it is graphical. Numerical results must be obtained by measurement of geometrical magnitudes. Furthermore, Jakob's system of specific heat curves, like Davis' system, is probably erroneous near the saturation limit. For these reasons Jakob's formulation may be considered the least satisfactory of the three. Heck's theory doubtless shows good agreement with experimental data, and a table constructed from it should be thoroughly satisfactory as regards accuracy and consistency. In the few cases in which comparisons have been made, the theory described in this bulletin appears to show somewhat closer agreement with experimental data than Heck's theory. Equation (A) for the pressure-temperature relation is evidently superior to Heck's modification of Marks' formula; the characteristic equation (B) represents the Knoblauch measurements more accurately than Heck's equation; and equation (D) for the heat content gives values of i at saturation that agree very closely with the points deduced by Davis from the throttling experiments, while Heck's curve for i runs appreciably higher.

As regards accuracy, the theory under discussion does not suffer in comparison with Heck's theory. In other respects it presents certain obvious advantages. (1) The range within which the formulation may be regarded as valid is somewhat greater. At saturation Heck's equations

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hold good up to 500° F., or perhaps a little higher; the equations (B) to (F) are valid up to nearly 600° F. (2) Heck's equations are excessively complicated and ill-adapted for computation. Equations (B) to (F) are in comparison relatively simple, and because of the repetition of certain terms in the several formulas the labor of computation is greatly abridged, and numerical results may be obtained with comparative ease.

39. *Effect of Future Experiments.*—The theory under discussion has been correlated with the most reliable experimental data available at the present time. It is, of course, probable that subsequent experiments of a higher order of precision will modify these data to some extent and possibly render necessary a revision of the formulation. It may be profitable in this connection to discuss briefly the sufficiency of existing data and indicate the possible effect of future experiments.

The pressure-temperature determinations may be regarded as final. It is not likely that further experiments will modify these to any extent. Further experiments on specific volumes are desirable; these should include a higher superheat and much greater pressure range than the Munich experiments. Such experiments would probably confirm the results of Knoblauch, Linde, and Klebe. The results of the experiments of Lanz and Schmidt on the specific heat of steam at the higher pressures are awaited with interest. A preliminary report indicates that the experiments of Knoblauch and Mollier are substantially confirmed. The experiments at higher pressures should settle the question of the proper course of the specific heat curves near the saturation limit. With reference to the specific heat of water, the results of Barnes and Callendar for the range 32°–212° should be accepted. For the range above 212° the experimental evidence is at present unsatisfactory and further experiments in this field are urgently needed. Finally, the questions raised by the various discrepancies in the throttling experiments should be settled by a new set of experiments conducted with proper precautions to insure the dryness of the steam and the accuracy of the measurements.

While the theory may require modification as additional experimental data become available, it is believed that such modifications as may be necessary can be made without essential changes in structure. The formulation is extremely sensitive to changes in the constants and any modifications that are likely to be made in the fundamental data, volumes, specific heats, etc., can be accommodated by slight changes in the constants.

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40. *Summary.*—In this bulletin is described the development of a general theory of the properties of superheated and saturated steam. For this theory the following claims are urged.

1. *Consistency.*—In the deduction of the fundamental equations the thermodynamic relations have been recognized, and the satisfaction of these relations assures consistency.

2. *Accuracy.*—The equations have been checked with the most reliable experimental data available, and in every respect the agreement is extremely close. It may be asserted that the formulation is at least as accurate as any other that has thus far appeared.

3. *Simplicity.*—The equations are relatively simple and well adapted for computation of numerical results.

4. *Flexibility.*—The structure of the formulation is such that modifications necessitated by further experimental data can probably be made by slight changes of the constants.

5. *Range of Validity.*—At the saturation limit the equations are valid throughout the range 32°–600° F., a range far wider than is at present employed in engineering practice.

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THE ANALYSIS OF COAL WITH PHENOL AS A SOLVENT

BY

S. W. PARR

AND

H. F. HADLEY

BULLETIN No. 76

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ENGINEERING EXPERIMENT STATION

BULLETIN No. 76 NOVEMBER, 1914

THE ANALYSIS OF COAL WITH PHENOL AS A SOLVENT
(WITH A STUDY OF THE RESULTING COMPOUNDS)

BY S. W. PARR, PROFESSOR OF APPLIED CHEMISTRY, AND
H. F. HADLEY, RESEARCH FELLOW, DEPARTMENT OF CHEMISTRY

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THE ANALYSIS OF COAL WITH PHENOL AS A SOLVENT

I. Introduction.

1. *Present Methods of Coal Analysis.*—There are two processes in vogue at the present time for the chemical examination of coal; one is the ultimate, and the other is the proximate method of analysis. In the first the organic or combustible part of the coal is separated into its elemental constituents, carbon, hydrogen, oxygen, and nitrogen. The mineral or non-combustible portion is separately determined under two items as ash and moisture.

In the proximate method the organic material is separated into two divisions, one being that portion which under high temperature and out of contact with the air passes off in the gaseous form, and the other that part which remains behind as the non-volatile or coke-forming carbon.

Each procedure has doubtless come into use as the result of a specific demand. For example, the engineer needed the data from which he could calculate the total heat of the coal and, in arriving at a heat balance, he must also have at hand any negative factors chargeable to the fuel, such as the quantity and character of the gaseous products of combustion. These items, therefore, would call for the data furnished by the ultimate methods of analysis.

The proximate method was developed as a natural accompaniment of the gas and coke industries, since it furnished in either case an index of the yield which might be expected from a given coal. Formerly, also, the quantity of volatile matter was made to serve as an index of the grade or quality of a coal. Thus the data from proximate analyses have been put into the form of "fuel ratios" or the ratio of the non-volatile to the volatile part of the coal, such ratios supposedly serving as an indication of the general class or type to which the coal belonged.

It is evident from this brief statement that the results obtained from ordinary methods of analysis are of rather narrow application and that the data from neither the ultimate nor the proximate methods

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convey much information which can be applied to the every-day problems connected with the handling and use of coal such as coking, storage, deterioration, oxidation, weathering, and spontaneous combustion.

It is to be noted in this connection that the ordinary methods of analysis give the percentage amounts of certain elements or compounds which result from destructive processes and do not even remotely take account of the actual constituents of the organic or active portions of the coal as they occur in unaltered form. But it is these unaltered constituents which have to do with such matters as coal storage and spontaneous combustion. Especially do they have to do with coking properties, with smoke formation, and the type of distillation products which they may be made to yield.

2. *Purpose of the Proposed Study.*—The purpose of this investigation has been to examine and enlarge upon certain processes of analysis which proceed in such a manner as to separate in unaltered form the fundamental substances of which coal is composed, and to study their characteristic properties in detail.

Efforts along this line have multiplied in recent years. Much suggestive information has come from the study of the types of vegetable matter which constituted the source of the initial coal substance. The microscopical studies of White,[1] of the United States Geological Survey, and Professor Jeffrey,[2] of Harvard, have developed the general proposition that the original organic constituents would fall into two general classes: those of the cellulose and those of the resinic type. A similar line of evidence has been developed by numerous investigators, who have made use of various solvents, which, in the main, have differentiated the compounds along the same general lines. The work of Frazer and Hoffman,[3] Clark and Wheeler,[4] and Professor Lewes[5] may be cited in this connection.

In conformity, therefore, to this general proposition as to the coal constituents this investigation proceeded upon the general hypothesis that in coal two types of compounds are to be met with: (1) the degradation products of the original cellulose substances, and (2) the less altered and perhaps more characteristic compounds of the resinic type. The first line of investigation, therefore, from the chemical standpoint, would seem to consist of a study of various solvents which might lend themselves to such a separation into the two main subdi-

1. Professional Paper 85-E, United States Geological Survey (1914).
2. Proc. Am. Acad. of Arts and Sciences, 46, 273 (1910).
3. Technical Paper 5, Bureau of Mines (1912).
4. Journal of Chemical Society, 103, 1704 (1913).
5. Progressive Age, 29, 1030 (1911).

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visions of the coal constituent, and, second, a study of the general properties of each branch.

3. *Outline of the Investigation.*—(1). Various extractive agents were used to determine the extent or completeness of their solvent properties. Solvents were selected which would seem to have the greatest ability to remove those bodies which were of the resinic type. The substances tried are arranged in the order of their solvent properties in so far as that would seem to be indicated in preliminary studies directly concerned with the extent of the solvent action of each. The list is as follows: phenol, ortho-cresol, low boiling tar distillate, para-cresol, pyridine, phenol-toluene mixture, aniline, methyl aniline, acetone, toluene, benzene, carbon disulphide, and turpentine. While some of the above solvents might warrant further study, there are evident advantages over the other compounds in the use of phenol and it was made the subject of extended study.

(2). A general survey of the various types of coal was made to determine the relative amounts of phenol-soluble extract contained in each. The average amount of material extracted from Pocahontas coal was 1.8 per cent and the results varied in the case of Illinois coals from 22 per cent to 39 per cent. These results were calculated on an ash and moisture free basis.

(3). By effecting a separation as indicated under (1), it was possible to study separately the insoluble residue and the extracted material, to determine which had the greater avidity for oxygen, also to which division should be credited the coking constituents, and what products of decomposition, such as gas, illuminants, tar, etc., could be assigned to each.

4. *Summary of Results.*—1. Phenol at 100°C. will dissolve certain constituents of bituminous coals in their natural state. The two subdivisions, designated as *insoluble residue* and *extraction material*, together make substantially 100 per cent or the amount of the original substance.

2. The amount of extractive material is definite and susceptible of quantitative determination.

3. Studies upon these two type substances indicate that the extract is the vital constituent concerned in the coking of coal. It conforms to the principle already enumerated[1] in that it has a sufficiently definite melting point and a decomposition temperature which is above that of the melting point.

4. Each subdivision is capable of absorbing oxygen. The effect upon the extract is to modify its coking properties by lowering or

1. Bulletin No. 60, U. of Ill. Eng. Exp. Sta., p. 20.

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greatly reducing its power to form a firm and coherent mass.

5. The oxygen taken up in either case is found to be chemically combined. The oxygen taken up by the fresh coal is similarly held. The insoluble portion has the greater avidity for oxygen. This relation to oxygen has a bearing upon other topics aside from that of coking, as the heating of coal in storage, spontaneous combustion, weathering, and deterioration.

II. Experimental.

A. Apparatus and Method of Procedure.

5. *Preliminary.*—In the preliminary part of the work, conducted to determine which solvent would be the best suited to the purpose, the adaptability of the solvent was determined by the amount of material which it would remove, and also, by the absence of chemical activity involved in its use. That is to say, the action must be that of a true solvent and must not develop chemical changes either in its own structure or that of the components of the coal. These features could be most readily indicated by noting (1) the amount of material dissolved, and (2) the sum of the extract and residue; this sum should correspond substantially to the weight of the original material, provided no chemical action had taken place. The refinements in analytical methods in the preliminary work were not developed beyond the point necessary to determine the applicability of the extractive agent to the work in hand. The solvents used are given in the order of their effectiveness, the most active being placed first: phenol, ortho-cresol, low boiling tar distillate, para-cresol, pyridine, phenol-toluene mixture, aniline, methyl aniline, acetone, toluene, benzene, carbon disulphide, and turpentine.

Para-cresol, ortho-cresol, and low boiling tar distillate decomposed during the extraction, thus apparently increasing the amount of extracted material. Ortho-cresol gave but little decomposition product and as a solvent it ranked next to phenol. Methyl aniline and aniline presumably underwent decomposition.[1] Pyridine and phenol-toluene mixture gave a certain amount of extract but it was not as large as that obtained by use of phenol alone. Furthermore, the increase in the amount of nitrogen which is found in both the extract and residue after treatment with pyridine must be taken as an index of the formation of substitution products not originally present in the coal substances.[2] Acetone, benzene, carbon disulphide and turpentine extracted only small amounts of material and were therefore discarded.

1. Lewes, Progressive Age, 29, 1033.
2. Lewes, Progressive Age, 29, 1033.
Clark and Wheeler, Journal Chem. Soc., 103, 1709.

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6. *Phenol and the Conditions which Influence the Amount and Character of Extract.*—As a result of the preliminary work, it was evident that phenol was the chemical best suited for use as an extractive agent. A study, therefore, of the ultimate requirements necessary in its use, was the next step to be undertaken. The fundamental conditions to be observed were found to be (a) time for the solvent act, (b) temperature, (c) presence or absence of air in any part of the process. The effect of these various conditions may be shown as follows:

(a) Time of Extraction.

In order to determine the amount of material extracted for various lengths of time, five grams of coal were treated for successive periods of five hours each, and the amount of extract determined. At the end of twenty hours the extraction was discontinued and the weight of the extract determined. The results of the extraction were as follows:

TABLE 1.

WEIGHT OF MATERIAL DISSOLVED FROM COAL USING PHENOL FOR FIVE HOUR PERIODS.

Weight of the extract after 1st five hours	1.188 grams
" " " " " 2nd " "	.110 "
" " " " " 3rd " "	.040 "
" " " " " 4th " "	.027 "

It will be seen from Table 1 that during the last five hours of extraction only .027 gram of material was obtained. To determine whether this last increment was true extract or decomposition products of the solvent, the phenol was distilled alone for ten hours as a blank without any coal in the extraction tube, and the phenol distilled off at the end of that time. The weight of the residue in the flask was .03 gram. This weight of decomposition product obtained after ten hours of distillation would represent an equivalent of .015 gram for five hours of distillation. When we consider that the amount of apparent extract during the last five hours was only .027 gram, if .015 gram of this were derived from the phenol, then only .012 gram would be the weight of the substance actually dissolved during that period. It was decided, therefore, to stop the extraction at the end of twenty hours and this length of time for the extraction process was used in all of the work.

(b) Temperature of the extraction.

In the earlier part of the work it was found that there was a

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larger amount of material extracted from the coal if the solvent were applied hot, as at 110°C,[1] instead of at 20° to 25°. The use of the higher temperature was accomplished by condensing the phenol at the temperature of boiling toluene, i. e., 110°. By causing the condensate at this temperature to flow upon the coal, which in turn was held in the vapors of the phenol, the temperature of the solvent was thereby always kept above 110°. The weight of extract and residue from two coals, when extracted with phenol at the two different temperatures, is given in the following table:

TABLE 2.

WEIGHT OF EXTRACT AND RESIDUE FROM COAL EXTRACTED AT DIFFERENT TEMPERATURES.

Coal	Divisions	Extracted at 25° C.	Extracted at 110° C.	Difference
A	Extract	1.564	1.691	+ 0.127 = + 2.56%
	Residue	3.543	3.451	— .092 = — 1.84%
B	Extract	1.326	1.458	+ 0.132 = + 2.64%
	Residue	3.788	3.674	— 0.114 = — 2.28%

It will be noticed that with both coals the weight of residue decreased and that the weight of extract increased when the extraction was carried out at the higher temperature. The discrepancies are to be referred to the decomposition resulting from the continuous boiling of the phenol. This topic is discussed more fully later. Although the additional amount of extract was not large, it was thought advisable to carry out the extraction at the higher temperature even though the apparatus which was necessary for doing it was slightly complicated.

(c) Presence of Air in the Extraction Apparatus.

In some of the preliminary experiments it was found that the weight of extract and residue combined was equal to more than the original weight of the coal. This would doubtless be due to the oxidation of some of the substances involved, i. e., extract, residue, or phenol. The extraction was, therefore, carried out in the absence of air, carbon dioxide being conducted through the apparatus. The following results show very plainly that there was an increase of weight in both the extract and residue when the distillation was carried out in the presence of air. The test was made on five grams of two different coals. These results are given in Table 3.

1. All temperature readings in this discussion are Centigrade.

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FIG. 1.
EXTRACTION APPARATUS
READY FOR OPERATING

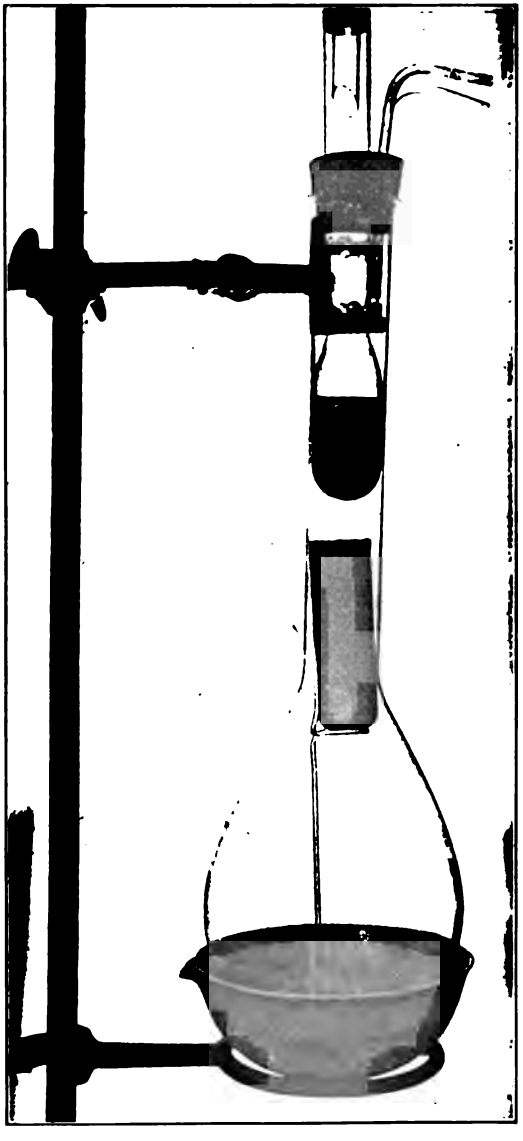

FIG. 2.
CONDENSER DEVICE FOR INCREASING THE TEMPERATURE OF THE SOLVENT

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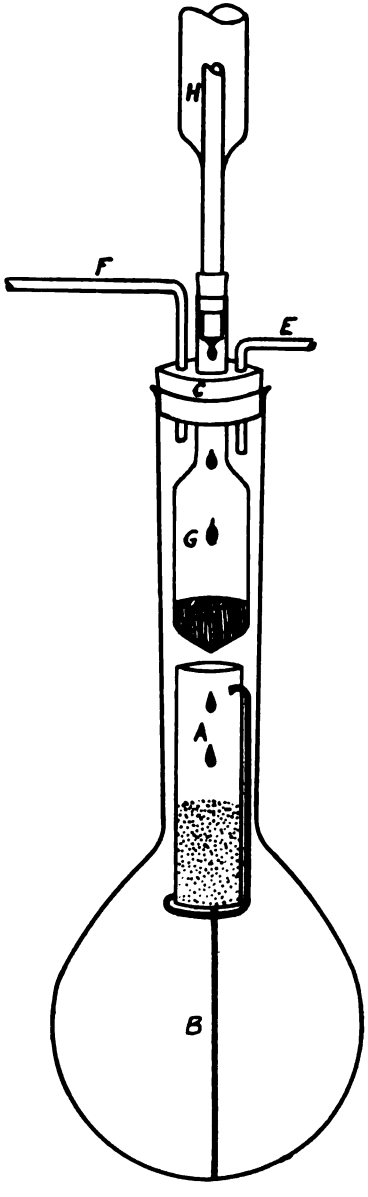

FIG. 3.
CONTINUOUS EXTRACTION
APPARATUS USING PHENOL AT 110°.

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TABLE 3.

COAL EXTRACTED WITH PHENOL IN THE PRESENCE OF AIR AND CARBON DIOXIDE.

Coal	Residual Substance	In CO_2	In Air	Increase
A	Extract	1.564	1.614	.050 = 1.00 %
	Residue	3.543	3.691	.148 = 2.96 %
	Total	5.107	5.305	.198 = 3.96 %
B	Extract	1.326	1.389	.063 = 1.26 %
	Residue	3.788	3.921	.133 = 2.66 %
	Total	5.114	5.310	.196 = 3.92 %

7. *Apparatus.*—From the preliminary work the apparatus shown in Figs. 1, 2, and 3 was decided upon as being the best adapted for carrying out the process. Figs. 1, 2, and 3 show different views of the apparatus. An extraction cone (A), 25 by 80 cm., was supported in the neck of a 750 cc. Kjeldahl flask by means of a bent glass rod (B). A No. 7 rubber stopper (C) which contained three holes was placed in the neck of the flask. Two of the holes were for small glass tubes (E and F) through which carbon dioxide could be conducted. The one large hole in the center was for the upper end of a glass bulb (G) which acted as a condenser for the phenol. This condenser-bulb was almost as large as the neck of the flask and was drawn out at the upper end to fit a water condenser. It was necessary that this bulb should be large enough to hold about 25 cc. of toluene. The toluene itself was condensed by an ordinary 70 cm. water condenser. A Woods' metal bath was used for boiling the phenol. The whole flask was protected from draughts by asbestos paper.

8. *Process of Extraction.*—One important problem which presented itself in the extraction of coal with phenol was the matter of freeing the residue and the extract from the solvent. Obviously, the extract and the residue must be treated differently because of the nature of each. In spite of the fact that all precautions as to oxidation, decomposition, etc., were observed during the extraction, it was found that the sum of the weight of extract and residue was larger than the original weight of coal. This fact seemed to indicate that phenol was being retained in either the extract or the residue by a physical or chemical union. If the phenol was being held chemically, it could not be used as a solvent. Therefore, it was necessary to determine whether such was the case or not.

After the extraction, the residue was washed first with alcohol and then with ether in a Soxhlet extractor. The residue was subsequently dried in the air for one hour at 100°. It was found that this method of freeing the residue from phenol could not be used for two

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reasons; even after the washing process and drying at 100° traces of phenol remained in the residue, and there was also the possibility of oxidation of the finely divided residue during the heating. In order to remove the adhering phenol the method would seem to be that of heating to a temperature above the boiling point of the solvent, i. e., to approximately 200°. Moreover by heating the residue which had been treated as above described to 200° traces of phenol were shown to be given off. The possibility of heating to this temperature raises the questions (1) of oxidation and (2) of decomposition of the material. The first point could easily be guarded by use of an inert atmosphere such as carbon dioxide. In the presence of such an atmosphere, heating to 200° could be carried on without decomposition as is shown by the following experiments.

TABLE 4.

SHOWING THE EFFECT OF HEATING COAL SUBSTANCE TO 200° IN AN ATMOSPHERE OF CO_2.

Residue	Weight of Residue	Time of Heating	Weight after Heating	Difference
A	4.125	3 hours	4.121	— .004
B	4.120	2 "	4.123	+ .003
C	4.200	1 "	4.200	.000
D	3.994	2 "	3.981	— .013

As a result of these experiments the following procedure was adopted. After being washed with ether, the residue was dried for a short time in the air to allow the ether to evaporate and was then transferred to a flask. It was kept at 200° for one hour with a slow stream of dry carbon dioxide passing through the apparatus. At the end of one hour the flask was allowed to cool and the carbon dioxide displaced by dry air. The flask was then weighed. Because of the very hygroscopic nature of the material, it was necessary to protect it from all possible sources of moisture during the drying process.

To free the extract from phenol the first method used was to distil at reduced pressure. After practically all of the phenol had been driven off under this condition the temperature was raised to 200° while a current of carbon dioxide was conducted through the the flask. It was thought that by simply bringing the mass to that temperature all of the phenol would be driven off. However, it was soon found that this was not the case. The heating of the extract was then continued for one hour in a stream of carbon dioxide, in a manner

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exactly similar to that used with the residue. It was found that this method of treatment gave a lower weight for the extract. Although phenol was given off during the heating, it was not known what effect heating to 200° would have upon the composition of the material. To test the matter, the extract after being dried at 200° for one hour was again heated for a certain length of time under the same conditions. The results from some of the extracts as given in Table 5 show for the length of time indicated, that the decomposition, if there was any, was not large.

TABLE 5.

SHOWING THE EFFECT OF HEATING THE EXTRACT TO 200° IN AN ATMOSPHERE OF CO_2.

Extract	Weight of Extract	Time of Heating	Weight after Heating	Difference
A	.895	1 hour	.893	—.002
B	1.027	1 "	.997	—.030
C	.932	2 "	.932	—.000

9. *Method of Procedure.*—As a result of the above experiments the following process was uniformly followed. Five grams of coal ground to 100 mesh, which had been dried at 105° for one hour, were extracted with 50 to 60 cc. of redistilled phenol. The extraction cones, previous to use, were dried and weighed with the exclusion of moisture. After extraction with phenol, the cone with the coal was placed in a Soxhlet extractor and washed for two hours with alcohol and then for two hours with ether. The ether was allowed to evaporate from the cone for a short time and the residue, when almost dry, was shaken out into a 100 cc. Erlenmeyer flask. The residue was then dried for one hour at 200° in a slow stream of carbon dioxide. The flask was allowed to cool and the carbon dioxide displaced by dry air. The flask was then weighed and the residue sealed in a small sample tube. The weight of the small amount of residue adhering to the cone was obtained by drying the cone at 105° and weighing. The phenol containing the extract was poured into a 200 cc. round-bottomed flask. The last trace of the extract was washed out of the Kjeldahl flask with pure phenol. The phenol was then distilled at reduced pressure (15 to 25 mm.) in the presence of carbon dioxide. The temperature of the Woods' metal bath was then raised to 200°, and carbon dioxide conducted through the flask to remove the last traces of phenol vapor. The flask was then transferred to an oven and maintained for one hour at 200°, while a slow current of carbon dioxide

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was continued through the flask. After cooling, the carbon dioxide was displaced with dry air and the flask weighed. The extract was pulverized and preserved in sealed tubes.

After all the details of the apparatus had been worked out and the sources of error in the method corrected as far as possible, a number of coals were extracted. The method just outlined was followed in detail. The extractions were carried out to test the constancy of the results and to find the limits of accuracy by comparing the sum of extract and residue with the amount of coal taken. The results are shown in Table 6.

TABLE 6.

WEIGHT OF EXTRACT AND RESIDUE FROM FIVE GRAMS OF COAL.

Table No.	Sample	Weight of Extract	Weight of Residue	Total Weight
1	Williamson Co., Ill.	1.120	4.006	5.126
2	Macoupin Co., Ill.	1.458	3.674	5.132
3	Madison Co., Ill.	1.691	3.451	5.142
4	Vermilion Co., Ill.	1.826	3.236	5.062
5	Madison Co., Ill.	1.446	3.642	5.088
6	Logan Co., Ill.	1.887	3.219	5.106
7	Jackson Co., Ill.	1.246	3.878	5.125
8	Sangamon Co., Ill.	1.959	3.139	5.098
9	Montgomery Co., Ill.	1.559	3.498	5.057
10	Williamson Co., Ill.	1.327	3.749	5.076
11	Vermilion Co., Ill.	1.937	3.104	5.041
12	Vermilion Co., Ill.	1.689	3.372	5.061
13	Williamson Co., Ill.	1.616	3.491	5.107
14	Pocahontas, W. Va.	.090	4.923	5.013
15	Lignite, Wyo.	.630	4.559	5.189

It will be noted in the above table that the weight of extract combined with the weight of residue was in all cases above the weight of coal taken. This is to be accounted for in the following manner. Experiments already given (page 7) showed that when 50 cc. of phenol were distilled for five hours that the amount of decomposition product was .015 gram. For twenty hours the amount of decomposition product would be .06 gram provided the decomposition was regular and that the presence of coal did not in any way tend to increase the rate of decomposition. It must be admitted that the alteration of phenol during all of the extractions would not be the same for the reason that the rate of distillation could not be controlled accurately and as a result the decomposition would vary somewhat in amount. On examining the results above it will be seen that the sum of the weight of extract and residue was from .013 to .189 gram in excess of the original weight of the coal. These two results are extreme cases. Now granting that the amount of decomposition of phenol would

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vary from one-half to two times the amount determined in the blank test, then the excess of weight from the process would be substantially accounted for by referring it to the phenol residue. Furthermore, oxidization or absorption of water by either the extract or residue would cause an increase in weight and would tend to augment the plus error observed.

It might be well to note in this connection that since the decomposition product from the phenol will appear in the extract only, a corrected weight of the extract would therefore be obtained by subtracting the weight of residue from the original weight of the coal. Thus the weight of the coal minus the insoluble residue is the corrected factor showing the degree of extraction.

B. Application of the Method to Different Types of Coals.

While the preliminary work was done on two coals and though the methods were not as accurate as those finally devised, it could be easily seen that the two coals contained markedly different amounts of material soluble in phenol. One coal was from the central and the other from the southern part of the state. As a means of comparison of coals from different localities it was decided to make an extraction of coals from the same and from different districts. Samples of Illinois coals were, therefore, obtained from local dealers. The most recent shipments and large lumps were selected as best suited to the purpose. Samples of Pocahontas coal and lignite were also obtained. The name, location, volatile matter, fixed carbon, and ash of the coals are given in the following table.

Table 7.

Showing Analysis of Coals. Dry Basis.

Table No.	Source of Sample	Ash per cent	Volatile Matter per cent	Fixed Carbon per cent
1	Williamson Co., Ill.	10.41	36.86	52.73
2	Macoupin Co., Ill.	13.54	40.12	46.34
3	Madison Co., Ill.	11.65	43.06	45.29
4	Vermilion Co., Ill.	8.68	44.36	46.96
5	Madison Co., Ill.	11.73	37.95	50.32
6	Logan Co., Ill.	8.74	41.99	49.27
7	Jackson Co., Ill.	6.01	34.93	59.06
8	Sangamon Co., Ill.	5.18	44.48	50.34
9	Montgomery Co., Ill.	11.84	42.12	46.02
10	Williamson Co., Ill.	7.88	36.78	55.34
11	Vermilion Co., Ill.	3.30	47.70	49.00
12	Vermilion Co., Ill.	6.74		
13	Williamson Co., Ill.	7.85	37.85	54.30
14	W. Va. Pocahontas	13.38	16.80	69.82

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The above coals were extracted with phenol. From the amounts of extract and residue as given in Table 6 the percentage amounts of each substance may be calculated, but on account of the variations in the ash it will be better to make such calculations on the ash free basis. It will be noted in Table 7 that the amounts of ash in these coals varied widely, from 3.30 to 13.54 per cent. Since the percentage of ash in the extract is seldom over 0.2 per cent it is quite evident that the phenol does not exert an appreciable solvent action on the mineral constituents in the coal.[1] Therefore, it is present simply as an inert material and serves to dilute the coal substance. If the amounts of residue and extract in one coal are to be compared with those same constituents in another coal, it can be done only after the residue and extract in each have been calculated to an ash free basis.

The following formula was used in this calculation:

$$\frac{\text{Per cent Residue as weighed} - \text{per cent ash in coal}}{100 - \text{per cent ash in coal}} = \text{Ash free Residue}$$

The corrected ash free extract was found as indicated on page 15 by subtracting the per cent of ash free residue from 100. As stated above the extract is not entirely free from ash, but as this was small in amount it was not considered in these calculations. The weight of residue and extract for Illinois coals were taken from Table 6 and calculated to an ash free basis. The results in the order of decreasing residue and increasing extract are presented in Table 8.

TABLE 8.

SHOWING PERCENTAGE OF INSOLUBLE RESIDUE AND EXTRACT REFERRED TO THE ASH FREE SUBSTANCE.

Table No.	Description of Sample	Percentage of Ash free Residue	Percentage of Ash free Extract
1	Williamson Co., Ill.	77.81	22.19
7	Jackson Co., Ill.	76.14	23.86
10	Williamson Co., Ill.	72.85	27.15
2	Macoupin Co., Ill.	69.33	30.67
5	Madison Co., Ill.	69.24	30.76
13	Williamson Co., Ill.	67.26	32.74
9	Montgomery Co., Ill.	65.93	34.07
12	Vermilion Co., Ill.	65.09	34.91
3	Madison Co., Ill.	64.94	35.06
4	Vermilion Co., Ill.	61.37	38.63
6	Logan Co., Ill.	60.97	39.03
8	Sangamon Co., Ill.	60.76	29.24
11	Vermilion Co., Ill.	60.80	39.20

1. Frazer and Hoffman, Bureau of Mines, Technical Paper 5, p. 8.

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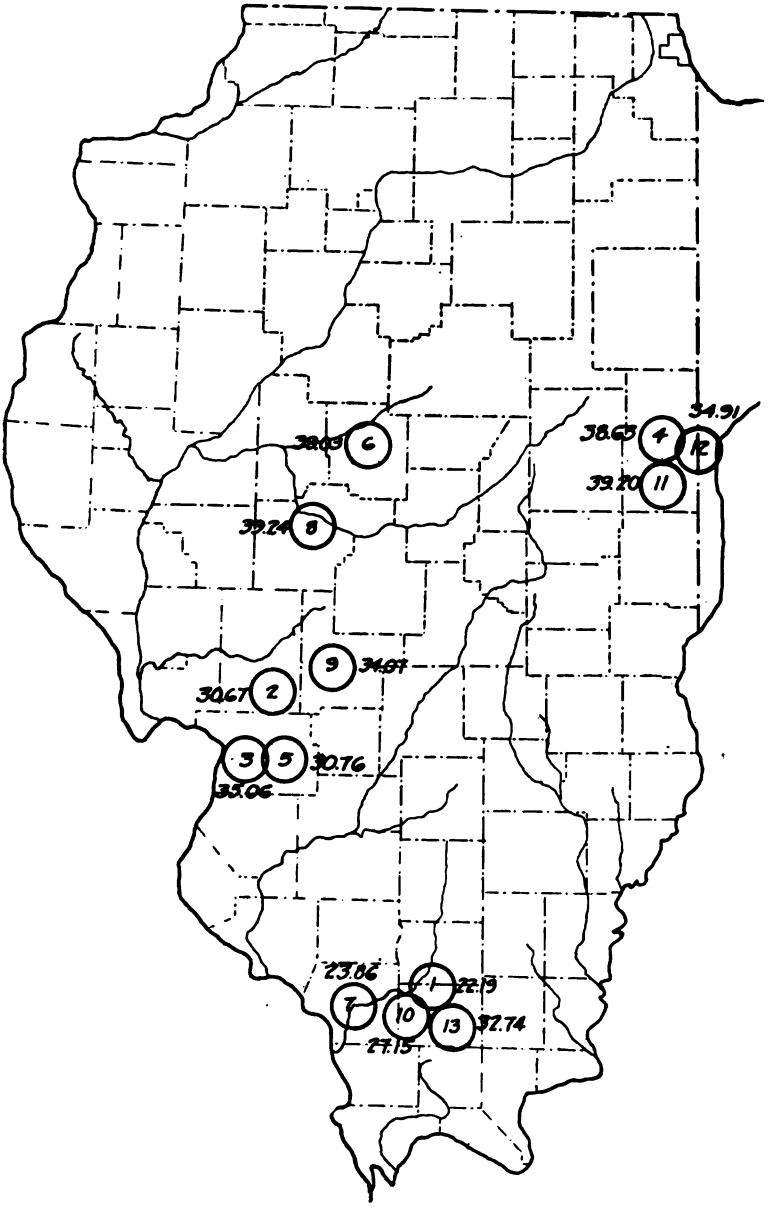

FIG. 4.
LOCATION OF THE ILLINOIS COALS.

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For purposes of geographical comparison the location of the Illinois coals together with the percentage of ash free extract has been shown on a map (page 17). The coals may be divided into three classes; (1) those low in extract; (2) those medium in extract; and (3) those high in extract. The class low in extract, i. e., from 20-30 per cent, is located in the extreme southern end of the State and is confined as far as the tests extend to Jackson and Williamson counties. The class of medium extract, i. e., 30-35 per cent., is located in Madison, Macoupin, and Montgomery counties. The class high in extract, i. e., above 35 per cent, extends through the middle of the State. The coals which gave the high amount of extract were from Sangamon, Logan, and Vermilion counties.

From this preliminary survey of the coals it seems that this method of analysis may lead to a much better comparison of coals or a closer subdivision of varieties than can be shown by the ordinary methods of analysis. Of course, the study must be much wider in scope before a fixed and definite conclusion can be drawn as to its applicability in this connection.

C. GENERAL PROPERTIES OF EXTRACT AND RESIDUE.

10. *Relation of Extract and Residue to Coking Properties.*—The relation of extract and residue to the coking properties was studied by making volatile matter determinations according to a specified method.[1] The volatile matter determination was useful in two ways: (1) the percentage of volatile matter was indicated, and (2) the nature of the coke produced could be roughly determined by examining the material remaining in the crucible. The percentage of volatile matter will be given in another connection.

The "coke" produced from the volatile matter determination of the insoluble residue showed very little coherence. The so-called "coke" was in fact not a coke at all. Its tendency to hold together depended to a certain extent upon the coal from which the residue had been obtained, for in some cases the material fell to pieces on being dumped out of the crucible and in all cases it could be disintegrated between the fingers. It was much duller in appearance than ordinary coke. The residue, therefore, could properly be designated as non-caking and in consequence non-coking.

The extract and residue differed widely in character. When heated to 300° to 350°, the extract softened but it did not produce a sticky material at first. Later, as the temperature was raised, it became sticky. On cooling it again assumed the compact brittle state, which

1. Stanton and Fieldner, "Methods of Analyzing Coal and Coke", Technical Paper 8, Bureau of Mines (1912).

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it originally possessed. The extract when subjected to the volatile matter test swelled to three or four times its original size and gave a large amount of volatile matter. The material remaining after the test possessed a bright, shiny appearance. It was very friable. However, it was a material entirely different from that produced from the residue.

If the residue and extract are mixed in the ratio in which they occurred in the coal and a volatile matter determination made, the material remaining is a coke about as strong in character and similar in appearance as that produced by the coal itself. From this it is evident that the coking constituent is the soluble extract and that its function is that of a cementing substance.

11. *Effect of Heating the Residue and Extract in the Air.*—It is well known that coal oxidizes readily in the air at 200°. It was desired to learn the behavior of the residue and extract when heated under the same conditions. The residue was weighed and heated in contact with air in an oven at 200° for successive one hour periods. The results are given in the following table.

TABLE 9.
SHOWING THE INCREASE IN WEIGHT OF RESIDUE WHEN HEATED AT 200°.

	Weight	Increase
Weight of residue	0.6782	0.0000
after 1st one hour period	0.6868	0.0086
after 2nd one hour period	0.6903	0.0121
after 3rd one hour period	0.6926	0.0144
after 4th one hour period	0.6952	0.0170
after 5th one hour period	0.6960	0.0178
after 6th one hour period	0.6973	0.0191
after 7th one hour period	0.6982	0.0200

The extract was also heated to 200° in the air. The results for the extract are given in Table 10. The time of the first heating was 30 minutes and after that each successive period was one hour.

TABLE 10.
SHOWING THE INCREASE IN WEIGHT OF THE EXTRACT WHEN HEATED TO 200°.

	Weight	Increase
Weight of extract	0.5718	0.0000
after heating 30 min.	0.5720	0.0002
after 1st one hour period	0.5745	0.0027
after 2nd one hour period	0.5770	0.0052

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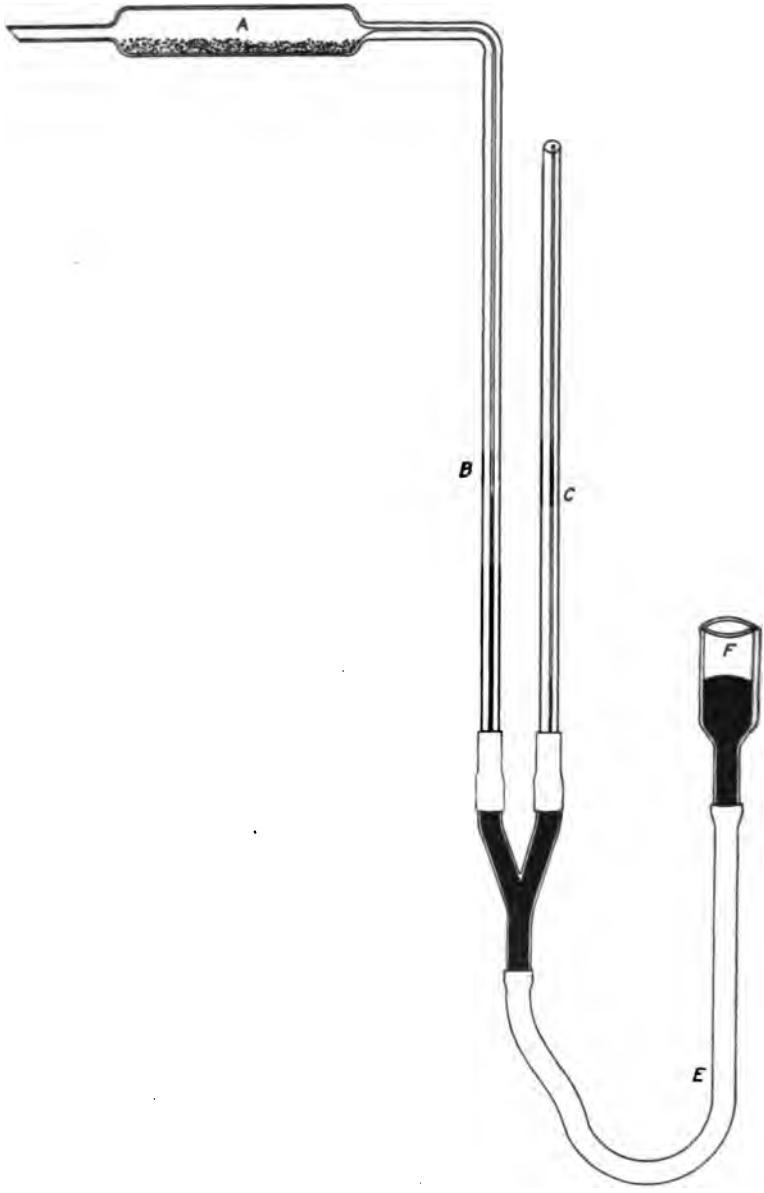

FIG. 5.
ABSORPTION OF OXYGEN BY COAL RESIDUES.

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It was thus shown that both the residue and extract increased in weight when heated and that the rate of increase was faster in the case of the insoluble residue. These experiments are taken to mean that the increase in weight is an indication of oxidation of residue and extract.

12. *Avidity of Residue and Extract for Water and Oxygen at Ordinary Temperature.*—The hygroscopic nature of the residue was one of the properties to be noticed first. On exposure to air it increased in weight rapidly, and on drying at 105° for half an hour most of the water was driven off. However, it did not return to its original weight on drying, so that the excess in weight was probably caused by oxidation of the material.

The hygroscopic nature of the extract was also noticeable but it was not as marked as that of the residue. The extract increased in weight when exposed to the air, and the moisture could be driven out again by drying at 105° for thirty minutes.

Table 11 shows the increase in weight of extract and residue when exposed to the moisture of the air and the decrease in weight when they were dried at 105° for 30 minutes.

TABLE 11.

EFFECT OF OXIDATION ON RESIDUE AND EXTRACT.

Substance	Weight	Weight after Exposure	Increase	Weight after Drying	Variation in Weight from Initial Material
Residue	.3029	.3129	.0100	.3037	+ .0008
Extract	.2467	.2485	.0018	.2462	— .0005

Oxidation of the residue and extract at ordinary temperature was also studied. Because of the hygroscopic nature of the residue and extract, it was difficult to determine whether oxidation was actually taking place or not. The rate of oxidation, if it occurred, would necessarily be slow at room temperature. Several experiments were performed for the purpose of determining whether or not the material did oxidize. Obviously, the experiment must be so carried out that moisture increases may be obviated in order to know definitely that the material had oxidized and had not taken up moisture.

The apparatus for showing the absorption of oxygen consisted of a barometer for measuring diminished pressure in a sealed space, which contained oxygen and the material to be tested. The plan of such an apparatus is shown in Fig. 5. The apparatus consists of

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a 10 cc. pipette (A) fused to a capillary tube (B) 900 mm. in length. The capillary tube is connected at the bottom by a Y-tube to another capillary tube (C) which serves as a leveling tube and through which oxygen can be introduced into the apparatus. The Y-tube is connected by a rubber tube (E) to a leveling bulb (F).

In setting up the apparatus the procedure was as follows: One gram of extract (or residue) was introduced into the pipette and the end sealed off. Mercury was then placed in E and F, and a pinch clamp placed on E. The height of the mercury was such that it did not disconnect B and C. The apparatus was then evacuated repeatedly and filled five or six times with commercial oxygen which was dried by passing it through sulphuric acid. This procedure was followed to displace the air in the apparatus with oxygen. Finally a very slightly diminished pressure was maintained in the apparatus, the pinch cock on E opened, and the leveling tube raised. If this procedure were carried out properly, the mercury in B would stand a few mm. above the rubber connection and the mercury in both the capillary tube C and the leveling bulb F would stand at the same level. At certain intervals simultaneous temperature and barometric readings were taken and the true pressure calculated.

Table 12 shows the results of an experiment on the extract.

TABLE 12.

SHOWING THE AVIDITY OF THE EXTRACT FOR OXYGEN MEASURED IN DIMINISHED PRESSURE (mm.).

Time after Standing.	Diminished Pressure in mm.
0	0
23 hours	36
64 "	56
88 "	76
112 "	91
136 "	96
208 "	125

TABLE 13.

SHOWING THE AVIDITY OF THE RESIDUE FOR OXYGEN MEASURED IN DIMINISHED PRESSURE (mm.).

Time after Standing.	Diminished Pressure in mm.
0	0
26 hours	86
49 "	123
90 "	154
114 "	188
138 "	209
160 "	220
232 "	269

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Fig. 7.
Coke from Vermilion County Coal.

Fig. 8.
Insoluble Residue from Vermilion County Coal—After Destructive Distillation.

Fig. 9.
Phenol Soluble Extract from Vermilion County Coal—After Destructive Distillation.

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FIG. 10.
COKE FROM VERMILION COUNTY COAL.

FIG. 11.
INSOLUBLE RESIDUE FROM WILLIAMSON COUNTY COAL—AFTER DESTRUCTIVE DISTILLATION.

FIG. 12.
PHENOL SOLUBLE EXTRACT FROM WILLIAMSON COUNTY COAL—AFTER DESTRUCTIVE DISTILLATION.

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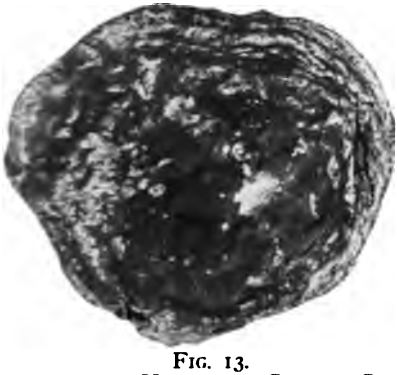

Fig. 13.
Coke from Vermilion County Coal
Without Previous Heating

Fig. 14.
Same as in Fig. 13.
Previously Heated 26 Hours at 105°.

Fig. 15.
Same as in Fig. 13.
Previously Heated 50 Hours at 105°.

Fig. 16.
Same as in Fig. 13.
Previously Heated 120 Hours at 105°.

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FIG. 17.
COKE FROM WILLIAMSON COUNTY COAL
WITHOUT PREVIOUS HEATING.

FIG. 18.
SAME AS IN FIG. 17.
PREVIOUSLY HEATED 26 HOURS AT 105°.

FIG. 19.
SAME AS IN FIG. 17.
PREVIOUSLY HEATED 50 HOURS AT 105°.

FIG. 20.
SAME AS IN FIG. 17.
PREVIOUSLY HEATED 120 HOURS AT 105°.

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Fig. 21.
DM Soluble Extract, Vermilion
UNTY Coal. No Previous
Heating of Extract.

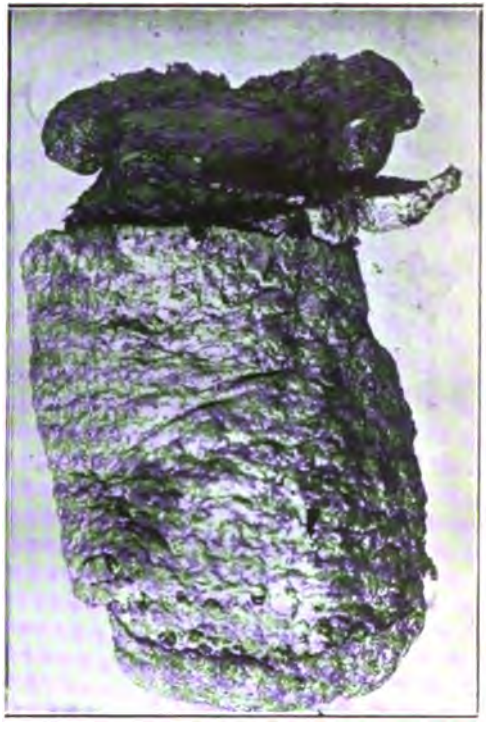

Fig. 22.
Same as in Fig. 21.
Previously Heated 50 Hours at 105°.

Fig. 23.
Same as in Fig. 21.
LY Heated 122 Hours at 105°.

Fig. 24.
Same as in Fig. 21.
Previously Heated 216 Hours at 105°.

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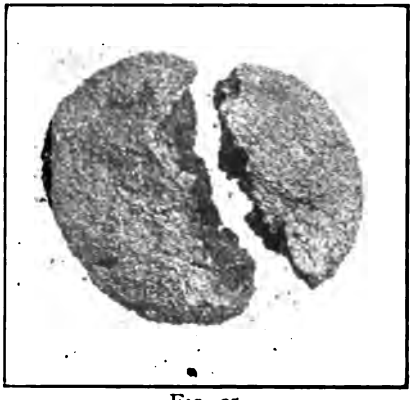

FIG. 25.
COKE FROM INSOLUBLE RESIDUE, VERMILION COUNTY COAL. NO PREVIOUS HEATING OF RESIDUE.

FIG. 26.
SAME AS IN FIG. 25.
PREVIOUSLY HEATED 50 HOURS AT 105°.

FIG. 27.
SAME AS IN FIG. 25.
PREVIOUSLY HEATED 122 HOURS AT 105°.

FIG. 28.
SAME AS IN FIG. 25.
PREVIOUSLY HEATED 216 HOURS AT 105°.

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FIG. 29.
COKE FROM SOLUBLE EXTRACT, WILLIAMSON COUNTY COAL. NO PREVIOUS HEATING OF EXTRACT.

FIG. 30.
SAME AS IN FIG. 29.
PREVIOUSLY HEATED 90 HOURS AT 105°.

FIG. 31.
SAME AS IN FIG. 29.
PREVIOUSLY HEATED 243 HOURS AT 105°.

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Fig. 32.
Coke from Insoluble Residue, Williamson County Coal. No Previous Heating of Residue.

Fig. 33.
Same as in Fig. 32.
Previously Heated 90 Hours at 105°.

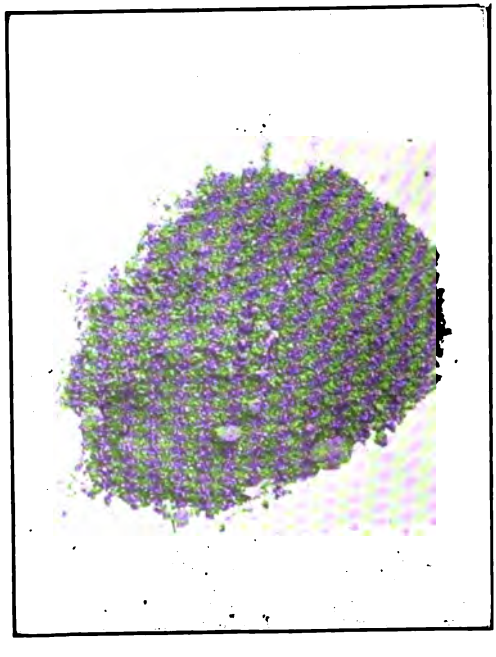

Fig. 34.
Same as in Fig. 32.
Previously Heated 242 Hours at 105°.

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For purposes of comparison of the absorption rates of extract and residue, the above results were plotted. Plotting time against diminished pressure in millimeters, it is seen that the rate of absorption was faster in the case of the residue.

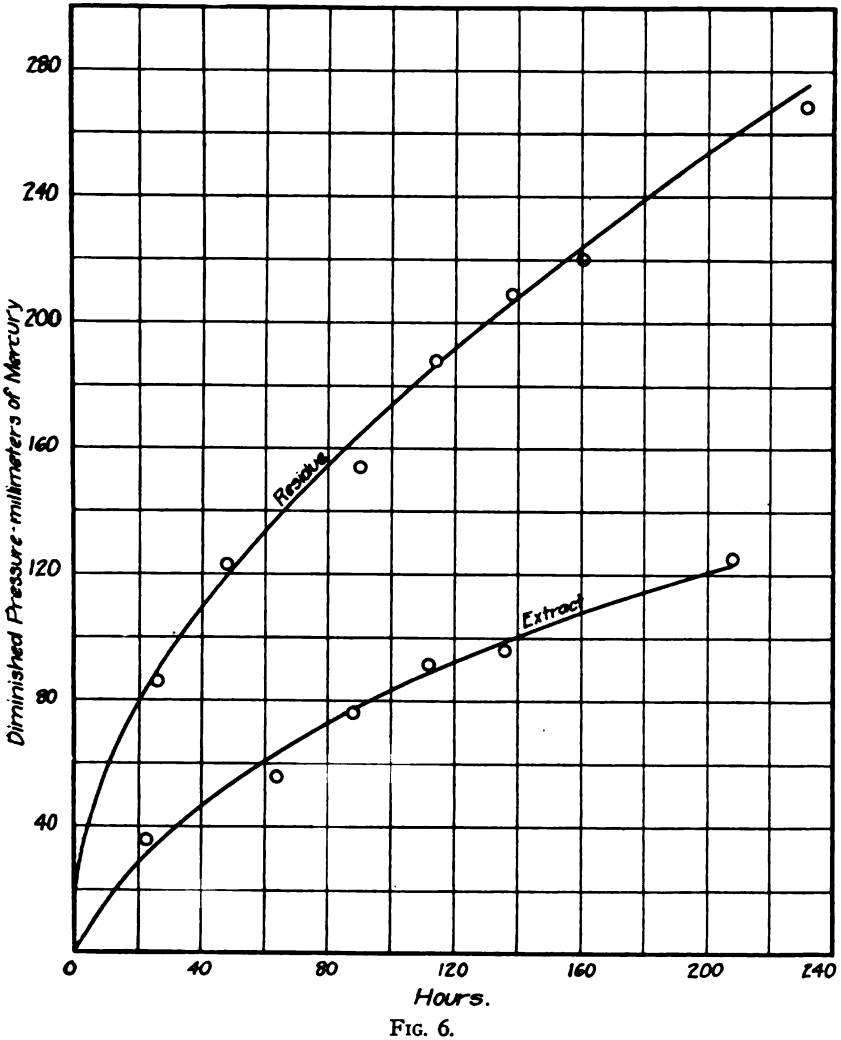

FIG. 6.
RATE OF ABSORPTION OF OXYGEN BY RESIDUE AND EXTRACT.

The results as given in Tables 12 and 13 show that oxygen was taken up by both the residue and extract. The amount of oxygen that

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had been absorbed when the experiment was stopped was about 4-5 cc. in the case of the residue, and 2-2½ cc. in the case of the extract. The experiment does not show absolutely that oxidation had taken place, but from our knowledge of coal we would be led to suspect that such was the case. At any rate it can be said that the coal was either oxidized or that it was holding the oxygen in an occluded state. Further experiments given below show that the reaction was one of oxidation.

13. *Volatile Matter Determination of Coal, Residue, and Extract.*—Something of the relation of residue and extract to coking properties has already been discussed. Further data as to the relative amounts of volatile matter in the two divisions were desired.

The percentage of volatile matter and fixed carbon in the residue and extract from different coals was therefore determined and for purposes of comparison, volatile matter determinations were also made on the coals under examination. These results are shown in Table 14.

The fuel ratio or the ratio of fixed carbon to volatile matter is also given in Table 14. The use of this ratio is advantageous for the reason that we have in it a method of comparison which does not readily appear from the percentages of fixed carbon and volatile matter alone.

TABLE 14.

SHOWING THE PERCENTAGE OF FIXED CARBON AND VOLATILE MATTER IN COAL, RESIDUE, AND EXTRACT.

Table No.	Substance	Fixed Carbon per cent		Volatile Matter per cent		Fuel Ratio	
6	Lincoln Coal	49.27		41.99		1.173	
	Residue		47.15		39.63		1.190
	Extract		47.25		52.39		.902
7	New Kentucky Coal	59.06		34.93		1.691	
	Residue		57.90		34.51		1.678
	Extract		51.52		48.42		1.064
8	Springfield Coal	50.34		44.48		1.132	
	Residue		50.26		41.90		1.200
	Extract		47.61		51.98		.916
10	Carterville Coal	55.34		36.78		1.504	
	Residue		54.83		34.77		1.577
	Extract		51.79		48.10		1.077
9	Montgomery Co. Coal	46.02		42.12		1.093	
	Residue		43.91		39.88		1.101
	Extract		48.82		50.45		.968
11	Catlin Coal	49.00		47.70		1.027	
	Residue		49.02		46.01		1.066
	Extract		47.35		52.30		.905
14	Pocahontas Coal	69.82		16.80		4.156	
	Residue		68.68		17.72		3.876

The fuel ratios of coal, residue, and extract are separately shown in Table 15. Here the results appear in such a form that the fuel

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ratio of one coal can easily be compared with that of another. Likewise, the fuel ratio of the residue and extract can be compared.

The fuel ratios for the several extracts are uniformly lower than the ratios for the residues. It is to be remembered that a decrease in the ratio number denotes an increase in the relative amount of volatile constituent. The table is made to show in the last column the relation between the various samples, the fuel ratio for the residue being used in each case as the basis of reference. It is quite possible that when reduced to this basis some idea may be had as to the coking property of a coal. While more data would be required on which to base a conclusion, it may be said that the high percentage differences as shown in the table are consistent in representing the more pronounced coking property of the samples listed.

TABLE 15.
SHOWING THE FUEL RATIO OF COAL, RESIDUE, AND EXTRACT.

Table No.	Description	Coal $\frac{C}{vol}$	Residue $\frac{C}{vol}$	Extract $\frac{C}{vol}$	Percentage of decrease of ratio in extract over ratio for residue referred to residue as basis
7	New Kentucky	1.705	1.678	1.062	40.3
10	Carterville Nut	1.504	1.578	1.077	38.3
9	Montgomery Co.	1.093	1.101	.968	26.5
6	Lincoln	1.173	1.190	.902	32.2
11	Catlin	1.027	1.066	.904	13.7
8	Springfield	1.133	1.200	.916	24.1

14. *Ultimate Analysis of Coal, Residue, and Extract.*—A difference in the chemical constitution of the residue and extract would be expected, when we consider the marked differences in their physical properties. In order to find out if there was a difference in the chemical constitution of these two substances, ultimate analyses were made of extract, residue, and coal from a number of mines as follows:

TABLE 16.
VERMILION COUNTY COAL.
SHOWING THE ULTIMATE ANALYSIS OF COAL, RESIDUE, AND EXTRACT.

Substance	Carbon	Hydrogen	Nitrogen	Sulphur	Ash
Coal	72.40	5.35	1.27	2.72	8.68
	72.36	5.34	1.16		
Residue	69.16	4.92	1.29	3.72	13.03
	69.08	4.90			
Extract	81.14	5.88	1.24	1.33	.53
	80.97	6.04			

These results were averaged and calculated to an ash free basis.

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TABLE 17.

VERMILION COUNTY COAL.

SHOWING THE ULTIMATE ANALYSIS OF COAL, RESIDUE, AND EXTRACT ON AN ASH FREE BASIS.

Substance	Carbon	Hydrogen	Nitrogen	Sulphur	Oxygen
Coal	79.26	5.86	1.32	2.97	10.59
Residue	79.48	5.68	1.48	4.28	9.12
Extract	81.48	5.99	1.25	1.34	9.94

The results of the ultimate analyses of Williamson County coal, residue, and extract are given in Table 18.

TABLE 18.

SHOWING THE ULTIMATE ANALYSIS OF COAL, RESIDUE, AND EXTRACT.

Substance	Carbon	Hydrogen	Nitrogen	Sulphur	Ash
Coal	74.58	4.72	1.63	2.35	7.85
	74.42	4.89			
Residue	72.46	4.70	1.53	2.52	11.35
	72.17	4.73			
Extract	82.69	5.54	1.73	1.42	.56
	82.60	5.54			

These results were averaged and calculated to an ash free basis.

TABLE 19.

WILLIAMSON COUNTY COAL.

ULTIMATE CONSTITUENTS OF COAL, RESIDUE, AND EXTRACT CALCULATED TO THE ASH FREE BASIS.

Substance	Carbon	Hydrogen	Nitrogen	Sulphur	Oxygen
Coal	80.85	5.21	1.77	2.55	9.62
Residue	81.57	5.32	1.72	2.84	8.55
Extract	83.11	5.57	1.74	1.43	8.15

On examining the results of the ultimate analysis, it will be seen that there was little difference in the proportion of the elements in the coal, residue, and extract. The percentage of carbon and hydrogen is slightly higher in the extract. However, because of the small variation no special significance can be attached to this fact.

The oxygen content was the chief point of interest to be consid-

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ered in the analysis. Previous work with solvents such as pyridine[1] and benzene[2] indicated that the material dissolved from coal by these solvents contained a smaller proportion of oxygen. Professor Lewes[3] states that "degradation products of the original vegetation are to be found in the bituminous coals, the residual body and humus forming the basis, which is luted together by the hydrocarbons and resins, and the characteristics of the various kinds of coal are dependent upon the proportion in which the four groups of the conglomerate are present". Assuming that the hydrocarbons and resins were the more soluble in phenol and would be dissolved by the phenol, it would be only natural to conclude that the extract would be low in the amount of oxygen it contained. Furthermore, if the bodies low in oxygen were dissolved, the residue would on that account contain a larger percentage of oxygen. When we consider that all the errors of the analysis are combined in the oxygen value, it would be impossible to state from the results of the above analyses that there was any difference in the percentage of oxygen in the extract from that of the residue.

It is to be noted that the percentage of sulphur in the residue is high. This is to be accounted for by the fact that the residue contained practically all of the ash that was present in the coal. That there was some sulphur in the extract was taken to indicate that a part of the sulphur in the coal was present in the organic form.

15. *Thermal Decomposition of Residue and Extract.*—The residue and extract were subjected to destructive distillation at 800° and the resulting gases analyzed. For purposes of comparison the coal was also treated in the same manner. The method for the destructive distillation of the material may be briefly described in the following manner.

One gram of the substance was placed in a hard glass test tube. Immediately above the substance was placed a short layer of asbestos which served the purpose of a tar scrubber and as a means of holding the extract in the lower part of the tube thus counteracting its tendency to foam. The tube was then connected by a rubber stopper, a glass stop cock, and a short piece of small rubber tubing to a 300 cc. gas holder which contained mercury. Previous to heating, the air was exhausted from the tube and from the connections by means of a filter pump. The test tube was then introduced into a gas furnace which had been previously heated to 800°.

The temperature was maintained between the temperature limits

1. Clark and Wheeler, J. of Chem. Soc., 103, 1709 (1913).
2. Pictet and Ramseyer, Arch. sci. phys. nat., 34, 234 (Chem. Abstracts 7, 1496) (1913).
3. Progressive Age, 29, 1032 (1911).

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of 780° and 810° throughout all of the experiments. The upper part of the test tube holding the rubber stopper was well insulated against the heat. As soon as the pressure in the tube was equal to the atmospheric pressure, the stop cock was opened and the gas collected. At the end of one hour the distillation was stopped. The gas was analyzed according to the usual procedure.

The average of duplicate determinations gave the following results for gases which were produced by the destructive distillation of Vermilion County coal, residue, and extract.

TABLE 20.

SHOWING THE COMPOSITION OF GASES PRODUCED BY DESTRUCTIVE DISTILLATION OF COAL, RESIDUE, AND EXTRACT.

Components of the Gas	Vermilion Co.	Residue	Extract
Carbon dioxide	18.1	15.1	12.9
*Ethylene	3.4	3.5	3.1
Oxygen	.5	.9	.1
Carbon monoxide	20.6	17.4	16.4
Hydrogen	35.6	39.2	47.8
Methane	17.4	18.1	13.4
**Ethane	1.6	1.5	2.8
Nitrogen	2.8	4.3	3.5

Coal, from Williamson County, residue and extract were also destructively distilled. The results of the analysis of the gas which was produced are given in Table 21.

TABLE 21.

SHOWING THE COMPOSITION OF GASES PRODUCED BY DESTRUCTIVE DISTILLATION OF WILLIAMSON COUNTY COAL.

Components of the Gas	Coal	Residue	Extract
Carbon dioxide	7.1	6.9	7.2
*Ethylene	3.1	2.9	2.6
Oxygen	.6	.8	.7
Carbon monoxide	12.8	16.5	11.8
Hydrogen	50.7	46.0	51.8
Methane	17.3	20.5	18.2
**Ethane	1.9	2.8	2.7
Nitrogen	6.5	3.6	5.0

* Including higher homologues and benzene.
** Including higher homologues calculated as ethane.

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The work of Clark and Wheeler[1] on the extract and residue which was obtained by the extraction of coal with pyridine, showed a marked difference in the percentage composition of the gases which were produced by the destructive distillation of these substances. The methane and ethane in the gas from the extract were larger in amount than that obtained from the gas of the residue.

From their results it may be supposed that similar results might be obtained by the destructive distillation of the residue and extract obtained by extraction of coal with phenol. The above results as indicated in Tables 20 and 21 do not show an increase in the amount of methane in the gas from the extract but instead there was a decrease in this constituent and an increase in the hydrogen content.

It will be observed in Table 20 that the amount of carbon dioxide was high in all of the gases. This fact taken in conjunction with the fact that the Vermilion County coal had been in a finely ground condition for some time suggested that oxidation had taken place. This led to a consideration of that problem which will be treated under a subsequent heading.

The properties of the material which remained in the test tube after thermal decomposition, were very characteristic.

The coal gave a good sound coke of shiny appearance.

The residue gave a very poor coke. It could be easily broken and possessed a dull appearance.

The extract gave a very light and friable material. It had a very smooth surface after the glass was broken away. The surface was much brighter than that of the coke produced from the coal.

Photographs of the residues are shown in Figs. 7-12. Fig. 7 is of Vermilion County coal; Fig. 8, residue; and Fig. 9, extract. Fig. 10 is of the Williamson County coal; Fig. 11, residue; and Fig. 12, extract.

16. *The Effect of Oxidation on the Volatile Matter Determination.*—From some of the previous work it became evident that oxidation was playing an important role in producing changes in the residue and extract. It is a well-known fact that when coal is exposed it gradually loses its coking properties, and that this is due to the absorption of oxygen. It was desired to know if this was oxidation or simple absorption of oxygen. The first experiments were made on the coal itself. It was desired to see what effect heating at 105° would have on the percentage of volatile matter.

Coals from Vermilion County, and Williamson County were heated in air at 105° for different lengths of time and volatile matter determinations were made at successive stages of oxidation. The results on these two coals are given in Table 22.

1. J. Chem. Soc., 103, 1710 (1913).

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TABLE 22.
SHOWING THE PERCENTAGE OF VOLATILE MATTER IN COAL AFTER CONTINUED HEATING AT 105°.

Time after Heating. Hours	Vermilion County Coal		Williamson County Coal	
	Volatile Matter in coal	Difference	Volatile Matter in coal	Difference
0	44.36	0.00	37.85	0.00
26	42.39	—1.97	39.22	+0.37
50	41.78	—2.58	36.75	—1.10
120	40.51	—3.85	35.91	—1.94

These results show that the volatile matter decreased with progressive heating. The plus result which was obtained in the case of the Williamson County coal was in all probability a variable in the volatile matter determination. From the results of these two coals the decrease in volatile matter was more rapid in the case of the coal which had the higher percentage in the beginning. The material remaining in the crucible after the determinations indicated very plainly that the length of time of heating affected the coking properties as shown below.

The Vermilion County coal gave a very poor coke after 120 hours' heating. Figs. 13 to 16 show the material after the volatile matter determination. Fig. 13, coal without heating; Fig. 14, after heating 26 hours; Fig. 15, after heating 50 hours; Fig. 16, after heating 120 hours.

The Williamson County coal after heating 120 hours would not coke. Figs. 17 to 20 show the material after the volatile matter determination. Fig. 17, coal without heating; Fig. 18, after heating 26 hours; Fig. 19, after heating 50 hours; Fig. 20, after heating 120 hours.

The study of the effect of oxidation on the amount of volatile matter was extended to the residue and extract. The residue and extract from the Vermilion County coal were heated for various lengths of time and volatile matter determinations made. The results are given in Table 23.

TABLE 23.
SHOWING THE PERCENTAGE OF VOLATILE MATTER IN THE RESIDUE AND EXTRACT AFTER HEATING AT 105°, VERMILION COUNTY.

Time of Heating. Hours	Volatile Matter in Residue	Difference	Volatile Matter in Extract	Difference
0	43.53	0.00	52.79	0.00
50	42.75	— 0.78	52.22	—0.57
122	61.08	+17.55	47.68	—5.11
216	50.45	+ 7.92	48.23	—4.56

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On examining these results it will be noted that the percentage of volatile matter in the extract decreased with the length of time of heating. It thus acted similarly to coal. On the other hand, the residue increased in the amount of volatile matter. This increase may be accounted for by the fact that during the volatile matter determination there was an excess of sparking. The material acted similarly to lignite coal when an attempt is made to coke a finely ground sample. There was evidence of considerable mechanical loss. The greater avidity of the residue for oxygen would also tend to increase the indicated amount of volatile matter by reason of the ultimate formation of CO_2 from the oxygen absorbed.

The material which remained after the volatile matter determination was of special interest. Figs. 21 to 24 show the extract after the volatile matter determination; Fig. 21, extract without previous heating; Fig. 22, after heating 50 hours; Fig. 23, after heating 122 hours; Fig. 24, after heating 216 hours. Heating the extract for 50 hours yields a more voluminous material by the volatile matter determination than is produced by the extract without heating. After heating 216 hours the extract had lost all tendency to swell but it still retained a slight coking tendency, i. e., the material did not fail to pieces.

Figs. 25 to 28 relate to the same source of material from Vermilion County, and show the material that remained after the volatile matter determination was made on the residue; Fig. 25 is the coke from residue without previous heating; Fig. 26, after heating 50 hours; Fig. 27, after heating 122 hours; Fig. 28, after heating 216 hours. If a volatile matter determination is made on the residue previous to heating a material is produced that will just adhere. After heating, the residue loses what little coking ability it originally possessed.

The residue and extract from Williamson County coal were also subjected to prolonged heating at 105°. The results are presented in Table 24.

TABLE 24.

SHOWING THE PERCENTAGE OF VOLATILE MATTER IN THE RESIDUE AND EXTRACT AFTER HEATING AT 105°, WILLIAMSON COUNTY COAL.

Time after Heating. Hours	Volatile Matter in Residue	Difference	Volatile Matter in Extract	Difference
0	37.44	0.00	47.21	0.00
90	56.40	+18.96	44.19	—3.02
242	67.33	+29.89	42.68	—4.53

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A discussion of these results on Williamson County coal will not be taken up for they are identical in character with those obtained from the Vermilion County coal.

Figs. 29 to 31 show the material which remained after the volatile matter determination of the extract from Williamson County coal. Fig. 29, extract without heating; Fig. 30, after heating 90 hours; Fig. 31, after heating 242 hours.

Figs. 32 to 34 show the material which remained after the volatile matter determination was made on the residue from Williamson County coal; Fig. 32, residue without heating; Fig. 33, after heating 90 hours; Fig. 34, after heating 242 hours.

Reviewing this phase of the study it is seen that Figs. 7 to 34 inclusive, illustrate the effect of oxidation such as may occur upon simple exposure to the air at room temperature and at 105°, and for varying lengths of time. The temperature employed for the destructive distillation was approximately 800°. Before applying this temperature the samples were subjected to a vacuum for the purpose of removing the surrounding oxygen as well as that which might have been occluded by the sample. The effect upon the insoluble residues of long exposure to a temperature of 105° is hardly noticeable so far as modifying the coking property of that material is concerned, for the reason that this substance, even without exposure to oxidation, is practically devoid of any tendency to coke when highly heated. Quite the contrary is true in the case of the soluble extract. Its tendency to coke and so to ultimately produce coke is marked, but after long exposure to oxidizing conditions this property is practically lost. This feature also characterizes the coal, the unseparated extractive material being sufficiently pronounced in its behavior to govern the effect of oxidation on the unheated coal.

17. *The Effect of Oxidation on the Amount of Material Extracted from Coal by Phenol.*—The further effect of oxidation was noticed in a number of different ways. For example, the amount of material which could be extracted from coal depended to a certain degree on the extent of oxidation.

Williamson County coal was heated for five days at 105° in an atmosphere of oxygen. The amount of material extracted from this coal is shown in Table 25.

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TABLE 25.

SHOWING THE EFFECT OF OXIDATION ON THE AMOUNT OF RESIDUE AND EXTRACT IN COAL, WILLIAMSON COUNTY.

	Coal not previously heated	Coal heated before extracting	Change
Weight of residue	3.777	4.102	+.325
Weight of extract	1.484	1.094	—.390

A finely ground sample of Vermilion County coal was kept in a closed jar for four months. This coal was extracted with phenol and the amount of residue and extract determined. The coal was also heated at 105° for varying lengths of time in oxygen and subsequently extracted with phenol. The results of these experiments are shown in Table 26.

TABLE 26.

SHOWING THE EFFECT OF OXIDATION ON THE AMOUNT OF RESIDUE AND EXTRACT IN COAL.

Condition of Coal	Weight of Residue	Increase in Weight of Residue	Weight of Extract	Loss in Weight of Extract
Fresh	3.236	0.000	1.826	0.000
Standing 4 mo.	3.494	+ .258	1.601	— .225
Heated 3 days	3.821	+ .585	1.325	— .501
Heated 6 days	3.860	+ .624	1.259	— .567
Heated 10 days	3.802	+ .566	1.242	— .584
Heated 15 days	3.795	+ .559	1.353	— .473

From the results in Tables 25 and 26 it is seen that the oxidation of coal increases the amount of residue insoluble in phenol, and that the decreased solubility becomes very marked when the coal is completely oxidized. Also it should be noted that in the sample of coal which was preserved in the sample bottle there was a decrease in the amount of soluble material. Thus the coal was being slowly oxidized although it was not directly exposed to the air. The fact that coal is so readily oxidized should be kept in mind in any work on coal and especially in work of the present nature.

18. *The Effect of Oxidation on the Composition of the Gases Produced by Destructive Distillation.*—During the work on the destructive distillation of coal as noted on page 28 one gas was produced which was high in the amount of carbon dioxide. This result, when considered in connection with the fact that the sample of coal was not

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fresh, and in the light of the preceding experiment, would indicate that oxidation had taken place. It was desired to see what effect oxidation of the extract and residue would produce in the composition of the gases when destructively distilled. In order to have a sample of coal which was not already partially oxidized, a fresh sample of Vermilion County coal was secured. The residue and extract from this coal were destructively distilled and the gases analyzed. The residue and extract were also heated at 105° for 24 hours and then destructively distilled. The results from the analyses are as follows:

TABLE 27.

SHOWING THE COMPOSITION OF GASES PRODUCED BY DESTRUCTIVE DISTILLATION OF RESIDUE AND EXTRACT, VERMILION COUNTY COAL.

Gas	From Residue not oxidized	From Residue oxidized by heating	From Extract not oxidized	From Extract oxidized by heating
Carbon dioxide	5.2	8.7	6.8	10.0
*Ethylene	4.8	4.6	5.1	4.2
Oxygen	.3	.3	.2	.4
Carbon monoxide	15.1	15.2	11.9	19.0
Hydrogen	44.1	43.0	40.5	44.8
Methane	26.2	24.0	24.0	17.4
**Ethane	3.6	3.3	7.3	3.2
Nitrogen	.7	.9	4.2	1.0

The residue and extract from the old sample of Vermilion County coal were heated for 48 hours at 105° in the air. They were then destructively distilled and the gases analyzed. For purposes of comparison some of the residue and extract which had not been heated were destructively distilled and the gases analyzed. The results are presented in Table 28.

TABLE 28.

SHOWING THE COMPOSITION OF GASES PRODUCED BY DESTRUCTIVE DISTILLATION OF RESIDUE AND EXTRACT, VERMILION COUNTY COAL.

Gas	From Residue oxidized by standing only	From Residue oxidized by heating	From Extract oxidized by standing only	From Extract oxidized by heating
Carbon dioxide	8.3	14.4	10.1	12.4
*Ethylene	5.2	3.5	4.0	3.2
Oxygen	.9	.1	.4	.3
Carbon monoxide	14.2	18.5	21.0	14.6
Hydrogen	39.8	40.5	40.8	50.0
Methane	23.1	20.4	16.3	15.6
**Ethane	3.4	1.4	2.8	2.5
Nitrogen	5.1	1.2	4.6	1.4

* Including higher homologues and benzene.
** Including higher homologues calculated as ethane.

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Extracts from the Vermilion County coal and also from Williamson County were heated for a much longer time and then destructively distilled. The extract from the Vermilion County coal was heated for 216 hours at 105° and that from Williamson County for 264 hours. The results are given in Table 29.

TABLE 29.
SHOWING THE COMPOSITION OF GASES PRODUCED BY THE DESTRUCTIVE DISTILLATION OF EXTRACT.

Gas	From Vermilion County Extract after being heated for 216 hours.	From Williamson County. Extract after being heated for 264 hours.
Carbon dioxide	9.1	7.9
Ethylene	3.7	3.8
Oxygen	.3	.4
Carbon monoxide	17.9	16.1
Hydrogen	44.2	47.5
Methane	19.9	20.9
Ethane	3.1	2.1
Nitrogen	1.8	1.3

On examining the data in Tables 27 and 28 it is to be noted that the amount of carbon dioxide in the gas is higher after the material has been heated. Also, the percentage of carbon monoxide increased in every case except one. The fact that the percentage of methane and ethane in the gas from the material that had been heated was reduced in every case, should also be noted. The above results seem to indicate that the oxygen taken up during the heating at 110° was not held in an absorbed condition but that it had entered into chemical combination with the material. Furthermore the apparatus was exhausted to 15-20 mm. before the destructive distillation was started, and when the temperature was raised, any oxygen which was merely occluded would be given off as oxygen before a temperature was reached at which active oxidation with the formation of CO_2 would take place. As a result of these experiments it may be stated that upon exposure to oxidizing conditions oxygen enters into chemical combination with both residue and extract.

The results in Table 29 seem to show that the oxidation may reach a point of saturation so far as the ability of the molecular structure is concerned and that beyond this point, if the oxidizing conditions are continued, a dissipation of the oxygen compounds may occur, presumably being discharged as H_2O. However, further proof of this reaction after such long continued periods of heating should be made.

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III. Summary and Conclusions.

(1) A study of solvents for the purpose of subdividing coal constituents without decomposition was carried out. Phenol was adopted as the most active solvent for the purpose of this investigation.

(2) An apparatus was devised for carrying out the extraction so that the temperature of the solvent would be above 110°.

(3) The application of the method to different types of coal showed that coal varied widely in the amount of material dissolved by phenol. The amount of soluble material seemed to differentiate sharply between subdivisions of coal types. On the ash and moisture free basis the high volatile coals of Vermilion County give 35-40 per cent of soluble material; the coals from Madison and Montgomery Counties give 30-35 per cent and the low volatile coals of Williamson County show 20-30 per cent of soluble material.

(4) Extraction of coal leaves a residue which will not coke. The coking constituent of the coal is in the extract.

(5) Residue and extract oxidize at room temperatures and more readily at 100°, the rate of oxidation being more rapid in the residue.

(6) Residue and extract possess an avidity for water as well as oxygen at ordinary temperatures. The residue shows the greater avidity in both cases.

(7) Volatile matter determinations show that the extract contains more volatile matter than the residue.

(8) The ultimate analysis of the coal, residue, and extract shows that the percentage composition of carbon, hydrogen, nitrogen, and oxygen is substantially the same.

(9) Destructive distillation of coal residue and extract gave gases of practically the same composition.

(10) Oxidation of coal, residue, or extract produces a lowering in the percentage of volatile matter.

(11) Oxidation decreases the amount of material which may be extracted from coal by phenol and the coking properties are decreased in proportion to the extent of the oxidation.

(12) Oxidation of coal, residue, and extract is shown by an increase in the percentage of carbon dioxide in the gases produced by thermal decomposition. It is concluded, therefore, that the oxygen absorbed is chemically held.

IV. Historical.

Professor Bedson[1] refers to work which was done by Dr. Smythe on coal at Göttingen in 1851. This work was published in a report

1. J. Soc. Chem. Ind., 27, 149 (1908).

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printed for the Commissioners of the 1851 Exhibition. A brown coal from Brühl, near Cologne, was used in the work. The following solvents were used in the extraction in the order named: benzene, chloroform, alcohol, ethyl ether, petroleum ether, and acetone. Benzene extracted 3 per cent of the coal, chloroform 1.8 per cent, and alcohol 2.4 per cent. The other extracts were very small. Attempts were made to obtain pure substances from these extracts by the ordinary methods of purification used in organic chemistry.

Reinsch[1] in 1885 supposed that coal was composed of two substances, which could be distinguished by their action towards alkaline solution. With an alkaline solution he was able to isolate substances which were very characteristic in not being attacked by mineral acids.

Baker[2] in 1901 examined the solvent action of pyridine on certain classes of coals. The extraction was carried out for 50 hours in a Soxhlet extractor. Anthracite coal from South Wales gave very little material soluble in pyridine. A bituminous coal from Durham was soluble in pyridine to the extent of 20.4 per cent. The pyridine extracts were dark brown in color and showed fluorescence in some cases. An ultimate analysis did not show a concordant change in the proportions of elements present in the coal, residue, and extract.

Andersen and Henderson[3] in 1902 extracted Bengal and Japan coal with pyridine. They selected samples from Boiaker (Bengal), Paronai and Yubari (Japan) and also, for comparison two Scotch coals: Linrigg Lower Dumgray and Bannockburn Main. They did not give the amounts which were dissolved. The pyridine extract, after the removal of the solvent, possessed a black lustrous appearance similar to bitumen. The extract from all of the coals was similar in character. Ultimate analyses of the extract were given and the results showed that there was little difference in the chemical constitution of the coals of Bengal and Japan, from those of Scotland. They stated that pyridine was the best solvent that had been used. The percentage of carbon, hydrogen, and nitrogen in the extract was about the same as that in the original coal. The coking properties of a poor coal could be entirely removed but that it was only partially removed in a strong coking coal.

Professor Bedson[4] in 1908 worked upon the pyridine extract from certain gas coals. The coals varied from 22 to 33 per cent of pyridine extract. Also, four Cannel coals were extracted giving from 7 to 29

1. Dingl. Poly. J., 256, 224. (Chem. Soc. A, 48, 876; 1885).
2. Trans. North Engl. Inst. Mining and Mech. Eng., 50, [2], 23 (1901). (J. Soc. Chem. Ind., 20, 789; 1901).
3. J. Soc. Chem. Ind., 21, 242 (1902).
4. J. Soc. Chem. Ind., 27, 147 (1908).

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per cent of extract. Bedson called attention to the fact that in the first six coals the percentage of volatile matter is not far removed from the percentage of pyridine extract. After commenting upon this fact he decided that it would be dangerous to base any definite conclusions upon this fact alone. Although those coals were "gas" coals and similar in character, frequently substances identical in chemical constitution, give marked differences in physical properties. So that in coal work any problem may not be considered from the purely statistical standpoint alone.

Through Bedson, Blair[1] became interested in the proximate constituents of coal. Blair extracted coal from the Busley Seam of the Botley Colliery. After freeing the extract from pyridine, it was extracted with solvents in the following order: petroleum ether, ethyl ether, absolute alcohol and chloroform. Only a scanty description of these compounds was given. A complete account of the results was reserved for another communication.

Professor Lewes[2] in 1912, in a series of lectures on coal carbonization has reviewed some of the work upon the extraction of coal with pyridine. He himself had done some work but the results with the exception of one analysis were not given. He states that in all probability the coking property of a poor coking coal can be entirely removed by pyridine and that such will not be the case with a strong coking coal, for the reason that in the latter case some of the resinic bodies resist the solvent action of pyridine. Lewes in his criticism of pyridine as a solvent, points out the following facts as objections to it. He has worked on some coals that gave a higher percentage of volatile matter after extraction than they did before. He gives some results of Andersen and Henderson and claims that they show the same property. However, the results which he compared were not apparently considered by the authors as comparable. Professor Lewes states that the increase in volatile matter in the residue over the original volatile matter in the coal can only be taken to mean that pyridine has formed an additional product in some way. However, Clark and Wheeler, as well as Wahl, in work to be mentioned later, found that the residue did not increase as to the percentage of volatile matter. It was noted that in some cases when an ultimate analysis was made of the extract, the amount of nitrogen was greater than it was in the coal, in spite of the fact that the extract had been carefully washed free from pyridine. Furthermore, the two most successful solvents, according to Lewes, were aniline and pyridine,—both organic bases. The same effect is produced

1. J. Soc. Chem. Ind., 27, 150 (1908).
2. Progressive Age, 29, 1030 (1911).

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upon coal by these solvents as with NaOH, i. e., a non-coking residue remains. Sodium hydroxide is known to saponify resinic substances, and it is probable that pyridine and aniline form a compound with some of the resin, which is soluble in an excess of the solvent. Of course, this would be shown in the high nitrogen content of the extract.

Lewes gives the following conclusions: "These considerations seem to make it clear that the resin constituents condition the coking of coal during destructive distillation, and that they are of at least two kinds—the one easily oxidizable, soluble in pyridine and saponifiable by alkalies, and which on weathering is oxidized into a humus body with the evolution of water and carbon dioxide, and is responsible for the heating of coal in storage; the other class non-oxidizable, not saponified by alkalies, and forming with pyridine a compound insoluble in excess of the reagent, and this class may be the hydrocarbons from decomposed resins, as the residue in which they are present yields rich liquid hydrocarbons, as tar and pitch, but not rich in gas".

A. Wahl[1] in 1912, extracted a number of coals with pyridine. Volatile matter determinations were made before and after the extraction. The percentage of volatile matter in the coal was from 13 to 55 per cent and after extraction, the volatile matter in the residue was slightly lower, in some cases as much as 5 per cent. The amount of material extracted was from 6 to 26 per cent. The coke which was produced from the residue was harder and more compact than that of the coal. The material dissolved by the pyridine was an amorphous brown solid, which produced a voluminous coke. The ultimate analyses of coal and pyridine extract were given in some cases, but these showed scarcely any difference in composition.

In 1912, Frazer and Hoffman[2] published a paper on "The Constituents of Coal Soluble in Phenol". They tried the effect of a number of reagents and organic solvents on coal and found that pyridine, aniline and phenol removed the largest amount of soluble material. The published report was concerned with those constituents of coal soluble in phenol. Illinois coal from Franklin County was used in the extraction. The amount extracted was 10.87 per cent, calculated on a moisture and ash free basis. After freeing the extract from phenol by distillation an analysis gave ash 2.16 per cent. In the light of the present investigation this amount of ash in the extract seems to be abnormally high. By the use of sodium hydroxide and organic solvents such as ether, acetic acid, methyl alcohol, acetone, benzene and petroleum ether, they were able to separate the extract into certain

1. Comtes rend., 154, 1094 (1912).
2. Technical Paper 5, Bureau of Mines.

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portions, which when distilled in vacuo, gave varying physical properties. Analyses of these substances were made. The authors concluded that in the absence of evidence to the contrary coal substance soluble in phenol was present as such in the coal itself. They also believed that some of the substances isolated were approximately pure compounds, but this could not be proven absolutely on account of the small amount of material which they had at their disposal. They intend to follow the investigation using pyridine as a solvent.

Pictet and Ramseyer,[1] in 1913, extracted large quantities of washed coal from Montrambert (Loire) with benzene. From 248 kg. coal, almost 244 grams of almost black, quite fluid oil were obtained. They distilled this at various temperatures and pressures and worked on the different fractions. From their studies they think that coal contains among other constituents polymerized hydroaromatic hydrocarbons.

In 1913, Clark and Wheeler[2] in a paper on the "Volatile Constituents of Coal" described some of their work on the extraction of coal with phenol. They contend that coal is composed of two different types of bodies, which possess different degrees of ease of decomposition. They called the two constituents, the "hydrogen-yielding" and the "paraffin-yielding", respectively. However, they did not mean to say that above a certain temperature, one type of body decomposes and that below that temperature it will not decompose. From their experiments on coal they concluded that because of an increase in the amount of hydrogen evolved between 750° and 800°, that there was a hydrogen-yielding constituent in coal. They did not mean to designate by a hydrogen-yielding constituent one which gave all hydrogen, or by a paraffin-yielding constituent one that gave all paraffin, but one which gave a predominance of hydrogen or paraffin. It was also shown by them that the temperature of the destructive distillation influenced to a large extent the kind of gas given off. Clark and Wheeler extracted coal with pyridine and then the pyridine extract was itself extracted with chloroform. It was noted from the ultimate analysis that the increase in the amount of nitrogen in the residue and extract was 2.75 per cent, showing that nitrogen was being added from some source. Data were given for the analysis of gases which were produced by the destructive distillation of the pyridine extract, pyridine residue, chloroform extract and chloroform residue, and for dehydrated cellulose. The gas which was produced from the pyridine extract contained a larger amount of methane than the gas from the residue. The results

1. Arch. sci. phys. nat., 34, 234 (Chem. Abs. 7, 1496; 1913).
2. J. Chem. Soc., 103, 1704 (1913).

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in the case of the present investigation did not show this difference. From the results obtained they did not hesitate to identify the pyridine residue as a degradation product of cellulose but they were doubtful if all of the pyridine extract was resinous in character. However, by extracting the pyridine soluble material with chloroform, they thought they were able to get out material consisting almost entirely of resinous matter. Detailed methods of extraction and of destructive distillation were given.

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A STUDY OF BOILER LOSSES

BY

A. P. KRATZ

75

BULLETIN No. 78

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ENGINEERING EXPERIMENT STATION

BULLETIN No. 78 APRIL, 1915

A STUDY OF BOILER LOSSES

BY A. P. KRATZ, ASSISTANT IN MECHANICAL ENGINEERING,
ENGINEERING EXPERIMENT STATION

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A STUDY OF BOILER LOSSES.

I. Introduction.

1. *Purpose of the Tests.*—The experiments herein described were undertaken to determine the conditions prerequisite for the continuous operation of the boilers in the new power plant at the University of Illinois, and to permit a detailed study of the boiler and furnace losses under varying conditions of load, depth of fuel bed, and draft. It was hoped that an analysis of the data obtained might render it possible to isolate and determine the amount of the several losses in the boiler, furnace, and setting, thus indicating where improvements could be made in order to increase the over-all operating efficiency.

No attempt was made to obtain record tests and in every case ordinary operating conditions prevailed, with the exception of the maintenance of approximately constant load on the boiler under test. The boiler was operated as one of a battery of two boilers which delivered steam directly into the mains connected with the old plant. The pressure maintained was thus governed by conditions prevailing in the system as a whole.

While a majority of the trials were made with fresh coal from the Mission Field Mine No. 3, at Danville, Illinois, a series was also made using weathered coals from mines in Sangamon, Williamson, and Vermilion counties, to permit a comparison with the tests previously made, and, if possible, to determine the effect of weathering upon the steaming qualities of the coal.

2. *Acknowledgments.*—The experiments described in this bulletin were undertaken with the approval of C. R. Richards, Professor of Mechanical Engineering and Acting Dean of the College of Engineering and Acting Director of the Engineering Experiment Station of the University of Illinois, and were carried out under his supervision. To him the writer is indebted for many valuable suggestions and also for his assistance in criticizing and revising the manuscript. Acknowledgment is also due to Mr. H. F. Geist for his assistance in running the tests and computing the results, and to Mr. W. E. Alley for his help in installing and maintaining the apparatus and running the tests.

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II. The Plant.

3. *Boilers.*—The boiler on which these tests were run is one of a battery of two Babcock and Wilcox boilers in the new power plant of the University of Illinois. It is designated as boiler No. 1. The second unit of the battery is placed between boiler No. 1 and the chimney. The battery, as a whole, has 10,160 sq. ft. of heating surface, equally distributed between the two boilers, and is designed to carry a working pressure of 160 lb. per sq. in. Each boiler has two 42 in. by 20.33 ft. drums, and 18 sections of 4-in. tubes 18 ft. long, each section containing 14 tubes. They are set so that the distance from the floor to the first row of tubes is 8½ ft. at the front end.

A cross section through the plant is shown in Fig. 1 and a view of the boiler fronts in Fig. 2. From Fig. 1 it may be seen that a style of baffling different from the standard Babcock and Wilcox type is in use. This construction provides a tile roof over the furnace and makes the attainment of a long combustion chamber possible.

The first pass for the gases is thus at the rear of the setting and

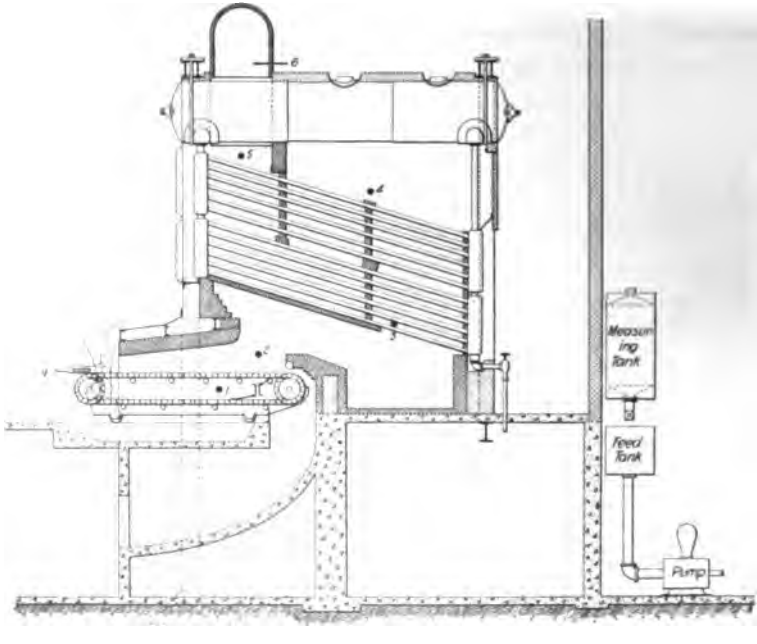

Fig. 1. Section Through Power Plant.

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FIG. 2. BOILER FRONTS.

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they are discharged into a breeching at the front instead of at the back of the boiler, as in the standard type.

4. *Stokers.*—The stokers are of the chain grate type, having an active grate surface of 90 sq. ft., and were built by the Green Engineering Company. At the time these tests were made, each stoker had a flat fire arch 6 ft. long, set 10 in. above the grates at the front end and 18½ in. at the back. This arch has since been changed so that at the present time it is 15 in. above the grates at the front and 33 in. at the back.

The draft is produced by means of a brick chimney, 175 ft. high, having an internal diameter of 10 ft. This was designed with the expectation that it was to serve an additional battery similar to the one under discussion, but at the time the tests were run the second battery had not been installed.

The feed water is delivered by means of two 14 in. by 8½ in. by 12 in. American steam pumps, the feed mains being arranged so that either pump can feed both boilers as a unit, or so that each can feed its corresponding boiler independently. The feed water consists of condensed steam from the heating system, with a small quantity of "make-up" water, supplied principally through the water backs of the stokers. The water is heated by means of a Webster "Star-Vacuum" heater before it reaches the pump.

5. *General Dimensions.*—The following is a table of the principal dimensions.

Number of steam drums	2
Length of steam drums, feet	20.33
Diameter of drums, inches	42
Number of tubes	252
Outside diameter of tubes, inches	4
Inside diameter of tubes, inches	3.75
Mean length of tubes exposed to gases, feet	18
Width of grate, feet	9.46
Effective length of grate, feet	9.50
Area of grate surface, square feet	90
Area of air space in grate, square feet	19.6
Ratio of air space to grate area	0.218
Height of ignition arch at front, inches	10
Height of ignition arch at back, inches	18.5
Length of ignition arch, feet	6
Height of chimney above grate, feet	175

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Sectional area of chimney at top, square feet	78.5
Height of gas passage over bridge wall, inches	33
Area of gas passage over bridge wall, square feet	26.8
Minimum area through 1st pass, square feet	27.7
Minimum area through 2nd pass, square feet	18.1
Area between front baffle and tile on lower tubes, square feet	23.2
Minimum area through 3rd pass, square feet	19.0
Total effective area of openings into breeching, square feet	15.2
Total water heating surface, square feet	5120
Ratio of water heating surface to grate surface	57.0

III. Methods of Conducting Tests.

With a few modifications, the methods employed in these experiments were those set forth in the A. S. M. E. code for conducting boiler trials.

6. *Starting and Stopping.*—As previously stated, the boiler tested forms part of the plant equipment, and as such was in continuous operation. Before starting each test, however, the gates were set for the predetermined thickness of fire, and the boiler operated for a period of from 1½ to 2 hours, carrying the load decided upon for that test. The coal bunker supplying the boiler under test was kept filled with Mission Field coal during the entire period over which tests were being made, thus obviating the necessity for emptying it before each run. In the case of the weathered coal, however, the fuel was piled on the floor of the boiler room and before starting the test the coal hopper on the stoker was allowed to empty. It was then filled with the fuel under test and the grates run until they were completely covered with the new fuel. This took from 45 minutes to one hour. It may, therefore, be seen that at the start of each test the grates were covered with the required fuel and the boiler had been running under the predetermined conditions of load and thickness of fuel bed for from 1½ to 2 hours.

On starting a trial the condition of the fuel bed was observed and recorded and an effort was made to have this condition duplicated at the close of the test. The start was made with the ash pit and drip pan clean and the fuel in the hopper just level with the top. The test was closed under the same conditions.

The condition of the fuel bed was maintained as nearly uniform as possible throughout a test and no adjustment was necessary at the end.

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7. *Coal.*—A small steel car resting on platform scales was used to weigh the coal. From this car it was shoveled into the hopper. The tests were started with a full car of coal on the scales, the drip pan and ash pit clean and with the fuel in the hopper just level with the top. At the end of each hour the drip pan and ash pit were again cleaned, the hopper leveled, and the coal remaining in the car weighed. Thus the coal fed to the grates was the total of the coal weighed during the hour minus that remaining on the scales at the end.

Before weighing a car of coal two samples were taken from it and placed in tight cans. At the end of the test one sample was quartered down in the usual manner to a weight of about 3,000 grams, placed in a shallow pan, weighed on a Troemen balance and set on top of the boiler to dry. After drying it was weighed a second time to determine the moisture loss, then quartered down to fill a quart jar and sent to the chemist for analysis. Proximate analyses were made on all of the samples. Ultimate analyses also were made on four samples of the Mission Field coal from the average of which ultimate analyses were calculated for the rest of the samples by making use of the individual proximate analyses. In the case of the weathered coal, however, ultimate analyses were necessarily made from each sample.

The second sample referred to above was spread out and allowed to dry, after which it was weighed and screened. The remainder was then weighed a second time and from the two weighings the percentage of dust was calculated. The screen used had ¼-in. round openings and what passed through it has been designated as "dust."

8. *Ash.*—The "ash and refuse" was raked from the pit into wheelbarrows and weighed on platform scales. A sample was taken from each load and placed in a tight can. At the end of the test this was quartered down until it could be contained in a quart jar and was then sent to the chemist for analysis. This analysis was used in determining the carbon loss in the ash. Analyses of both coal and ash were made in the Chemical Laboratory at the University of Illinois.

9. *Water.*—The feed water was measured by means of two cylindrical tanks about 4 ft. in diameter and of 4,600 lb. capacity. These tanks, as shown in Fig. 3, had conical heads tapering to the size of a standard 6-inch nipple and were emptied through quick opening valves at the bottom. By this arrangement the error in filling was reduced to a minimum. They were calibrated at about 155° F., the calibration including all of the volume from the valve to the top of the nipple on the upper head. Making use of these data, a curve was calculated, taking

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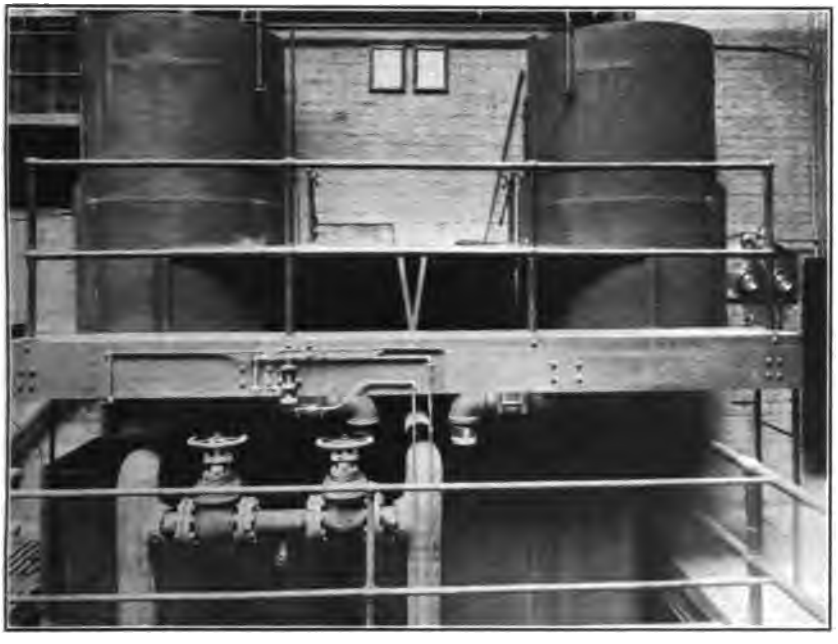

FIG. 3. FEED WATER TANKS.

into account the change in volume of the tank as well as the change in density of the water, from which the weight of water contained in the tank could be determined when its temperature was known. The temperature was then taken just before emptying a tank.

The measuring tanks emptied into a rectangular feed-tank 40 in. deep and of about 8,600 lb. capacity, from which the water gravitated to the feed pump supplying the boiler under test. This tank was also calibrated and a curve plotted giving the weight of water per inch of depth at any given temperature. The height of water in the tank was shown by a pointer which was attached to a float and moved over a scale having 0.25-in. divisions.

The boiler itself was calibrated and a family of curves plotted such that when the height of water in the gage glass and the pressure were known the total weight of water in the boiler could at once be determined.

Both boilers were fed by a single main, the two feed pumps discharging into it at points within about 3 feet of each other. Two valves were inserted in the main between the points where the pumps discharged and a bleeder was placed between the valves. Since the

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bleeder was always open during a test any leakage through either valve could be detected immediately. In this way the two parts of the main were completely isolated from each other and one pump could be used to deliver the water from the weighing tanks to the boiler under test, while the other was being used to feed the remaining boiler and to deliver water from the heater to the weighing tanks.

In the installation under discussion the water-back formed a circulating system entirely separate from the boiler, the water passing through it being measured by means of a calibrated meter and afterwards discharged into the feed water heater. It is not unusual for the water-back to be connected into the boiler in such a way as to form a closed system and to utilize the natural circulation taking place in the latter in which case it forms part of the heating surface the same as one of the tubes. If the water-back is disconnected from the boiler and a positive means of circulation adopted, the circulating water being taken from the feed water and returned to it subsequent to taking the weight and the temperature of the latter, the result is essentially the same as though the closed system had been used, and no account need be taken of the water thus circulated. In the present case, however, the temperature was taken close to the boiler and after the circulating water had gone into the feed water. The water-back has, therefore, been considered as part of the heating surface of the boiler and the heat absorbed by the circulating water has been reduced to terms of equivalent water evaporated and added to the boiler output as discussed under feed water corrections in the Appendix.

10. *Flue Gas.*—Flue gas samples were collected over half-hour periods. One sample was taken at the breeching about 2 ft. below the dampers and a second one at a point about 8 in. behind and below the end of the tile roof of the combustion chamber. This point was about 34 in. back of the bridge wall at the point where the gases turned to go up through the first pass. It was close enough to the furnace so that no air leakage chargeable to the boiler setting had as yet occurred, and far enough back so that combustion was complete, as may be seen from a comparison of the CO in the gas there and at the dampers.

The sampler used in the breeching is shown in Fig. 4 and the breeching itself in Fig. 5. In the latter it may be seen that the boiler had 3 nozzles opening from the last pass into the breeching with a damper in each one, the dampers being operated simultaneously from one control rod. A set of sample tubes cross connected as shown in Fig. 4 extended into each one of these passages. The two gate valves shown at A and B

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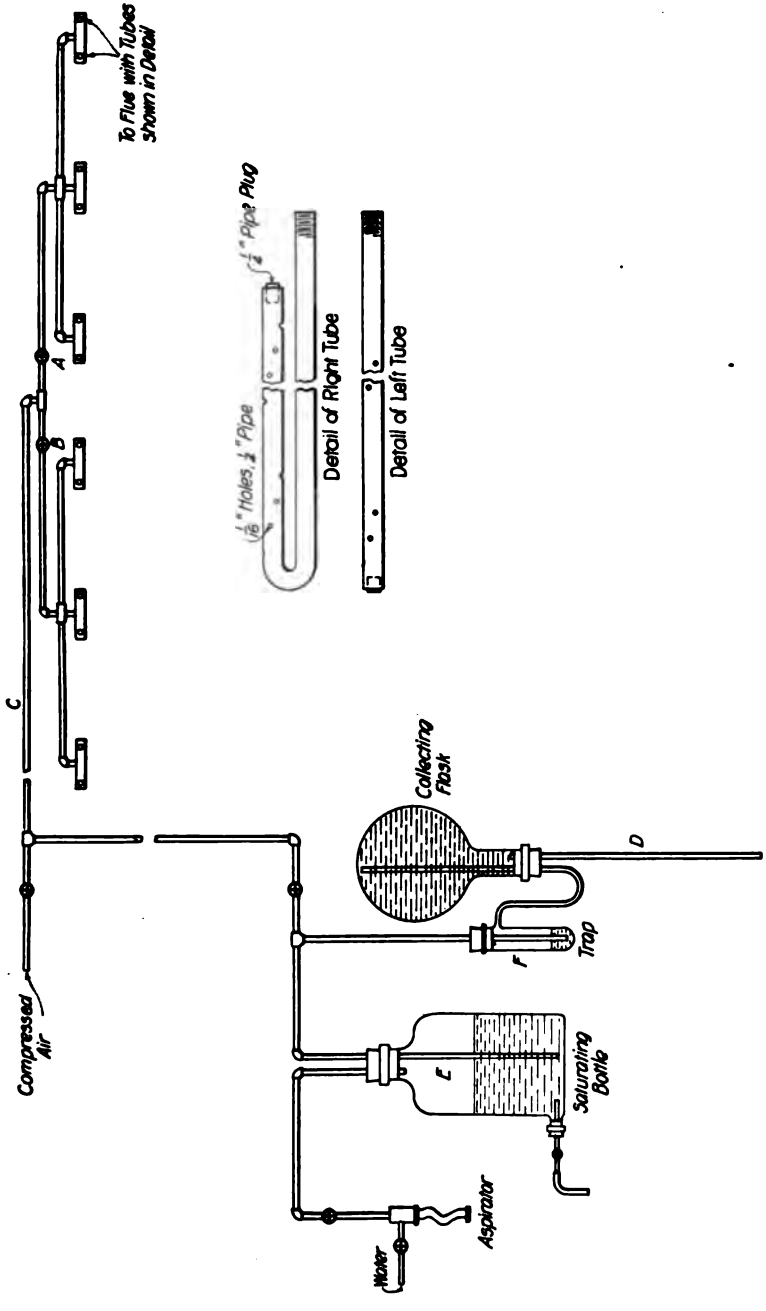

FIG. 4. GAS SAMPLING APPARATUS FOR FLUE.

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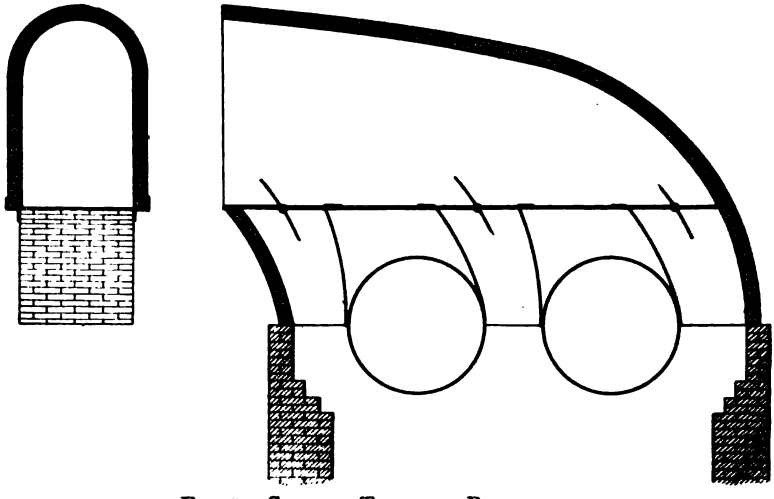

FIG. 5. SECTION THROUGH BREECHING.

made it possible to obtain a sample from either boiler. Before starting the tests the valve at B was closed and the whole apparatus tested for leaks by means of compressed air. The valve B then remained closed during the period over which tests were run. In order to keep the small holes open the tubes were blown occasionally with compressed air. The system was tested frequently to see that it did not leak. A continuous flow of gas was maintained through the pipe C by means of a small laboratory aspirator placed near the floor line. The sample bottle was attached at a tee near this point and any back flow of gas from the bottle was prevented by placing a small mercury trap (F) in the line between the latter and the tee.

The sampler used for the furnace gases is shown in Fig. 6. This consisted of a 2½-in. pipe extending in to the center of the furnace. Five small tubes were brazed into the pipe and water was kept flowing through the apparatus in order to prevent it becoming hot enough to react on the gas. It was assumed that the composition of the gas varied the same either side of the center toward the furnace walls and also that it was fairly well mixed by turning the corner at the point where the sample was taken. While this does not insure a strictly average sample, it is, however, believed to be within the limits of accuracy of the rest of the observation in boiler trials.

The gas samples were taken in one liter flasks as indicated in Fig. 4. Before being attached the latter were filled with water and inverted.

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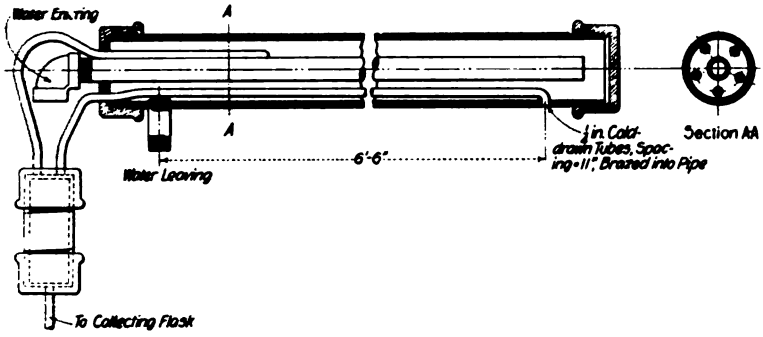

FIG. 6. GAS SAMPLING APPARATUS FOR FURNACE.

They were then allowed to empty under a head of about 3 ft., the rate of drawing the sample being regulated by means of a glass stop cock in tube D so that as nearly uniform flow as possible was maintained over the half-hour periods. A bottle of water was placed at E, between the sampler and aspirator, and all of the gas was drawn through it, thus keeping a supply of gas-saturated water for use in the flasks.

11. *Quality of Steam.*—The quality of steam was taken by means of a throttling calorimeter and the steam pressure was taken by a calibrated gage at this point.

12. *Temperature.*—Calibrated mercury thermometers were used for taking all temperatures, with the exception of that of the gas leaving the boiler. The latter was taken at points about 2 ft. below the dampers by means of 3 copper-constantan thermo-couples and a Siemens-Halske millivoltmeter. These were calibrated with the cold junctions at 100 degrees F. This temperature was read during each test and a correction applied in case it was not 100° F., the correction in no case being larger than about 5° F. The couples were calibrated before the tests and were then left in place for the rest of the series, a period of about 3 months. On being re-calibrated at the end of this time the new points fell on the curve of the previous calibration. It is, therefore, believed that the accuracy of the apparatus is within 5° F.

13. *Drafts.*—Drafts were taken at the points indicated in Fig. 1 by means of Ellison differential gages. The draft at (6) was taken at a point about 2 ft. below the center damper.

14. *Observation Periods, etc.*—All readings of pressure, temperature, drafts, etc., were taken at 15-minute intervals. The tests were of sufficient length to give a total coal consumption of 250 lb. per sq. ft.

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of grate surface, with the exception of the weathered coal tests, which were necessarily limited by the amount of the samples. The heat balance shows a normal unaccounted-for loss for the latter, however, and it is believed that the accuracy has not suffered materially on this account.

IV. Discussion of Results.

The general results of the tests are shown in Tables 1 to 16. The items in these tables are in accordance with the Boiler Test Code of

Table 1.

General Conditions.

Test Number	Date	Duration, Hours	Thickness of Fuel Bed, Inches	Pressure, Lb. Per Sq. In.			Draft, Inches of Water	
				Barometric	Steam (Gage)	Steam (Abs.)	Between Damper and Boiler	At End of 3rd Pass
Col. No.	2	3	4	5	6	7	8	9
* a	1	2	81		11		12	
‡ b	2	3		6	8	9	10	11
1	11-11-'13	13.0	6.0	14.4	128.2	142.6	0.091	0.134
2	11-18-'13	13.0	7.0	14.3	132.0	146.3	0.053	0.102
3	11-13-'13	13.0	8.0	14.4	131.4	145.8	0.077	0.118
4	11-20-'13	13.0	10.0	14.3	133.0	147.3	0.263	0.310
5	11-25-'13	9.0	6.0	14.4	129.4	143.8	0.379	0.433
6	11-29-'13	9.0	7.0	14.4	137.9	152.3	0.254	0.307
7	12- 2-'13	9.0	8.0	14.5	132.5	147.0	0.358	0.397
8	12- 9-'13	9.0	9.3	14.5	132.0	146.5	1.038	1.056
9	12- 4-'13	9.0	9.8	14.4	135.1	149.6	0.726	0.767
10	12- 5-'13	9.0	6.0	14.3	138.1	152.4	0.522	0.568
11	12-12-'13	9.0	6.0	14.5	134.2	148.7	0.984	1.011
12	12-13-'13	9.0	7.0	14.5	138.7	153.2	1.124	1.138
13	12- 6-'13	9.0	7.0	14.2	136.3	150.5	0.500	0.540
14	12- 8-'13	9.0	8.0	14.6	131.4	146.0	0.591	0.637
15	12-16-'13	9.0	9.4	14.4	131.4	145.8	1.126	1.127
16	12-19-'13	9.0	6.0	14.3	138.3	152.6	1.090	1.094
17	12-18-'13	9.0	7.0	14.4	133.9	148.3	1.147	1.153
18	12-15-'13	9.0	8.0	14.5	135.2	149.7	1.139	1.149
19	1-29-'14	7.0	7.0	14.4	141.0	155.4	1.124	1.160
20	12-30-'13	5.5	7.0	14.4	132.7	147.1	1.090	1.108
21	1- 2-'14	3.5	6.5	14.2	138.0	152.2	1.048	1.110
22	1-10-'14	4.5	7.0	14.4	134.0	148.4	1.250	1.240
23	1- 3-'14	6.0	7.0	14.3	134.7	149.0	1.055	1.159
24	1- 5-'14	6.0	7.0	14.5	133.0	147.5	1.225	1.206

* (a) Numbers correspond to the A. S. M. E. Code.
‡ (b) Numbers correspond to Calculating Code given in Appendix.

the American Society of Mechanical Engineers, with the exception that some items have been added. These consist of three efficiencies, viz.: The efficiency of furnace and grate, the efficiency of the furnace, and the efficiency of the boiler exclusive of furnace and grate, and also some items referring to gas analysis at the furnace and to weights of air and

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gas per lb. of coal. The heat balance has also been considerably extended and is based on 1 lb. of dry coal fired instead of on combustible.

The efficiencies given in the A. S. M. E. code are the over-all efficiencies for the boiler and furnace together and do not afford a means of determining whether a loss in efficiency of the unit is due to poor furnace construction or conditions or to dirty tubes or faults in the

TABLE 2.

GENERAL CONDITIONS (CONTINUED).

Test Number	Draft, Inches of Water				Temperature, Degrees Fahrenheit				
	Between 1st & 2nd Pass	At End of Combustion Chamber	Over Fire in Furnace	In Ash Pit	Outside Air	Fire Room	Steam	Feed Water at Boiler	Flue Gases
Col. No.	10	11	12	13	14	15	16	17	18
a			13	14	15	16	17	20	21
b	12	13	14	15	16	17	18	19	21
1	0.045	.....	0.088	0.002	36.0	74.0	354.5	137.3	451.0
2	0.020	0.079	0.072	0.001	64.8	78.1	356.5	123.2	445.7
3	0.048	0.117	0.108	0.004	58.8	73.9	356.1	127.3	435.3
4	0.168	0.217	0.184	0.009	69.6	83.5	357.0	114.0	460.7
5	0.208	0.219	0.171	0.032	48.8	78.6	355.2	128.8	551.0
6	0.141	0.179	0.149	0.015	59.9	79.8	359.7	125.2	515.0
7	0.247	0.274	0.232	0.008	53.8	70.7	356.9	128.8	532.3
8	0.721	0.654	0.549	0.047	34.9	83.1	356.6	146.5	573.7
9	0.499	0.449	0.389	0.029	51.2	74.5	358.2	125.9	555.0
10	0.320	0.289	0.200	0.042	52.3	73.1	359.7	126.4	596.6
11	0.631	0.524	0.330	0.051	47.7	84.1	357.8	136 5	638.3
12	0.748	0.656	0.421	0.053	47.9	79.1	360.1	138.8	644.4
13	0.320	0.290	0.227	0.023	54.2	77.8	358.8	126.0	544.8
14	0.404	0.391	0.327	0.015	26.5	71.4	356.3	130.4	557.5
15	0.818	0.748	0.540	0.020	40.4	78.7	356.2	142.5	563.5
16	0.712	0.594	0.320	0.078	37.3	76.2	359.8	154.0	612.5
17	0.779	0.667	0.364	0.043	37.2	78.2	357.6	147.7	620.3
18	0.778	0.666	0.416	0.041	39.6	84.2	358.3	140.5	616.4
19	0.746	0.546	0.375	0.017	45.7	70.1	361.2	147.0	632.8
20	0.624	0.537	0.416	0.108	32.6	77.0	357.0	154.3	602.0
21	0.664	0.590	0.467	0.085	34.7	72.6	359.6	144.4	613.8
22	0.732	0.587	0.482	0.070	23.4	77.0	357.6	166.9	618.3
23	0.705	0.649	0.540	0.099	34.3	63.1	357.9	142.1	597.0
24	0.716	0.572	0.441	0.089	30.3	77.5	357.1	149.4	621.0

boiler itself. It was for this reason that the efficiencies indicated above were added and that the heat balance was extended and divided so as to separate from one another the losses chargeable to boiler and furnace alone. In order to do this it was necessary to obtain an analysis of gas at the furnace and to include some items on weights of air and gas. A discussion of all these items may be found in the Appendix.

15. *Fuel.*—The fuel used on tests 1 to 18 inclusive was Vermilion

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County screenings from Mission Field Mine No. 3. The coal was of the same quality as that used for the normal operation of the plant and was all obtained from the same mine with the expectation that the ash and dust content would not vary much. So far as the ash was concerned this expectation was fairly well fulfilled but the dust varied considerably.

The fuel used on tests 20 to 24 inclusive was weathered coal. This

TABLE 3.

COAL AND ASH.

Test Number	Kind of Fuel (District from which mined)	Condition of Fuel	Commercial Size	Amount Passing a ¼-in. Screen Per Cent
Col. No.	19	20	21	22
a		22	23	
b				
1	Mission Field	Fresh	Screenings	35.0
2	" "	"	"	34.8
3	" "	"	"	40.0
4	" "	"	"	38.0
5	" "	"	"	37.8
6	" "	"	"	19.6
7	" "	"	"	36.3
8	" "	"	"	50.5
9	" "	"	"	35.7
10	" "	"	"	30.6
11	" "	"	"	33.2
12	" "	"	"	40.3
13	" "	"	"	36.3
14	" "	"	"	42.8
15	" "	"	"	22.3
16	" "	"	"	34.1
17	" "	"	"	34.7
18	" "	"	"	29.5
19	" "	"	Pea	7.2
20	Sangamon Co.	Weathered	Nut	31.9
21	" "	"	Screenings	45.1
22	Williamson Co.	"	Nut	13.9
23	" "	"	Screenings	42.9
24	Vermilion Co.	"	Nut	35.0

had been exposed in bins since January, 1908, and consisted of one lot of each of the following: Sangamon County nut, Sangamon County screenings, Williamson County nut, Williamson County screenings, and Vermilion County nut. This coal had disintegrated considerably and when dry it presented a white appearance as though coated with clay. It was very easily broken and the surface thus exposed was black and rather oily.

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16. *Relations Between Thickness of Fire, Capacity, and Efficiency.*—Curves are shown in Figs. 7 to 11 in which efficiency is plotted against thickness of fuel bed. For the sake of convenience in reference the points have been numbered to correspond to the tests from which they were taken. From the curves it becomes evident that the best efficiency under full load conditions was obtained with a fire of about 7 or 7½

TABLE 4.

COAL AND ASH (CONTINUED).

Test Number	Total Coal as Fired Lb.	Total Dry Coal Lb.	Combustible Consumed Lb.	Calorific Value by Calorimeter		Total Ash and Refuse Lb.	Percentage of Ash and Refuse in Dry Coal Per Cent
				Per Lb. Dry Coal B. t. u.	Per Lb. Combustible B. t. u.		
Col. No.	23	24	25	26	27	28	29
a	25	27	30	50	51	28	31
b	22	24	29	62	63	25	30
1	21964	18026	13026	10979	14177	4087	22.67
2	17935	14813	11061	11537	14287	3925	26.49
3	19135	15966	11661	11629	14318	3771	23.62
4	19840	16352	12742	11815	14276	3445	21.07
5	21805	17959	14061	11933	14381	3819	21.27
6	21035	17398	12418	11947	14437	4159	23.91
7	21512	18061	14328	12408	14478	3826	21.18
8	23325	19493	14982	11821	14409	3832	19 66
9	19257	16266	13164	12292	14439	2801	17.22
10	25397	21181	17421	12335	14452	4394	20.75
11	30173	25587	18255	10954	14377	5671	22.16
12	28490	23798	18369	11949	14320	4905	20.61
13	25678	21405	16549	11876	14230	4852	22.67
14	25390	21246	16474	12184	14462	4799	22.59
15	23416	19847	15320	11748	14350	4021	20.26
16	30955	26176	20872	12186	14381	5342	20.41
17	29235	24753	18924	11921	14324	5454	22.03
18	29309	24778	18183	11472	14260	5881	23.73
19	25087	21612	17617	12622	14465	5968	27.61
20	18226	14517	11743	11179	13469	2351	26.19
21	12658	9868	7618	10497	12707	1718	17.41
22	15660	13710	11009	12103	14067	2382	17.37
23	16917	14029	11002	12153	13911	2271	16.19
24	21383	17102	14193	11965	13787	2098	12 27

inches in thickness. This seems to give a fuel bed resistance such that the normal amount of coal can be burned without excessive draft. If a thinner fire than this is used there is a greater probability that there will be holes or thin places in the fuel bed, forming paths of least resistance through which an excessive amount of air passes with a consequent decrease in efficiency. On the other hand if the thickness of the fire is materially increased it becomes necessary to use more draft in order to maintain the load. The increased draft causes an increase in the air leakage through the setting, etc., thus causing the efficiency

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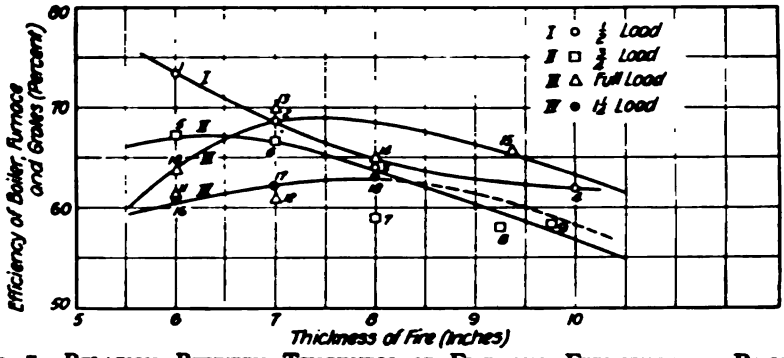

FIG. 7. RELATION BETWEEN THICKNESS OF FIRE AND EFFICIENCY OF BOILER, FURNACE, AND GRATES.

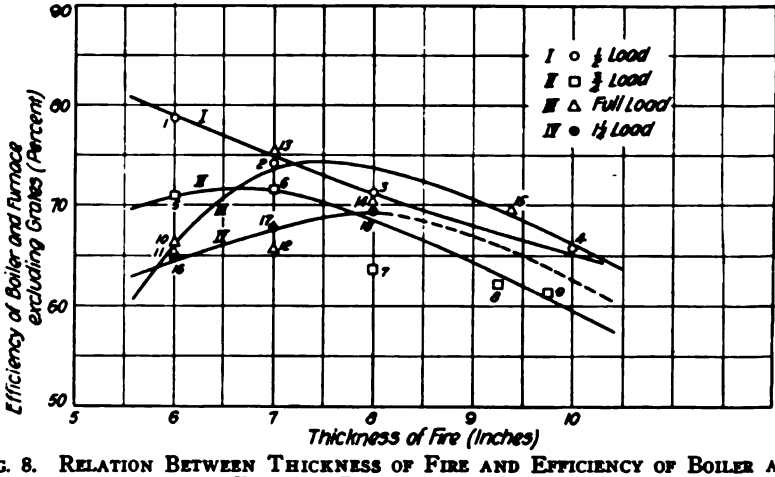

FIG. 8. RELATION BETWEEN THICKNESS OF FIRE AND EFFICIENCY OF BOILER AND FURNACE, EXCLUDING GRATES.

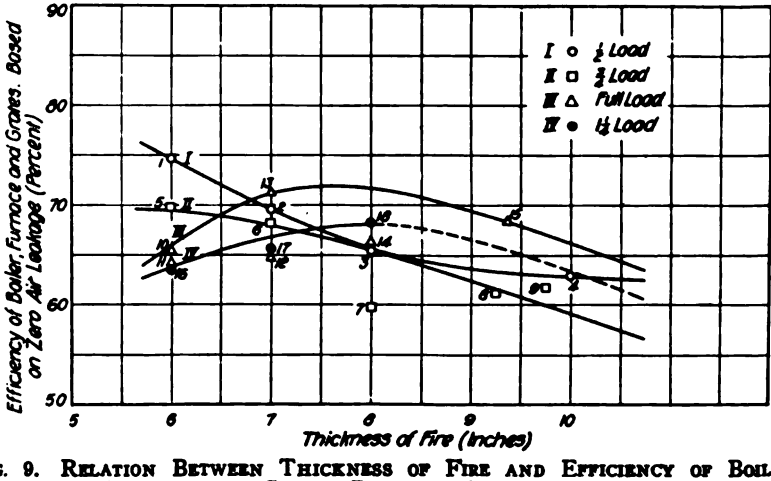

FIG. 9. RELATION BETWEEN THICKNESS OF FIRE AND EFFICIENCY OF BOILER, FURNACE, AND GRATES, BASED ON ZERO AIR LEAKAGE.

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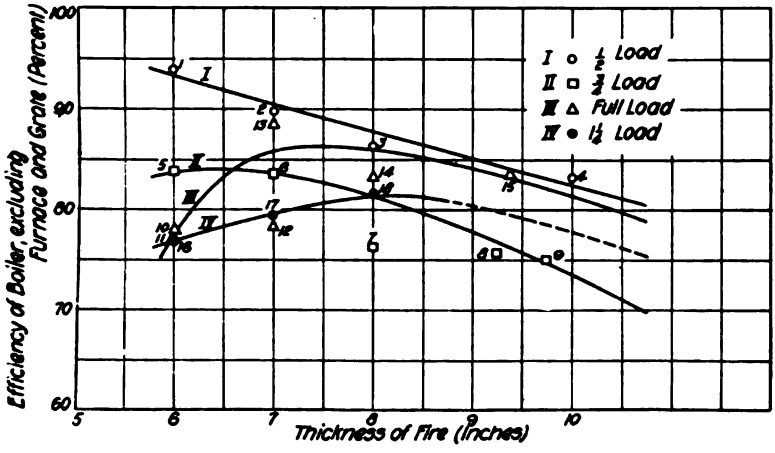

FIG. 10. RELATION BETWEEN THICKNESS OF FIRE AND EFFICIENCY OF BOILER, EXCLUDING FURNACE AND GRATE.

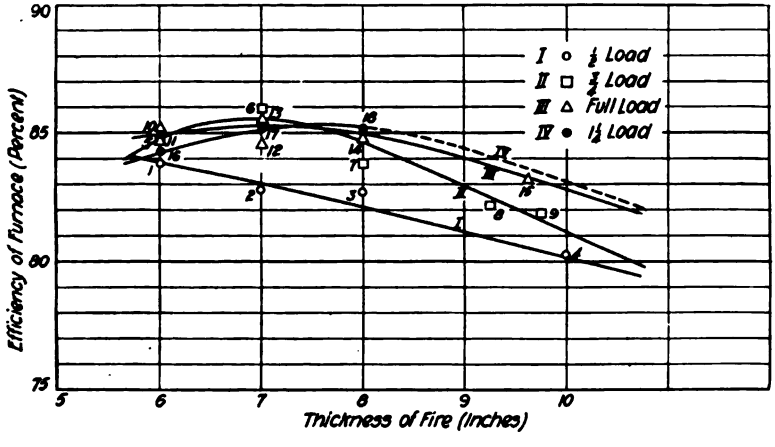

FIG. 11. RELATION BETWEEN THICKNESS OF FIRE AND EFFICIENCY OF FURNACE.

curve to drop. The above discussion is based on the assumption that there is no more CO formed when using a thick fuel bed than with a thin one. An inspection of the gas analyses will show that the CO was never more than 0.24 per cent as a maximum and that this value was obtained on but two tests. For this reason it has been disregarded.

Another explanation of the decrease in efficiency when the thickness of fire is increased above 8 inches is offered by the fact that the arch

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is very low, it being only 10 inches above the grate at the front end. This causes an excessive draft loss toward the front and makes the combustion poor over that part of the grate thus reducing the effective grate surface. On this account it was found impossible to obtain loads as high as 1¼ capacity with a fire 9½ inches in thickness. That part of the 1¼ load curve to the right of the point representing the 8-in.

TABLE 5.

STEAM AND WATER.

Test Number	Quality of Steam	True Factor of Evaporation	Total Water to Weighing Tanks Lb.	Total Weight of Water in Boiler, Lb.		Steam Pressure, Lb. per Sq. In. (Gage)	
				Initial	Final	Initial	Final
Col. No.	30	31	32	33	34	35	36
a		60					
b	66	111	67	83	84	75	77
1	0.981	1.104	128849	34075	33000	128.0	135.0
2	0.958	1.098	101703	34700	34520	104.0	132.0
3	0.965	1.100	106005	32940	34300	140.0	111.0
4	0.959	1.109	106843	33800	32870	137.0	133.0
5	0.966	1.099	134113	33600	34970	140.0	138.0
6	0.969	1.107	121025	34420	33000	140.0	135.0
7	0.969	1.102	120655	33650	33765	135.0	126.0
8	0.962	1.078	124928	31650	32985	143.0	136.0
9	0.966	1.103	106858	33540	33300	131.0	138.0
10	0.972	1.108	153213	33860	33415	145.0	141.0
11	0.965	1.091	153191	34940	31750	126.0	135.0
12	0.965	1.090	162271	33215	34255	144.0	139.0
13	0.969	1.106	162830	33510	34960	147.0	139.0
14	0.966	1.098	158007	33070	34340	130.0	120.0
15	0.963	1.082	143285	33595	34450	140.0	123.0
16	0.985	1.091	180054	33440	34010	140.0	133.0
17	0.965	1.079	170912	33880	34380	143.0	129.0
18	0.968	1.090	166713	34265	34880	138.0	131.0
19	0.968	1.101	164818	33985	34805	149.0	138.0
20	0.963	1.088	101619	31500	33790	125.0	137.0
21	0.986	1.102	55580	36500	35170	123.0	137.0
22	0.985	1.077	105364	30755	33975	128.0	135.0
23	0.986	1.104	97300	34610	34595	125.0	140.0
24	0.982	1.093	124576	34375	34300	130.0	135.0

fire has therefore been shown as a dotted line and has been made to parallel the full load curve.

A study of the curves in Figs. 7 to 11 will show that for each load there was a well marked thickness of fire which gave the best efficiency and that as the load decreased this also became less. Thus for 1¼ load the maximum efficiency occurred with 8 to 8½-in. fires, for full load with 7 to 7½ inches and for ¾ load with about 6½ inches. It is probable that for ½ load the fire could be decreased to 5 inches before

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the efficiency would drop materially. This can only be inferred, however, from the action at other loads, since no data were taken using fires of less than 6 inches.

17. *Draft.*—When analyzed from the standpoint of draft the above condition is to be expected. For each load, or rate of combustion, a given amount of air is necessary. If the fuel bed is too thick, or

TABLE 6.

STEAM AND WATER (CONTINUED).

Test Number	Total Weight of Water to Water-back Lb.	Temperature of Water to Water-back Deg. F.		Feed-Water Corrections, Lb.			Total Water Fed to Boiler, Corrected, Lb.	Equivalent Evaporation from and at 212°, Lb.
		Entering	Leaving	For Feed Tank Level	For Pressure and Gage Glass	For Water-back		
Col. No.	37	38	39	40	41	42	43	44
a							57	61
b	90	91	92	72	102	105	110	112
1	88075	58.7	123.2	+ 318	+ 974	+ 5305	135446	149532
2	97760	60.3	109.1	+ 3086	+ 694	+ 4477	109960	120736
3	108095	59.5	96.0	+ 3083	— 1594	+ 3696	111190	122309
4	99970	59.7	88.7	+ 1067	+ 648	+ 2695	111253	123380
5	70245	58.3	113.0	— 2019	— 1100	+ 3602	134596	147921
6	68310	59.3	117.8	+ 3512	+ 1020	+ 3721	129278	143111
7	68715	58.7	104.7	+ 319	— 251	+ 2955	123678	136293
8	51930	58.5	110.8	+ 1796	— 1174	+ 2597	128147	138142
9	65250	58.1	96.6	— 319	+ 309	+ 2347	109195	120442
10	62810	58.3	112.8	— 1382	+ 277	+ 3234	155342	172119
11	53640	59.0	136.3	+ 2332	+ 2679	+ 3917	162119	176872
12	52785	59.1	122.6	— 1377	— 906	+ 3169	163157	177841
13	55125	59.5	137.5	— 213	— 1262	+ 4008	165363	182891
14	53550	57.4	126.3	— 3080	— 1181	+ 3464	157210	172617
15	54630	58.3	106.4	+ 423	— 826	+ 2502	145384	157306
16	58275	58.1	123.2	+ 316	— 579	+ 3583	183374	200061
17	56925	58.2	123.3	+ 1319	— 648	+ 3538	175121	188956
18	53370	58.8	127.7	+ 318	— 614	+ 3477	169894	185184
19	50960	60.6	121.7	+ 1686	— 822	+ 2913	168595	185623
20	44633	58.4	108.4	— 2951	— 1634	+ 2113	99147	107872
21	26863	59.7	113.2	+ 2801	+ 1331	+ 1344	61056	67284
22	35392	60.4	140.1	— 575	— 2669	+ 2699	104819	112890
23	48480	59.7	104.9	— 4070	+ 280	+ 2046	95556	105494
24	46320	59.5	121.0	— 3368	+ 151	+ 2687	124046	135582

dirty, it requires a heavy draft, with a consequent increase in air leakage through places other than the fire, thus causing a loss in efficiency. If it is too thin the draft may be reduced, but in this case the air leakage occurs through the fuel bed itself with the same result as before.

The curves shown in Fig. 9 have been calculated on the assumption that no air leakage occurred. In this discussion air leakage has been defined as the air infiltration occurring in that portion of the setting

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from the bridge wall to the breeching and is chargeable against the boiler, and not against the furnace. It is believed that this efficiency forms a better basis for comparing the action of two different boilers and furnaces than the over-all efficiency. It was deemed advisable to calculate this because after the tests had been completed a place was found which permitted some air to leak into the second pass. On account

TABLE 7.

FUEL AND WATER PER HOUR.

Test Number	Dry Coal, Lb.		Combustible Consumed, Lb.		Equivalent Evaporation, Lb.	
	Per Hour	Per Sq. Ft. of Grate Surface per Hour	Per Hour	Per Sq. Ft. of Water-heating Surface per Hour	Per Hour	Per Sq. Ft. of Water-heating Surface per Hour
Col. No.	45	46	47	48	49	50
a	46	48	47	49	63	64
b	58	60	59	61	113	114
1	1287	15.41	1002	0.196	11502	2.25
2	1139	12.66	851	0.166	9287	1.81
3	1228	13.64	897	0.175	9408	1.84
4	1258	13.98	980	0.191	9491	1.85
5	1995	22.17	1565	0.306	16436	3.21
6	1933	21.48	1491	0.291	15901	3.11
7	2007	22.30	1592	0.311	15144	2.96
8	2166	24.07	1665	0.325	15349	3.00
9	1807	20.08	1462	0.286	13383	2.61
10	2353	26.14	1936	0.378	19124	3.74
11	2843	31.59	2028	0.396	19652	3.84
12	2644	29.38	2041	0.399	19760	3.86
13	2378	26.42	1839	0.359	20321	3.97
14	2361	26.23	1830	0.357	19180	3.75
15	2206	25.51	1702	0.332	17479	3.41
16	2908	32.31	2319	0.453	22229	4.34
17	2750	30.56	2103	0.411	20995	4.10
18	2753	30.59	2020	0.395	20576	4.02
19	3088	34.31	2517	0.492	26516	5.18
20	2639	29.32	2135	0.417	19613	3.83
21	2819	31.32	2177	0.425	19234	3.75
22	3047	33.86	2446	0.478	25087	4.90
23	2338	25.98	1834	0.358	17582	3.43
24	2850	31.67	2366	0.462	22597	4.41

of its inaccessible position this place had escaped detection from the time the boilers were installed. A close watch was kept and precautions were taken to prevent the development of leaks during the tests, and it was supposed that the setting was as tight as possible. With the possible exception of one or two tests, the difference between the percentage of CO_2 at the furnace and that at the breeching was not enough to arouse suspicion, since the setting was not a plastered one, and the

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ordinary brick setting is susceptible to some unavoidable air seepage. A comparison of this set of curves with the other efficiency curves indicates that the leakage was not sufficient to detract from any of the conclusions drawn from the data. A discussion of the method of calculation for the efficiency used in Fig. 9 may be found in the Appendix.

The curves in Figs. 12 to 15 inclusive show the relation existing

TABLE 8.

BOILER PERFORMANCE.

Test Number	Horse Power Developed	Builder's Rating, Horsepower	Percentage of Builder's Rating Developed	Water Apparently Evaporated per Lb. of Coal as Fired, Lb.	Equivalent Evaporation from and at 212°. Lb.		
					Per Lb. of Coal as Fired	Per Lb. of Dry Coal	Per Lb. of Combustible Consumed
Col. No.	51	52	53	54	55	56	57
a	65	66	67	68	69	70	71
b	115	116	117	118	119	120	121
1	333.4	508	65.7	6.17	6.81	8.30	11.48
2	269.2	"	53.0	6.13	6.73	8.15	10.92
3	272.7	"	53.7	5.81	6.39	7.66	10.49
4	275.1	"	54.2	5.61	6.22	7.55	9.68
5	476.4	"	93.8	6.17	6.78	8.24	10.51
6	460.9	"	90.8	6.15	6.80	8.23	10.67
7	439.0	"	86.5	5.75	6.33	7.55	9.51
8	444.9	"	87.6	5.49	5.92	7.09	9.22
9	387.9	"	76.4	5.67	6.25	7.40	9.15
10	554.4	"	109.1	6.12	6.78	8.13	9.88
11	569.6	"	112.1	5.37	5.86	6.91	9.69
12	572.7	"	112.7	5.73	6.24	7.47	9.68
13	589.0	"	115.9	6.44	7.12	8.55	11.05
14	555.9	"	109.4	6.19	6.80	8.13	10.48
15	506.6	"	99.7	6.21	6.72	7.93	10.27
16	644.3	"	126.8	5.92	6.46	7.64	9.59
17	608.5	"	119.8	5.99	6.46	7.63	9.99
18	596.4	"	117.4	5.80	6.32	7.47	10.18
19	768.6	"	151.3	6.72	7.40	8.59	10.54
20	568.5	"	111.9	5.44	5.91	7.43	9.19
21	557.2	"	109.7	4.82	5.32	6.82	8.83
22	727.1	"	143.1	6.69	7.21	8.23	10.25
23	509.6	"	100.3	5.65	6.24	7.52	9.59
24	655.0	"	128.9	5.80	6.34	7.93	9.55

between the various efficiencies and capacity or load in boiler horse power. They bring out about the same facts as those already discussed.

On the whole, the over-all efficiencies obtained are not surprisingly high. Under normal conditions of load and fire the over-all efficiency averaged about 65 per cent. As stated before, however, no attempt was made to obtain record tests, so that this represents the actual operating efficiency and as such is in accord with good practice. A study of column 60 in Table 9 will show that the efficiency of the furnace and

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grate was fairly constant for all loads and depths of fuel bed. Column 61 indicates that the loss in efficiency at higher loads was due to reduced heat absorption by the boiler. Also, a comparison of the values in this

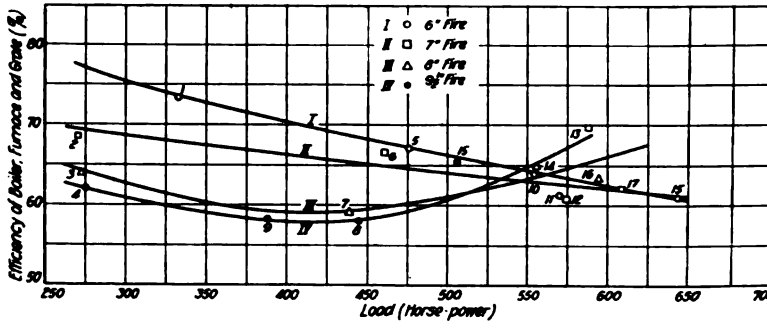

FIG. 12. RELATION BETWEEN LOAD AND EFFICIENCY OF BOILER, FURNACE, AND GRATE.

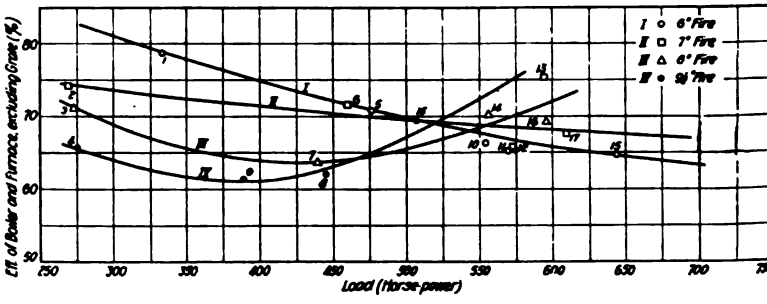

FIG. 13. RELATION BETWEEN LOAD AND EFFICIENCY OF BOILER AND FURNACE, EXCLUDING GRATE.

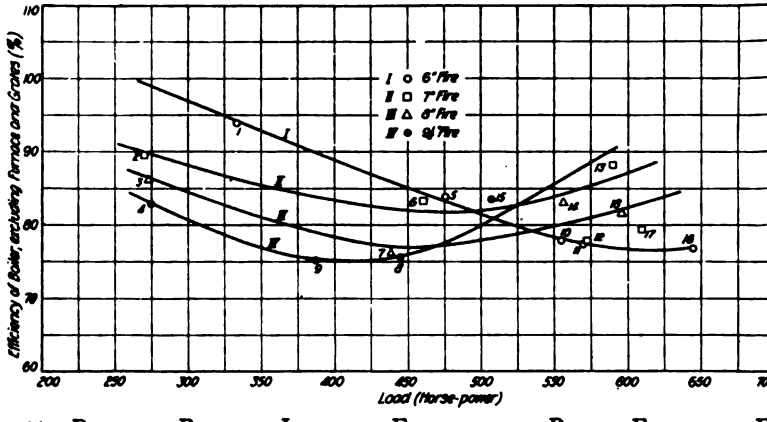

FIG. 14. RELATION BETWEEN LOAD AND EFFICIENCY OF BOILER, EXCLUDING FURNACE AND GRATES.

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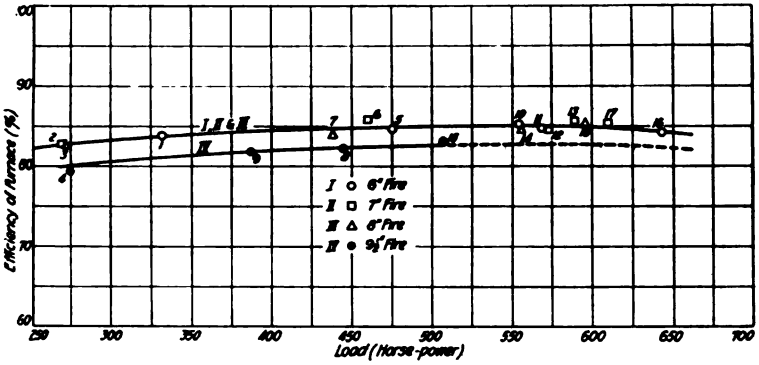

FIG. 15. RELATION BETWEEN LOAD AND EFFICIENCY OF FURNACE.

column with the temperature of the flue gas given in column 18, Table 2, indicates that in general the lower boiler efficiencies correspond with the higher temperatures.

The curves in Fig. 16 give the drafts throughout the boiler setting. The positions plotted as abscissae refer to the draft gage positions indicated in Fig. 1. It may be noticed that there is an unexpected drop in the curves at positions 4 and 6. At first it was supposed that this was due to error either in the gage itself or in the position of the tube. All the gages were then calibrated and found to be well within the limits of accuracy. The positions of the tubes were also changed, and they were moved backward and forward across the channels, with the same result each time. On comparing the cross-sectional areas of the various passes it was found that the area at point 4 was greater than that at point 3. Also that the wall forming part of the last pass was corbeled so that the area at point 6 was greater than that at point 5. This peculiarity in the curves, therefore, has been attributed to the dynamic effect of the flowing gases.

Referring to Fig. 17, it may be seen that the reading of a draft gage (h'_1), when connected to a chamber in which the pressure is less than atmospheric, indicates the difference between the barometric pressure (H) and the true pressure or static head (h_1) measured above absolute zero. If a Pitot tube is placed at the same point, and there is a flow of gas, the velocity head (h_3) will add to the static head (h_1), and the reading of the Pitot tube (h'_2) will be less than that of the draft gage (h'_1). According to Bernouilli's theorem, the total head at any point is the pressure head, plus the velocity head, plus the head lost up to that point. The atmospheric pressure (H) is the total head in this

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case, and h_2 is the sum of the pressure and velocity heads. Therefore h'_1 must represent the lost head or, in other words, the friction head. Proceeding from the front of the furnace to the breeching the quantity h'_2 increases more or less uniformly, and, since it is a loss, it would deduct

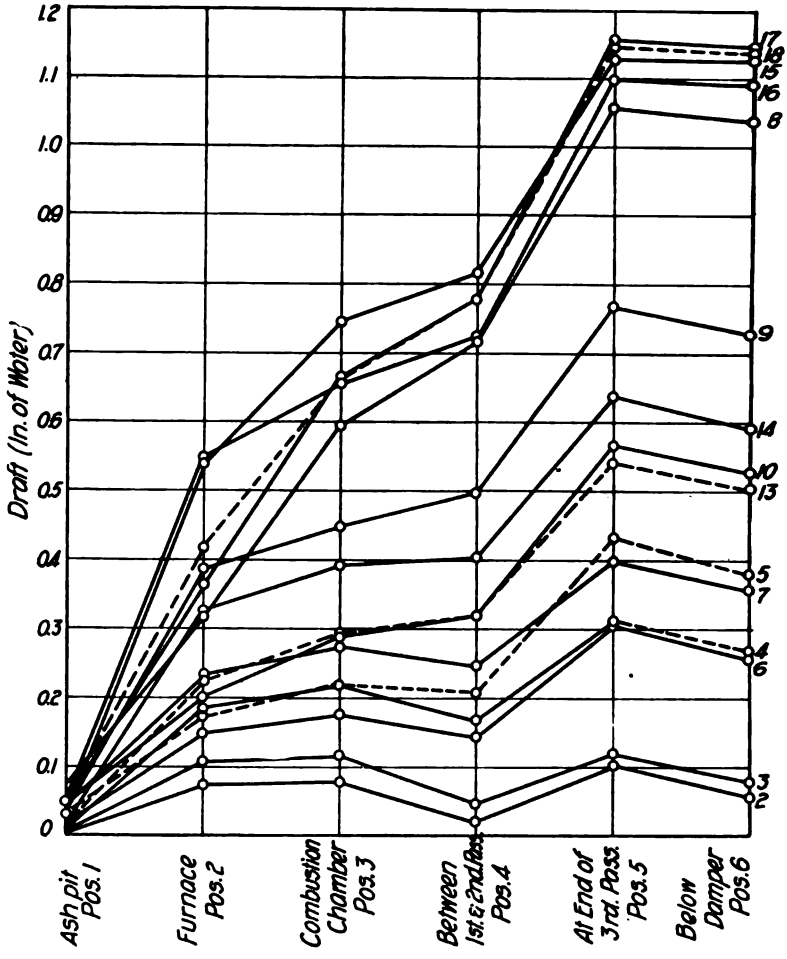

FIG. 16. DRAFTS THROUGHOUT THE BOILER SETTING.

from the value of h_1 and increase the gage reading (h'_1). In view of this fact, the natural expectation would be to find the gage readings increasing for the successive positions from the furnace to the breeching. Any decrease in velocity head (h_3), however, goes into an increase in

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pressure head (h_1) and therefore makes the gage reading (h'_1) less. Consequently, if a material increase in section, or reduction of gas volume occurs at any point, the decreased velocity head might diminish h'_1 enough to make it read less than the gage preceding it. This explains the dip in the curves at positions 4 and 6 in Fig. 16, since at position 4 there was an increase in section over that at position 3, and also at position 6 over that at position 5. It may, therefore, be seen that

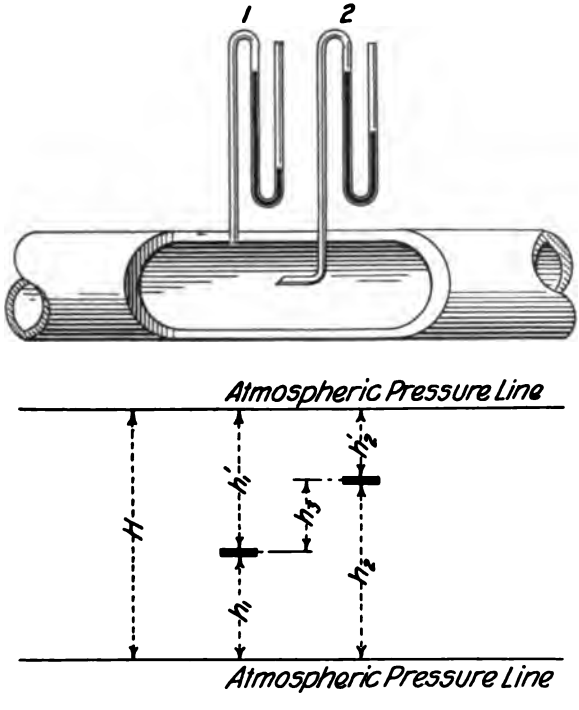

FIG. 17. DRAFT READINGS.

the practice of subtracting draft gage readings and calling the result the friction loss is incorrect unless the area of cross-section is the same at both points where the readings were taken. If this is not true a correction for velocity should be made.

Three factors determine the amount of draft necessary. They are the thickness of fire, the amount of dust in the coal, and the horsepower developed. Taking each load separately it may be seen from Fig. 16 that the 9½-in. fire invariably required the maximum draft for any

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given load. With the thinner fires sometimes one and sometimes another required more draft, depending largely upon the amount of dust, and slightly varying horsepower; conditions that seemed to affect the thinner fires to a larger extent than the thicker ones.

The curves in Fig. 18 show the relation existing between the draft loss from the ash-pit to the furnace, and the coal burned per square foot

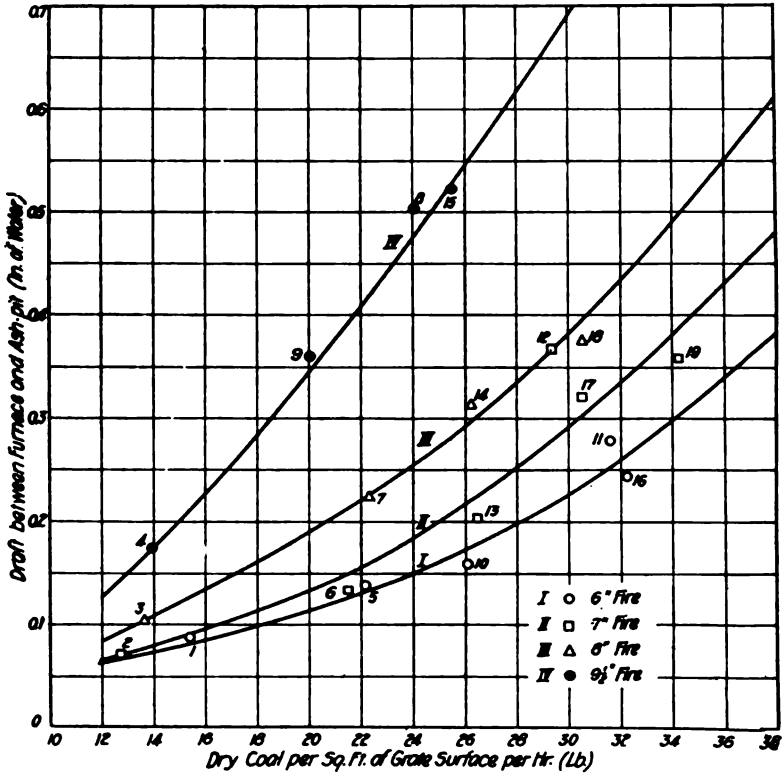

FIG. 18. RELATION BETWEEN DRAFT LOSS FROM ASH-PIT TO FURNACE, AND COAL BURNED PER SQ. FT. OF GRATE SURFACE PER HOUR WITH VARYING THICKNESS OF FIRE.

of grate surface per hour with varying thickness of fire. The amount of dust in the coal influences a number of these points to some extent. For instance, on test No. 12 the coal had a large percentage of dust. Points 17 and 19 were, therefore, given more weight in determining the end of the curve. From these curves it is evident that between the limits of 24 to 32 pounds per square foot of grate surface and with a normal

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thickness of fire of about 7½ inches, it requires a draft of approximately 0.01 in. of water per lb. of coal per square foot of grate surface per hour to burn Illinois screenings. This figure increases slightly at higher rates of combustion. With fires above 8 inches in thickness, the draft necessary for the coal to burn at a given rate becomes high enough to cause excessive air leakage, and it increases very rapidly with higher rates of combustion. The same draft required for the combustion of 38 lb. of coal per square foot per hour with a 7-inch fire is only sufficient for the combustion of 24 lb. per square foot per hour with a 9½-inch fire.

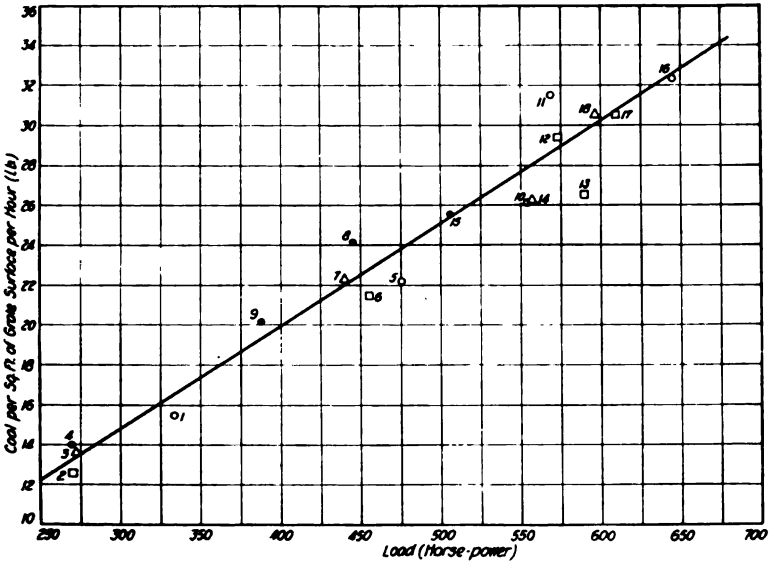

FIG. 19. RELATION BETWEEN LOAD AND COAL PER SQ. FT. OF GRATE SURFACE PER HOUR.

Fig. 19 shows the relation between rate of combustion and the horsepower developed. This relation may be represented by a straight line. The fact that the heating values of the coal varied somewhat, and also, that it was impossible to run all the tests with the tubes in exactly the same condition as regards soot deposits, tends to throw some of the points off of the curve.

Fig. 20 gives the draft requirements for a given horsepower using different thicknesses of fire. These curves necessarily assume the same form as those in Fig. 18, since the load is a straight line function of the rate of combustion. The points do not fall on these curves as they do

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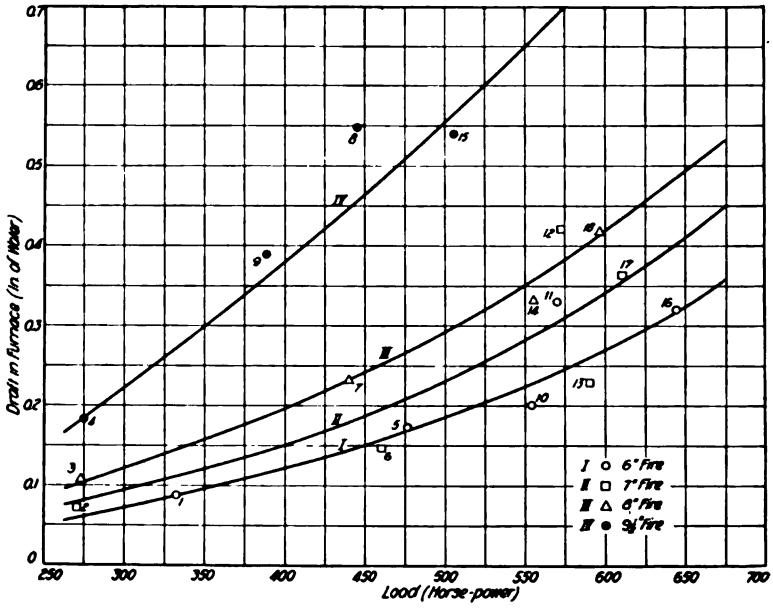

FIG. 20. RELATION BETWEEN LOAD AND DRAFT IN FURNACE.

on those of Fig. 18, however, because some factors such as soot on the tubes and the variable heating value of the coal influence the horsepower developed and do not influence the rate of combustion. Also they were plotted using draft over the fire instead of the difference between this and the draft in the ash pit.

18. *Excess Air.*—The curves in Fig. 21 give the relations between excess air, thickness of fire, and load. There are a number of other factors which affect the excess air, and the curves shown are at best only approximate. They are influenced by draft conditions, which in turn are somewhat dependent upon the dust content of the coal. For instance, it required practically the same draft to give ¾ load on test No. 9 as it did to give full load on test No. 15, due to the fact that on the former the dust content was higher than on the latter. This placed point 9 practically on the full load curve. In tests Nos. 11, 12, and 14 the dust was high, while in Nos. 10 and 13 it was normal. Hence more weight was given to the latter points in drawing the curves.

Taking the curve for each load separately it is evident that the percentage of excess air decreases to a minimum and then increases again

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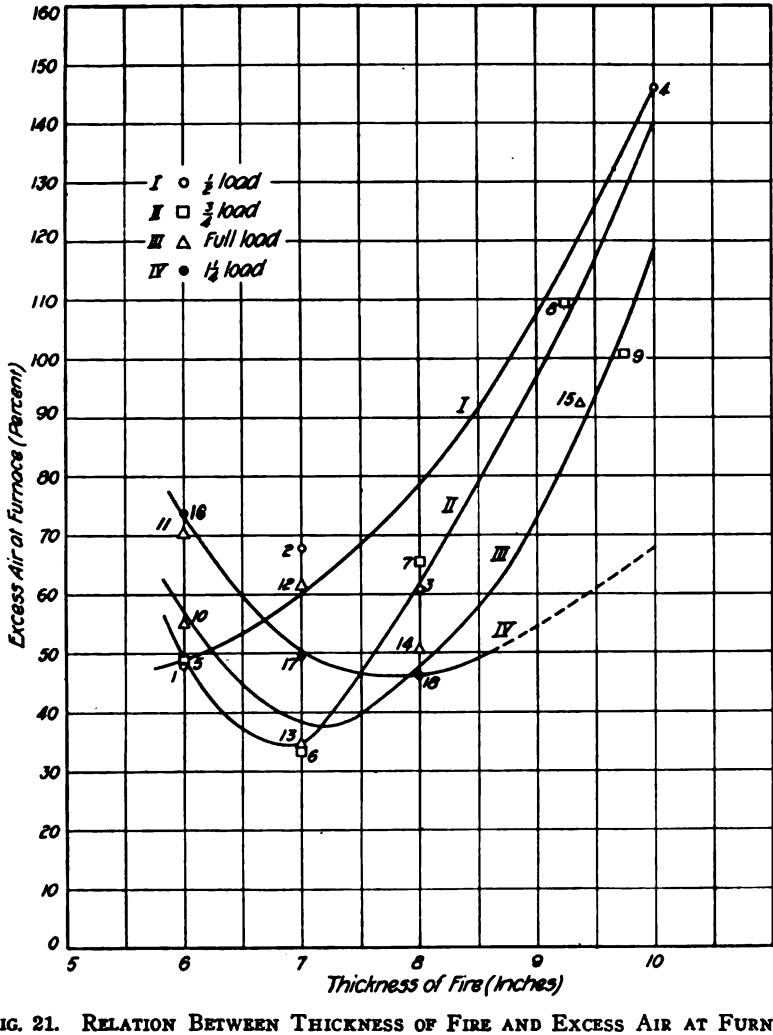

FIG. 21. RELATION BETWEEN THICKNESS OF FIRE AND EXCESS AIR AT FURNACE.

as the thickness of fire increases. These minimum points correspond very closely to the maximum ones shown on the efficiency curves in Figs. 7 to 11. With thin fires there is a marked tendency toward the formation of holes in the fuel bed, through which an excessive amount of air can pass. Also the entire fuel bed is more open and porous, and the small channels formed by the voids are shorter. Hence the air is not brought into as intimate contact with the incandescent surface of the

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coal as it otherwise would be. When the thickness of the fire is increased the passage becomes more tortuous and thus a larger proportion of the volume of air passing through the fuel bed comes into contact with the surface of the coal. At the same time the total area of the incandescent surfaces increases. Hence the excess air decreases. As the depth of fuel bed increases, however, the ash, as it forms, interferes more with the processes of combustion. At a thickness of about 7½ inches it appears that the ash begins to protect the incandescent surface from the action of the air, thus reducing the total area exposed and serving to mitigate the effect gained by the increased turbulence of the air. Therefore, increasing the depth of fuel on the grate beyond this point results in an increase in the per cent of excess air.

Inspection of the curves in Fig. 21 will show that for fires less than 7½ inches thick, the excess air increases when the load or rate of combustion is increased if a constant depth of fuel bed is maintained, while for fires above 7½ inches thick the reverse is true. When the load is made greater a larger volume of air is drawn through the fuel bed. That part actually coming into contact with the fuel also increases, but, in the case of thin fires, not so rapidly as the total. Hence in this case the excess air will increase with load. As the fires become thicker the part coming into contact is a larger proportion of the whole and it is not necessary to increase the total volume as much in order to obtain a given rate of combustion. At about 8 inches the tendency for the ash to choke the fuel bed becomes manifest and with fires above this thickness, when the draft is made greater, the increased velocity of the air tends to sweep the surfaces clean and to permit better combustion. Hence the percentage of excess air is apt to be lower at the higher rates when using thick fires.

The trend of these curves is also influenced somewhat by air leakage along the sides and at the back of the stoker. This would have had a more marked effect when the heavier drafts were used. A study of the per cent CO_2 at the furnace, however, from which the excess air was calculated will prove that this was at no time excessive. On all but two of the tests the CO_2 ran from 8 to 13 per cent, representing an excess of from 100 to 25 per cent, and in most cases was about 50 per cent.

In Fig. 22 the percentage of CO_2 in the flue gas has been plotted against the excess air expressed in per cent. Most of the points for this curve were taken from the analysis of the gas at the furnace, a

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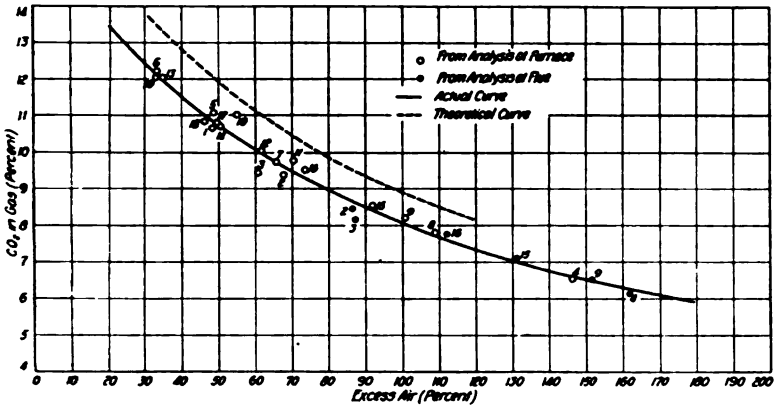

FIG. 22. RELATION BETWEEN EXCESS AIR AND CO_2 IN GAS.

few of the analyses at the flue being used, however, to establish some of the more remote points.

The theoretical curve shown in Fig. 22 was calculated from the following analysis, which was obtained by averaging the analyses for tests 1 to 18 inclusive:
C=65.77%, H_2=4.17%, O_2=6.27%, N_2=1.65%, S=4.52%, and Ash=17.62%. On the assumption that complete combustion took place, a gas analysis was computed for the above coal using 100, 75, 50, and 25 per cent excess air, the results of which are shown as a dotted line in Fig. 22.

The fact that the curve plotted from the analyses so closely parallels the theoretical curve may be taken as an indication of the accuracy of the gas analyses made on the tests. The explanation for the theoretical curve falling above the other lies in the fact that some of the carbon in the coal actually used went through the grates, some went to form soot and a small amount to form CO. This made the carbon actually appearing in the gas per lb. of coal about 5 per cent lower than that used for the theoretical curve.

An inspection of the curve will show that all of the points in the group representing about 35 per cent, or the minimum excess air, were obtained from 7-inch fires. Of those in the 50 per cent excess air group, 4 were obtained from 6-inch, one from 7-inch, and one from 8-inch fires. This seems to verify the conclusion that a fire approximately 7 inches thick is the best to use for this kind of coal.

19. *CO_2 and CO Relations.*—In Fig. 23 the O_2 in the flue gas is plotted against per cent excess air. This gives the same results as Fig.

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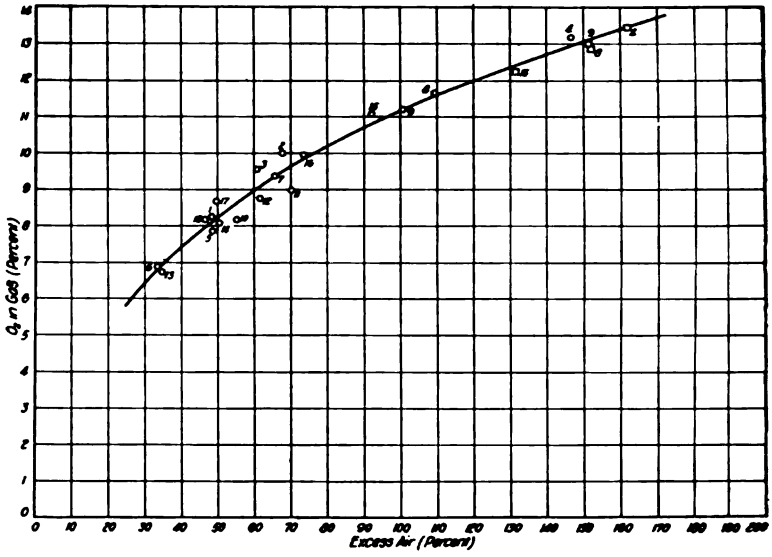

FIG. 23. RELATION BETWEEN EXCESS AIR AND O_2 IN GAS.

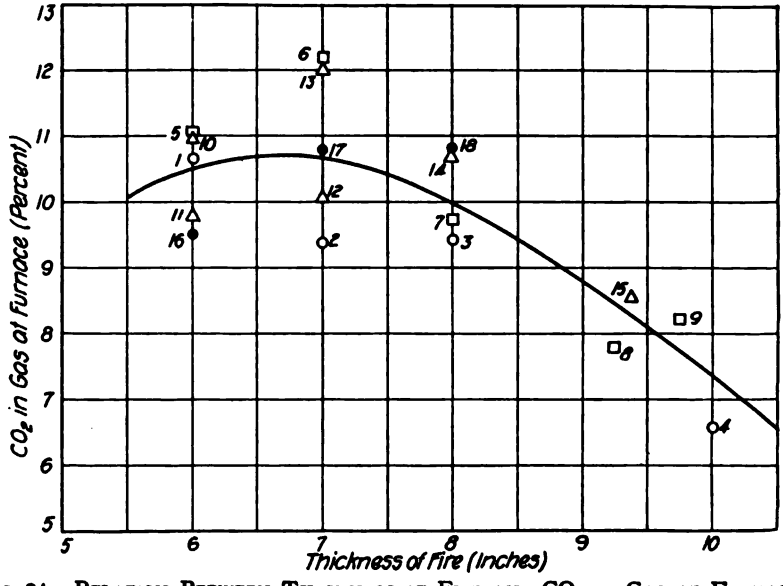

FIG. 24. RELATION BETWEEN THICKNESS OF FIRE AND CO_2 IN GAS AT FURNACE.

22, and of the two, the latter is the more valuable, since readings of CO_2 may be easily obtained in practice, while O_2 is not so readily determined. This curve, however, is given to serve as a check on the CO_2 curve.

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Fig. 24 was plotted in an attempt to obtain a relation between the thickness of the fire and the percentage of CO_2. The latter seems to have a tendency to reach a maximum on the 7-inch fire, from which point it falls off rapidly as the thickness of the fire is increased.

In Figs. 25 and 26, the points fall with sufficient irregularity to require a slight stretch of the imagination in order to draw a curve. They

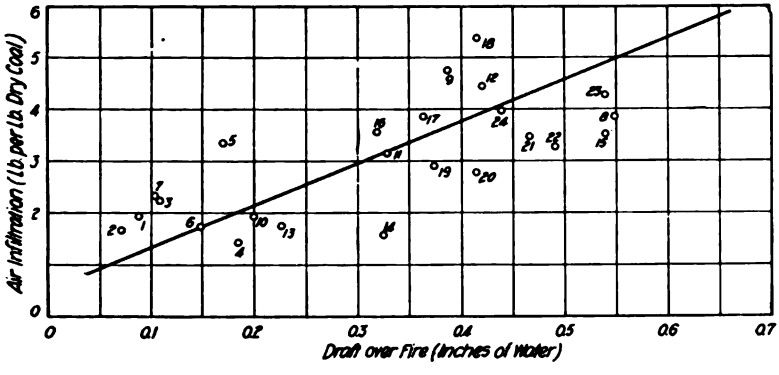

FIG. 25. RELATION BETWEEN DRAFT OVER FIRE AND AIR INFILTRATION.

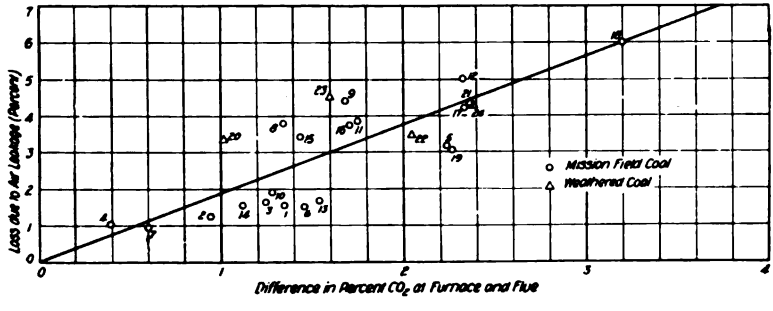

FIG. 26. RELATION BETWEEN LOSS DUE TO AIR LEAKAGE AND DIFFERENCE IN PER CENT CO_2 AT FURNACE AND FLUE.

seem to indicate straight line relations, however, the points being about equally distributed on both sides of a straight line.

Fig. 25 indicates that the air leakage through the setting is roughly proportional to the draft over the fire. This curve is affected by the variation in the pounds of gas per pound of coal, which is in turn influenced by a number of factors previously discussed.

In Fig. 26 the loss due to air leakage is plotted against the difference in the CO_2 readings at the furnace and at the flue. This curve is also

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affected by the variation in the pounds of gas per pound of coal, but it shows the importance of stopping all the air leaks.

An attempt was made to find a relation between the loss due to excess air and the loss due to CO formed, but it was found impossible to deduce such a relation from the data obtained.

The curves in Fig. 27 and Fig. 28, however, have been calculated by making use of the coal analysis given under the discussion of Fig. 22. This analysis is fairly representative of the coals used. From these curves some idea may be gained of the relation between the losses caused

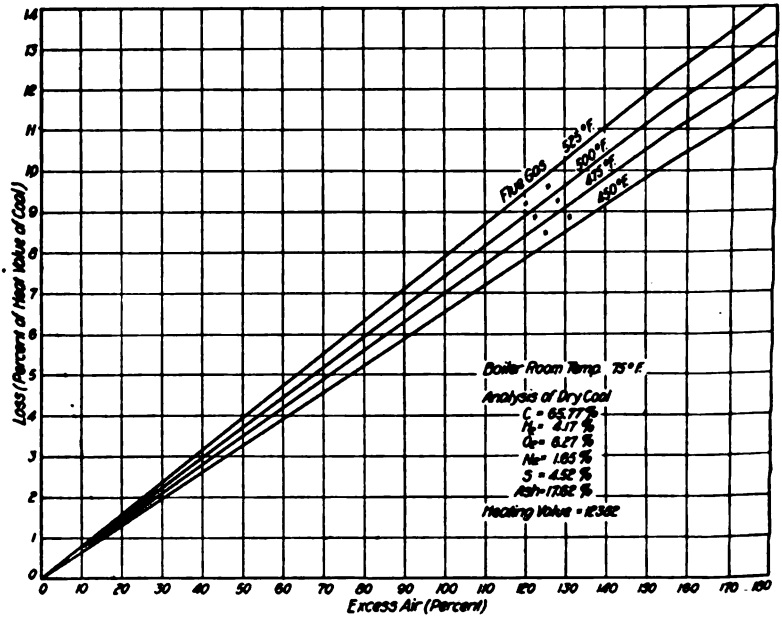

FIG. 27. LOSS CAUSED BY EXCESS AIR.

by excess air and CO. Fig. 27 has been calculated for the different flue gas temperatures indicated, on the assumption that the gases enter the furnace at 75° F., and that the temperature of the flue gas would be the same for the different percentages of excess air.

Assume that under certain operating conditions 11.5 per cent CO_2 is being obtained, and that there is no trace of CO in the gas. Reference to Fig. 22 will show that 40 per cent excess air is being used. Assume that the air supply is cut down, and when the per cent of CO_2 becomes 11.9, indicating 35 per cent excess air, a trace of CO begins to appear.

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If the air supply is further reduced, so that the per cent of CO_2 becomes 12.9, indicating that only 25 per cent excess air is being used, reference to Fig. 27 will show that the loss has been cut down from 2.61 per cent when using 35 per cent excess air, with the flue gas at 500° F., to 1.88 per cent when using 25 per cent excess, with the flue gas at the same temperature, giving a difference of 0.73.

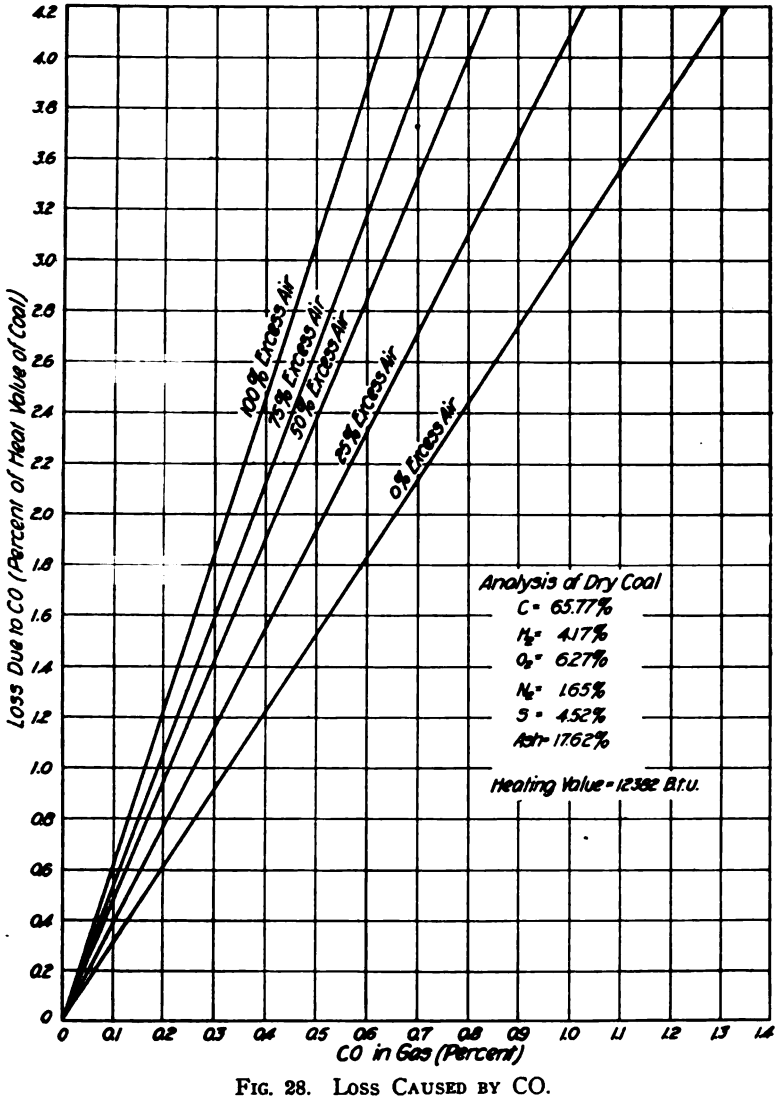

FIG. 28. LOSS CAUSED BY CO.

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From Fig. 28 the percentage of CO which represents a loss equal to 0.73 is 0.19 when 25 per cent excess air is being used. Hence, in the case assumed, if 0.19 or more of CO appears in the gas when the excess air is reduced from 35 per cent to 25 per cent, the gain from reducing the air supply is more than offset by the loss due to the formation of CO.

For every furnace and for every coal, therefore, there will be well defined limits beyond which the air supply can not be reduced without causing a greater loss due to incomplete combustion. Just how much CO is permissible for any given coal and installation may be determined by plotting a set of curves similar to the above, and by finding by experiment at what value of CO_2 a trace of CO begins to appear.

Fig. 29 gives the relation between loss due to combustible in ash and loss due to excess air. This is influenced by so many factors, and the points fall so irregularly that it is impossible to draw any very exact conclusions from it. The general tendency shown by the points may be fairly well represented by the straight line. This shows that for the chain grate stoker the tendency is for the loss due to excess air to increase as the combustible in the ash is decreased, and that the sum of the losses remains fairly constant at from 12 to 14 per cent. This means

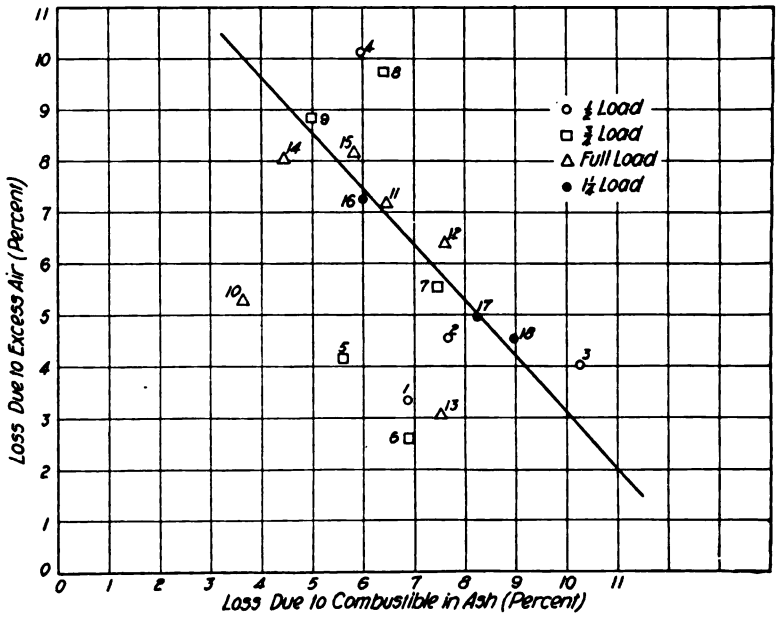

FIG. 29. RELATION BETWEEN LOSS DUE TO COMBUSTIBLE IN ASH AND LOSS DUE TO EXCESS AIR.

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that in order to reduce the carbon appearing in the ash it is necessary to increase the air supply, which in turn entails a corresponding loss. It is probable that the combustible loss, being a visible one, will cause more concern than the invisible excess air loss, but the above, in some measure, indicates the futility of trying to cut down on the former at the expense of the latter. Since this conclusion is based entirely upon data obtained

TABLE 9.

EFFICIENCY.

Test Number	Efficiency, Per Cent					
	Of Boiler and Furnace Excluding Grates	Of Boiler, Furnace and Grates	Of Furnace and Grates	Of Boiler	Of Furnace	Over-all Based on Zero Air Leakage
Col. No.	58	59	60	61	62	63
a	72	73				
b	123	124	127	128	130	133
1	78.58	73.36	78.07	93.97	83.82	74.85
2	74.17	68.55	76.46	89.66	82.80	69.69
3	71.10	63.92	74.27	86.06	82.74	65.35
4	65.80	62.01	74.54	83.20	79.26	62.92
5	70.92	67.01	80.00	83.75	84.72	69.68
6	71.72	66.85	80.10	83.46	85.99	68.09
7	63.75	59.05	77.55	76.14	83.81	59.76
8	62.09	58.20	76.92	75.66	82.16	61.08
9	61.49	58.42	77.80	75.09	81.87	61.74
10	66.34	63.96	82.04	77.95	85.16	65.48
11	65.40	61.21	79.33	77.16	84.79	64.16
12	65.59	60.67	78.07	77.71	84.49	64.59
13	75.36	69.87	79.15	88.27	85.59	71.35
14	70.32	64.75	77.97	83.04	84.76	66.04
15	69.45	65.50	78.40	83.54	83.22	68.41
16	64.71	60.84	79.23	76.79	84.26	63.75
17	67.68	62.11	78.15	79.48	85.18	65.45
18	69.28	63.19	77.53	81.50	85.16	68.16
19	70.71	66.04	80.05	82.50	85.73	68.60
20	66.21	64.50	78.62	82.03	80.83	67.24
21	67.44	63.05	75.85	83.12	81.99	66.62
22	70.71	65.98	79.33	83 18	85.21	68.87
23	66.90	60.04	72.46	82.86	81.14	63.75
24	67.22	64.20	80.40	79.86	84.44	67.66

from a chain grate it is impossible to state whether or not it will hold for other types of stokers or methods of firing.

20. *Heat Balance.*—The heat balance as used on these tests has been based on one pound of dry coal and has been considerably extended over the one given in the A. S. M. E. code.

In considering the question of steam generation, there are two separate processes which require attention. First, the heat must be evolved from the coal, and second, it must be absorbed by the water. The first

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is the function of the furnace and the second is a function of the boiler. Each of these processes entails losses, some of which are mutually independent and some of which are more or less dependent one upon the other. Since the efficiency of the unit should be the product of the efficiency of the furnace and the efficiency of the boiler alone, it is evident that all

TABLE 10.

WEIGHT OF AIR AND GASES PER LB. OF DRY COAL.

Test Number	Weight of Gas, Lb.			Weight of Air, Lb.				Weight of Excess Air, Lb.		Per Cent Excess Air	
	Theoretical for Complete Combustion	Actual, at Furnace	Actual, at Flue	Theoretical for Complete Combustion	Actual, at Furnace	Actual, at Flue	Leakage through Setting	At Furnace	At Flue	At Furnace	At Flue
Col. No.	64	65	66	67	68	69	70	71	72	73	74
a											
b	153	162	168	174	177	180	181	182	184	183	185
1	8.76	12.64	14.41	8.35	12.37	14.30	1.93	4.02	5.95	48.14	71.26
2	9.23	15.05	16.57	8.78	14.73	16.38	1.65	5.95	7.60	67.77	86.56
3	9.29	14.45	16.50	8.84	14.21	16.45	2.24	5.37	7.61	60.75	86.09
4	9.49	22.58	23.94	9.04	22.25	23.68	1.43	13.21	14.64	146.13	161.95
5	9.52	13.73	17.04	9.06	13.45	16.81	3.36	4.39	7.75	48.45	85.54
6	9.48	12.37	13.95	9.03	12.02	13.73	1.71	2.99	4.70	33.11	52.05
7	9.90	15.92	16.91	9.43	15.63	16.67	1.04	6.20	7.24	65.75	76.78
8	9.39	18.97	22.71	8.95	18.72	22.54	3.82	9.77	13.59	109.16	151.84
9	9.80	19.06	23.75	9.34	18.76	23.49	4.73	9.42	14.15	100.85	151.50
10	9.84	14.88	16.74	9.38	14.54	16.46	1.92	5.16	7.08	55.01	75.48
11	8.86	14.49	17.57	8.40	14.29	17.43	3.14	5.89	9.03	70.12	107.50
12	9.60	14.99	19.34	9.14	14.76	19.20	4.44	5.62	10.06	61.49	110.06
13	9.60	12.63	14.39	9.15	12.35	14.14	1.79	3.20	4.99	34.97	54.54
14	9.68	14.08	15.63	9.21	13.84	15.47	1.63	4.63	6.26	50.26	67.97
15	9.37	17.45	20.84	8.92	17.13	20.64	3.51	8.21	11.72	92.04	131.39
16	9.73	16.42	19.91	9.27	16.09	19.68	3.59	6.82	10.41	73.57	112.20
17	9.65	14.09	17.82	9.18	13.71	17.57	3.86	4.53	8.39	49.35	91.39
18	9.22	13.12	18.38	8.78	12.84	18.23	5.39	4.06	9.45	46.24	107.63
19	9.89	12.95	15.83	9.49	12.65	15.55	2.90	3.16	6.06	33.30	63.86
20	8.45	19.13	22.11	8.03	18.82	21.79	2.79	10.79	13.76	134.37	171.35
21	8.44	12.57	15.96	7.99	12.19	15.66	3.47	4.20	7.67	52.56	96.00
22	9.68	14.58	17.91	9.23	14.37	17.61	3.24	5.14	8.38	55.69	90.80
23	9.63	17.66	21.77	9.19	17.20	21.45	4.25	8.01	12.26	87.16	133.40
24	9.54	14.56	18.41	9.08	14.10	18.08	3.98	5.02	9.00	55.28	99.12

losses should be tabulated under but two heads, viz., boiler losses and furnace losses.

Since it requires a difference in temperature between the hot gases and the water for any heat to flow from the former into the latter, it is manifestly impossible to utilize any of the heat remaining in the gases after their temperature has fallen below the temperature of the steam,

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and hence of the water in the boiler. This introduces a set of unavoidable losses. In considering these unavoidable losses, and the loss due to excess air taken in at the furnace, it was not entirely self-evident under which head they logically belonged. Owing to reasons which will subsequently be discussed, the following division was finally decided upon as being the most convenient:

DEBIT.

1. Heat supplied by one pound of dry coal.

CREDIT.

2. Heat absorbed by boiler.

Furnace Losses (unavoidable).

3. Loss due to heat in theoretical weight of dry gas required for complete combustion, up to the temperature of the steam in the boiler.

4. Loss due to moisture in the coal, up to the temperature of the steam.

5. Loss due to moisture formed from H_2 in the coal, up to the temperature of the steam.

6. Loss due to moisture in the air, up to the temperature of the steam.

Furnace Losses (controllable).

7. Loss due to excess air up to the temperature of the steam in the boiler.

8. Loss due to CO formed.

9. Loss due to combustible in the ash.

10. Loss due to high temperature of the ash.

Boiler Losses.

11. Loss due to theoretical weight of dry gas for complete combustion, above the temperature of the steam in the boiler.

12. Loss due to moisture in coal, above the temperature of the steam.

13. Loss due to moisture formed from H_2 in coal, above the temperature of the steam.

14. Loss due to moisture in the air, above the temperature of the steam.

15. Loss due to excess air, above the temperature of the steam.

16. Total loss due to air leakage into the boiler setting.

17. Radiation, conduction, and unaccounted-for losses.

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In making up this heat balance, excess air was defined as the air taken into the furnace in excess of that necessary for complete combustion, and was determined from the analysis of the gas at the furnace. Air leakage was defined as the air leaking into the gases after they had passed the bridge-wall. It was found by comparing the gas analyses at the flue and at the furnace. Therefore, all of it was charged against the boiler.

TABLE 11.

CHEMICAL ANALYSES.

Test Number	Proximate Analysis of Coal as Fired					Ultimate Analysis of Dry Coal					
	Fixed Carbon Per Cent	Volatile Per Cent	Moisture Per Cent	Ash Per Cent	Sulphur Separately Determined Per Cent	Carbon Per Cent	Hydrogen Per Cent	Oxygen Per Cent	Nitrogen Per Cent	Sulphur Per Cent	Ash Per Cent
Col. No.	75	76	77	78	79	80	81	82	83	84	85
a	32	33	34	35	36	37	38	39	40	41	42
b	38	39	41	42	43	50	51	52	53	54	55
1	30.50	33.09	17.89	18.52	4.29	60.31	4.06	6.71	1.13	5.23	22.56
2	32.63	34.06	17.41	15.90	4.11	63.87	4.10	6.15	1.65	4.98	19.25
3	33.34	34.43	16.56	15.67	4.13	64.29	4.13	6.19	1.66	4.95	18.78
4	33.01	35.20	17.58	14.21	3.81	65.86	4.23	6.35	1.70	4.62	17.24
5	32.67	35.68	17.64	14.02	3.83	66.02	4.24	6.36	1.71	4.65	17.02
6	32.13	36.31	17.29	14.27	3.94	65.74	4.22	6.33	1.70	4.76	17.25
7	31.20	40.75	16.04	12.01	3.19	69.03	4.43	6.65	1.79	3.80	14.30
8	32.63	35.93	16.43	15.01	3.99	65.13	4.18	6.27	1.69	4.77	17.96
9	33.75	38.16	15.53	12.56	3.55	68.22	4.38	6.57	1.76	4.20	14.87
10	33.22	37 96	16.60	12.22	3.28	68.63	4.40	6.61	1.78	3.93	14.65
11	30.25	34.36	15.20	20.19	3.99	61.96	3.47	4.52	1.53	4.71	23.81
12	33.68	36.01	16.47	13.84	3.55	66.75	4.28	6.43	1.73	4.25	16.56
13	32.41	37.16	16.64	13.79	3.52	66.79	4.29	6.43	1.73	4.22	16.54
14	31.84	38.66	16.32	13.18	3.77	67.21	4.31	6.48	1.74	4.51	15.75
15	32.73	36.66	15.24	15 37	4.06	64.97	4.17	6.26	1.68	4.79	18.13
16	34.43	37.23	15.44	12.90	3.77	67.67	4.34	6.52	1.75	4.46	15.26
17	34.17	36.29	15.33	14.21	3.35	67.59	4.11	6.06	1.51	3.95	16.78
18	32.16	35.85	15.46	16.53	3.91	63.92	4.10	6.16	1.65	4.62	19.55
19	36.56	38.61	13.85	10.98	3.09	67.75	5.30	8.54	2.08	3.59	12.74
20	35.88	30.23	20.35	13.54	2.99	59.19	4.55	13.44	2.07	3.75	17.00
21	34.06	30.35	22.04	13.55	2.97	58.87	4.53	13.25	2.15	3.81	17.39
22	45.41	29.92	12.45	12.22	1.89	68.04	4.66	8.79	2.39	2.16	13.96
23	43.36	29.09	17.07	10.48	1.81	67.75	4.90	10.77	1.75	2.19	12.64
24	38.54	30.99	20.02	10.45	1.26	67.62	4.68	10.59	2.47	1.58	13.06

On the basis of a rigid analysis, regarding the furnace as being entirely separate from the boiler, the unavoidable loss might have been included under boiler losses. We can conceive of a unit so built that the water comes into it at room temperature, and is gradually passed through and heated up to steam temperature. In this kind of a unit the gas might also be made to leave at room temperature. In this hypothetical

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apparatus any heat in the coal which was not given to the gas would be chargeable to the furnace, while any that was not taken from the gas down to room temperature would be charged against the boiler. This conception leads to a difficulty with the excess air. The amount of excess air depends upon the design and operation of the furnace, and as a boiler furnace, the air should be charged against it. But under the above

TABLE 12.

CHEMICAL ANALYSES (CONTINUED).

Test Number	Analysis of Ash and Refuse		Analysis of Dry Gases at Furnace Per Cent by Volume				Analysis of Dry Gases at Flue Per Cent by Volume			
	Carbon Per Cent	Earthy Matter Per Cent	CO_2	O_2	CO	N_2 By Difference	CO_2	O_2	CO	N_2 By Difference
Col. No.	86	87	88	89	90	91	92	93	94	95
a	44	45					84	85	86	88
b	56	57	139	140	141	142	143	144	145	146
1	22.84	77.16	10.69	8.23	0.23	80.85	9.34	8.98	0.18	81.50
2	22.93	77.07	9.39	9.94	0.18	80.49	8.45	10.49	0.20	80.86
3	34.67	65.33	9.43	9.52	0.24	80.81	8.18	10.58	0.20	81.05
4	22.95	77.05	6.54	13.18	0.12	80.16	6.15	13.47	0.12	80.26
5	21.49	78.51	11.06	7.84	0.16	80.94	8.82	10.15	0.14	80.89
6	23.55	76.45	12.20	6.87	0.06	80.87	10.74	7.88	0.05	81.33
7	30.07	69.93	9.74	9.31	0.08	80.87	9.14	9.82	0.08	80.96
8	26.36	73.64	7.80	11.61	0.02	80.57	6.45	12.86	0.05	80.64
9	24.37	75.63	8.20	11.16	0.13	80.51	6.51	12.97	0.13	80.39
10	14.96	85.04	11.00	8.14	0.04	80.82	9.72	9.23	0.04	81.01
11	21.86	78.14	9.78	8.99	0.05	81.18	8.03	10.90	0.02	81.05
12	30.34	69.66	10.07	8.75	0.01	81.17	7.74	11.21	0.00	81.05
13	27.12	72.88	12.03	6.72	0.05	81.20	10.49	8.34	0.05	81.12
14	29.72	70.28	10.70	8.06	0.05	81.19	9.58	9.03	0.06	81.33
15	23.10	76.90	8.54	11.01	0.04	80.41	7.10	12.21	0.04	80.65
16	24.52	75.48	9.50	9.91	0.01	80.58	7.79	11.46	0.00	80.75
17	30.72	69.28	10.78	8.67	0.04	80.51	8.44	10.75	0.03	80.78
18	29.78	70.22	10.81	8.18	0.05	80.96	7.61	11.39	0.04	80.96
19	20.81	79.19	12.02	6.84	0.03	81.11	9.75	9.38	0.02	80.85
20	13.00	87.00	7.37	12.34	0.01	80.28	6.35	13.50	0.01	80.14
21	31.08	68.92	10.58	9.17	0.07	80.18	8.22	11.36	0.09	80.33
22	33.02	66.98	10.68	7.95	0.01	81.36	8.63	10.75	0.01	80.61
23	55.20	44.80	8.23	11.92	0.04	79.81	6.63	13.17	0.03	80.17
24	32.15	67.85	10.96	8.84	0.01	80.19	8.58	10.93	0.01	80.48

analysis so far as the furnace is concerned it merely takes this air in and heats it. All the heat that is put into it appears in the gas leaving the furnace, the only difference being that there is a larger volume at a lower temperature. A failure to utilize this heat would then be due to imperfections in the boiler.

The arrangement given was, therefore, considered the more practical, because the furnace in question is not a separate piece of apparatus for

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the purpose of heating gases, but is an integral part of the unit, and the heat in the gas below steam temperature is simply not available any more than that in the CO, which would ignite and burn if it were not cooled by the boiler tubes. On this basis any excess air taken by the furnace would increase the amount of heat unavailable, and this amount should

TABLE 13.

HEAT BALANCE, BASED ON 1 LB. OF DRY COAL.

Test Number	Heating Value of 1 Lb. of Dry Coal B. t. u.	Heat Absorbed by Boiler B. t. u.	Furnace Losses, B. t. u. per Lb. of Dry Coal					
			Unavoidable Loss, up to Temperature of Steam			Controllable Loss		
			Due to Theoretical Weight of Dry Gas	Due to Moisture in Coal	Due to H_2 in Coal	Due to Excess Air	Due to CO in Gas	Due to Combustible in Ash
Col. No.	96	97	98	99	100	101	102	103
a	50							
b	62	186	189	196	198	206	209	211
1	10979	8054	590	257	431	271	106	753
2	11537	7909	617	248	434	398	135	884
3	11629	7433	629	234	439	364	136	1190
4	11815	7337	623	250	446	367	118	704
5	11933	7996	632	252	449	292	97	664
6	11947	7966	637	246	447	201	28	819
7	12408	7327	680	236	472	426	55	926
8	11821	6880	616	230	441	641	47	753
9	12292	7181	667	217	466	641	127	611
10	12335	7889	676	236	469	355	28	451
11	10954	6705	582	210	366	387	14	706
12	11949	7249	647	232	454	379	0	909
13	11876	8297	647	235	455	216	29	894
14	12184	7889	662	231	459	317	39	976
15	11748	7695	624	211	441	547	34	680
16	12186	7414	662	216	461	464	0	728
17	11921	7404	647	213	435	304	22	984
18	11472	7249	607	214	432	267	30	1028
19	12622	8336	691	191	566	221	13	836
20	11179	7210	568	301	482	725	9	305
21	10497	6618	581	335	483	290	59	787
22	12103	7986	652	168	494	346	7	835
23	12153	7297	681	245	526	568	27	1300
24	11985	7695	640	295	496	337	7	574

be charged against the furnace; that part above steam temperature not absorbed by the boiler should be charged against the latter. The air leakage loss should be charged to the boiler, because this air went through the setting, and not through the furnace.

This treatment might have been made slightly more rigid by in-

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cluding the unavoidable losses as a separate set of items charged up to the unit as a whole, but in this case the efficiency of the unit would not have been the product of the boiler efficiency and furnace efficiency. Any arrangement that is made, however, is merely a matter of distribution and in no way affects the calculation of the losses themselves.

Two items are given in the above heat balance which have not been

TABLE 14.

HEAT BALANCE (CONTINUED).

Test Number	Boiler Losses, B. t. u. per lb. of Dry Coal					
	Above Temperature of Steam				Due to Air Leakage	Radiation, Conduction and Unaccounted-for
	Due to Theoretical Weight of Dry Gas	Due to Moisture in Coal	Due to H_2 in Coal	Due to Excess Air		
Col. No. a	104	105	106	107	108	109
b	216	218	219	221	224	226
1	203	11	18	93	175	+ 17
2	198	9	16	127	146	+ 416
3	177	8	15	102	194	+ 708
4	236	11	20	329	129	+ 755
5	447	21	37	206	381	+ 459
6	353	16	29	112	179	+ 894
7	417	17	35	261	115	+ 1451
8	490	21	41	509	450	+ 702
9	463	18	39	445	545	+ 872
10	560	23	47	293	241	+ 1067
11	596	25	44	397	418	+ 505
12	655	28	55	383	603	+ 355
13	429	19	36	143	200	+ 276
14	467	20	39	224	190	+ 671
15	466	19	39	409	409	+ 174
16	590	23	49	414	462	+ 703
17	608	24	49	286	502	+ 443
18	571	24	48	251	688	+ 63
19	645	23	65	206	392	+ 437
20	495	31	50	635	374	− 6
21	515	36	52	256	451	+ 34
22	606	19	55	322	421	+ 192
23	553	25	53	460	544	− 126
24	604	33	56	318	519	+ 411

calculated for this set of tests. They are the heat loss due to moisture carried in by the air, and the heat loss due to the temperature of the leaving ash. The moisture loss forms a very small percentage, and can not be determined with any very great degree of accuracy. It is true that the relative humidity may be found by means of a wet and dry bulb hygrometer, but this varies through a considerable range in different parts

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of the room, and is so affected by purely local conditions that it was not considered of much importance.

It was not possible to obtain a satisfactory temperature of the ash. As it leaves the grate the ash is probably in the neighborhood of 1800° F. Part of this heat, however, is given back to the setting and to the

TABLE 15.

HEAT BALANCE (CONTINUED.)

Test Number	Heat Absorbed Per Cent	Furnace Losses, Per Cent of Heat in Dry Coal					
		Unavoidable Loss, up to Temperature of Steam			Controllable Loss		
		Due to Theoretical Weight of Dry Gas	Due to Moisture in Coal	Due to H_2 in Coal	Due to Excess Air	Due to CO in Gas	Due to Combustible in Ash
Col. No.	110	111	112	113	114	115	116
a							
b	227	228	229	230	232	233	234
1	73.36	5.37	2.34	3.93	2.47	0.97	6.86
2	66.55	5.35	2.15	3.76	3.45	1.17	7.66
3	63.92	5.41	2.01	3.78	3.13	1.17	10.23
4	62.01	5.27	2.12	3.77	7.34	1.00	5.96
5	67.01	5.30	2.11	3.76	2.45	0.81	5.56
6	66.85	5.33	2.06	3.74	1.68	0.23	6.86
7	59.05	5.48	1.82	3.80	3.43	0.44	7.46
8	58.20	5.21	1.95	3.73	5.42	0.40	6.37
9	58.42	5.43	1.77	3.79	5.21	1.03	4.97
10	63.96	5.48	1.91	3.80	2.88	0.23	3.66
11	61.21	5.31	1.92	3.34	3.53	0.13	6.44
12	60.67	5.41	1.94	3.80	3.17	0.00	7.61
13	69.87	5.45	1.98	3.83	1.82	0.24	7.53
14	64.75	5.43	1.90	3.77	2.60	0.32	8.01
15	65.50	5.31	1.80	3.75	4.66	0.29	5.79
16	60.84	5.43	1.77	3.78	3.81	0.00	5.97
17	62.11	5.43	1.79	3.65	2.55	0.19	8.25
18	63.19	5.29	1.87	3.77	2.33	0.26	8.96
19	66.04	5.47	1.51	4.48	1.75	0.10	6.62
20	64.50	5.08	2.69	4.31	6.49	0.08	2.73
21	63.05	5.53	3.19	4.60	2.76	0.56	7.50
22	65.98	5.39	1.39	4.08	2.86	0.06	6.90
23	60.04	5.60	2.02	4.33	4.67	0.22	10.70
24	64.20	5.34	2.46	4.14	2.81	0.06	4.79

air entering the furnace. The ash leaving the pit was at a temperature of about 600° F., but the heat down to this point had not all been utilized because a large part of it was dissipated through the walls and door of the pit. The true temperature was probably somewhere between these two, but owing to the small size of the loss and the uncertainty regarding the temperature it was not considered worth while to calculate the loss.

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Fig. 30 will give some idea as to the relative magnitude of the several losses. In this figure the heat balances for five typical tests have been plotted, from which it appears that the controllable losses due to the gases escaping at a temperature higher than that of the steam are in all cases less than the unavoidable losses up to the temperature of the steam. About the only method of diminishing these losses is by reducing the

TABLE 16.

HEAT BALANCE (CONTINUED).

Test Number	Boiler Losses, Per Cent of Heat in Dry Coal					
	Above Temperature of Steam				Due to Air Leakage	Radiation, Conduction and Unaccounted-for
	Due to Theoretical Weight of Dry Gas	Due to Moisture in Coal	Due to H_2 in Coal	Due to Excess Air		
Col. No. a	117	118	119	120	121	122
b	236	237	238	240	241	242
1	1.85	0.10	0.16	0.85	1.59	0.15
2	1.72	0.08	0.14	1.10	1.27	3.60
3	1.52	0.07	0.13	0.88	1.67	6.08
4	2.00	0.09	0.17	2.78	1.09	6.40
5	3.75	0.18	0.31	1.72	3.19	3.85
6	2.95	0.13	0.24	0.94	1.50	7.50
7	3.37	0.14	0.28	2.10	0.93	11.70
8	4.14	0.18	0.35	4.31	3.81	5.93
9	3.77	0.15	0.32	3.62	4.43	7.09
10	4.54	0.19	0.38	2.38	1.95	8.64
11	5.44	0.23	0.40	3.62	3.82	4.61
12	5.48	0.23	0.46	3.21	5.05	2.97
13	3.62	0.16	0.30	1.20	1.68	2.33
14	3.83	0.16	0.32	1.84	1.56	5.51
15	3.97	0.16	0.33	3.48	3.48	1.48
16	4.84	0.19	0.41	3.40	3.79	5.77
17	5.10	0.20	0.41	2.40	4.21	3.71
18	4.97	0.21	0.42	2.19	6.00	0.54
19	5.12	0.18	0.52	1.63	3.11	3.47
20	4.43	0.28	0.45	5.68	3.35	
21	4.91	0.34	0.50	2.44	4.30	0.32
22	5.01	0.16	0.45	2.66	3.48	1.58
23	4.55	0.21	0.44	3.79	4.48	
24	5.04	0.28	0.47	2.65	4.33	3.43

excess air and eliminating air leakage, since these are the only factors which are not at a minimum.

The largest loss seems to be due to the combustible in the ash. In a general way, however, where this is high, the excess air loss is low, tests No. 3 and No. 4 indicating this very well.

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Since the unaccounted-for loss includes the effects of various experimental errors, as well as radiation and conduction, it has no definite relation to the other losses. It can be reduced, however, by adequate heat insulation for the setting.

A study of the boiler efficiency and furnace efficiency in Table 9 will show that these two are very nearly the same, thus indicating that the losses are about equally distributed between them. This should afford an easy means of checking them against each other, as any considerable loss in either one would immediately become apparent.

V. Tests on Weathered Coal.

Five tests were run on weathered coal. As previously stated, this fuel had been placed in bins in January 1908, and formed part of the series reported in Engineering Experiment Station Bulletin No. 38, "The Weathering of Coal," by S. W. Parr and W. F. Wheeler. In Table 17 may be found a statement of the kind of coal, the conditions of storage, heating value, and dust content of this fuel. The data concerning original size and heating value have been taken from Bulletin No. 38.

From Table 17 it may be seen that from the physical standpoint, and in heating value, the coal had deteriorated considerably from its original condition, and that at the time of the tests it was in about the same class as the Mission Field screenings. A discussion of the physical and chemical

Table 17.

Deterioration Due to Weathering.

Test No.	Coal	How Stored	Jan. 1908		Jan. 1914	
			Per Cent Passing a ¼" Screen	B. t. u. per Lb. Dry Coal 7 Days After Mining	Per Cent Passing a ¼" Screen	B. t. u. per Lb. Dry Coal at Time of Test
20	Sangamon Co. Nut	Open bin	1.2	11800	31.9	11179
21	Sangamon Co. Screenings	Covered bin	26.3	11684	45.1	10497
22	Williamson Co. Nut	Covered bin	1.4	12341	13.9	12103
23	Williamson Co. Screenings	Covered bin	38.8	12287	42.9	12153
24	Vermilion Co. Nut	Open bin	13.6	12412	35.0	11985

properties, however, is outside of the scope of this work, it being concerned merely with the behavior of this fuel when used in a boiler furnace.

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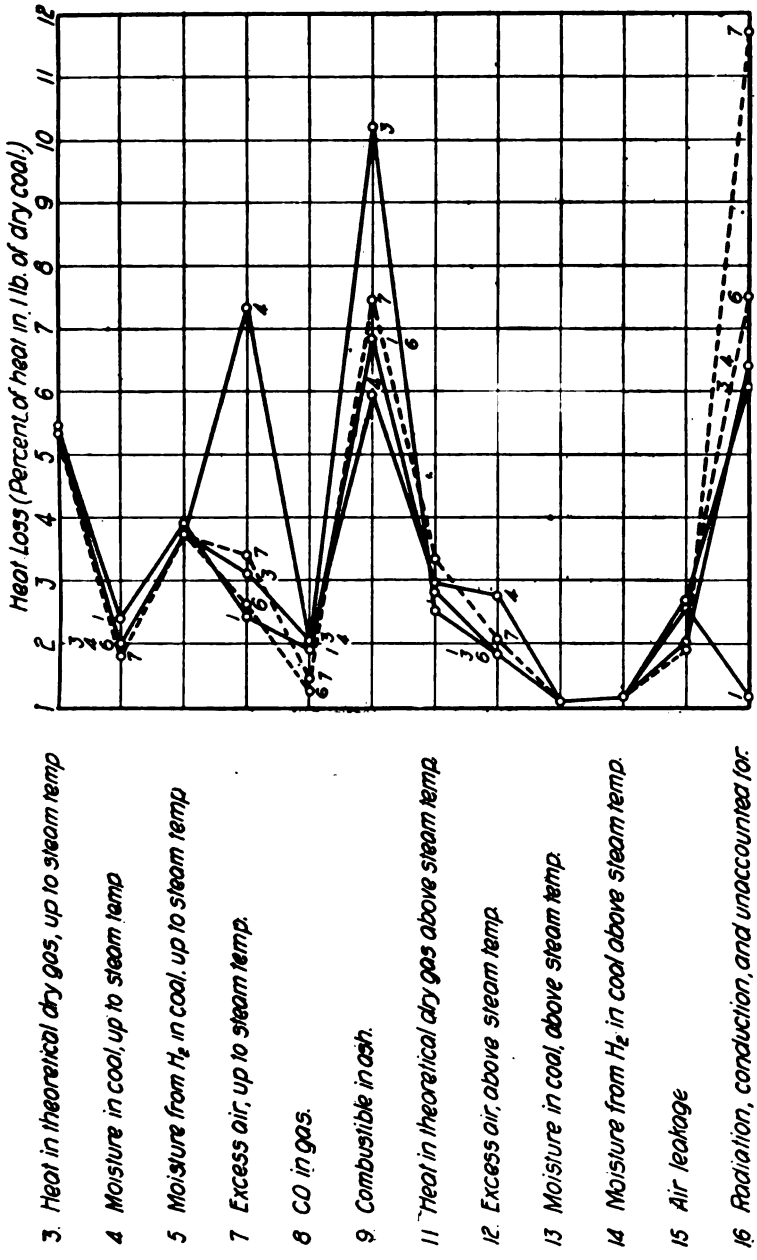

FIG. 30. RELATIVE MAGNITUDE OF VARIOUS LOSSES.

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Since the samples were limited in quantity, these were necessarily short tests, but every effort was made to maintain uniform conditions, and particularly to have the fuel bed in the same condition at the start and close, thus reducing the chances of error as much as possible. An inspection of the heat balance will prove that no very serious error arose.

As the coal appeared to be covered with a coating of ash or clay, it was expected that considerable trouble would be experienced in burning it. This, however, did not prove to be the case. On the first test the coal banked slightly at the water-back, and the whole amount on the grate became clinkered. It immediately became evident that in order to run at all, the coal had to be kept away from the water-back. After the clinker had been removed, a fresh start was made and care was taken to keep the fuel bed from four to six inches away from the water-back. When this was done no further trouble was experienced.

All of the tests on the weathered coal were run with a 7-inch fire with the exception of test No. 21, which was run with a 6½-inch fire because the coal seemed to be broken up finer than the rest. In all cases but one the fire burned white and did not require much attention. The exception was test No. 23. On looking up the firing record, it was found that on this test the fire was rather cold and gave evidence of being too thick. It was found also that the fire required considerable attention, having to be barred and raked much oftener than the rest. This process, of course, let in air and lowered the CO_2. An inspection of the efficiencies in Table 9 will show that while the boiler efficiency compares favorably with the rest, the efficiency of the furnace and grate was low. Also, it appears from the heat balance that the loss due to combustible in the ash was high. A comparison of the dust in this coal with that in test No. 21 discloses the fact that they were very nearly the same. In that test a 6½-inch fire was used and the results were satisfactory. These facts all lead to the conclusion that a 7-inch fire was too thick in the case of test 23, and that the trouble encountered on this test should be charged against the lack of experience in handling the coal, rather than against the coal itself. The capacity obtained was also lower than in the other weathered coal tests.

A comparison of the draft required for a given rate of combustion for the weathered coal with that for the Mission Field screenings is made in Fig. 31. Referring to this figure it becomes evident that more draft is required to produce a given rate of combustion with the former than

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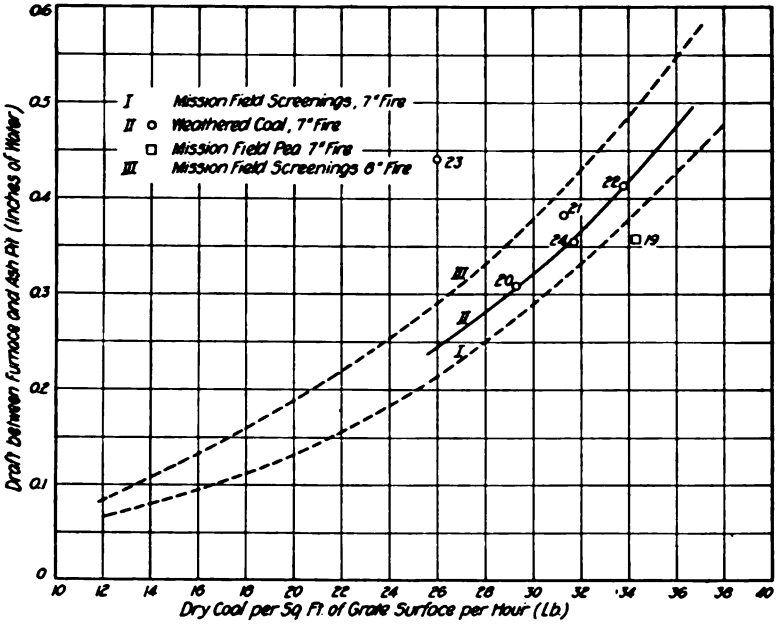

FIG. 31. DRAFT REQUIRED FOR A GIVEN RATE OF COMBUSTION FOR WEATHERED COAL AND MISSION FIELD SCREENINGS.

TABLE 18.

COMPARISON OF DUST CONTENT, MISSION FIELD SCREENINGS AND WEATHERED COAL.

Mission Field Screenings		Weathered Coal	
Test No.	Per Cent Dust	Test No.	Per Cent Dust
2	34.8	20	31.9
6	19.6	21	45.1
12	40.3	22	13.9
13	26.3	23	42.9
17	24.7	24	35.0

with the latter. That this is not caused by dust in the coal may be shown by Table 18 and Fig. 18.

The dust content of the coals from which the two curves for the 7-inch fires were plotted is practically the same. In each case there were one or two tests in which the dust content was high, and the points representing these fell higher than the curve; i. e., point 12 was off of the

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curve in Fig. 18, while in Fig. 31 points 21 and 23 were not on the curve. In Fig. 18 the points of low dust content are below the curve for 7-inch fires, while in Fig. 31 all the points are above this curve, which is shown dotted. This seems to lead to the conclusion that the dust is not responsible for the high draft requirement. A possible explanation is that this coal is harder to ignite, owing to the partially oxidized condition indicated by the gray coating on the outside, and therefore it requires more draft than the Mission Field coal for a given rate of combustion.

The efficiencies obtained from the weathered coal compare very favorably with those obtained from the Mission Field coal. For the sake of convenience these have been tabulated for corresponding loads, and are shown in Table 19. The average for each set is about 64 per cent, the weathered coal showing an advantage at heavier loads. It is also worthy of note that greater capacity was obtained with the weathered coal than with the screenings, the heat value of both being about the same.

On comparing firing sheets, it was also found that on the whole, when weathered coal was being used, the fires required less attention, such as slicing and raking, than they did when fresh screenings were being used.

TABLE 19.

COMPARISON OF BOILER PERFORMANCES ON MISSION FIELD AND WEATHERED COAL.

Mission Field Coal			Weathered Coal		
Test No.	Boiler H. P. Developed	Efficiency of Boiler, Furnace and Grate	Test No.	Boiler H. P. Developed	Efficiency of Boiler, Furnace and Grate
10	554.0	63.96	20	568.5	64.50
11	569.6	61.21	21	557.2	63.05
12	572.7	60.67	22	727.1	65.98
13	589.0	69.87	23	509.6	60.04
14	555.9	64.75	24	655.0	64.20
15	506.6	65.50			
16	644.0	60.84			

VI. SUMMARY AND CONCLUSIONS.

General Conclusions.—1. The over-all efficiency of the unit tested, under the average conditions which prevailed during the tests, and when

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developing in the neighborhood of 550 boiler horsepower, is about 64 per cent.

2. The best thickness of fire for Mission Field screenings is about 7 or 7½ inches, depending upon the amount of fine material in the coal.

3. The maximum capacity developed was 768 boiler horsepower. This was obtained from pea coal with a 7-inch fuel bed and a draft over the fire of 0.375 inches of water.

4. The best rate of combustion is about 26 pounds of dry coal per sq. ft. of grate surface per hour. The drafts over the fire required to give this rate are as follows: 0.18 in. water for a 6-in. fire, 0.22 in. water for a 7-in fire, 0.29 in. water for an 8-in. fire, and 0.52 in. water for a 9½-in. fire.

5. The greatest losses were due to combustible in the ash, excess air, and air leakage.

6. The smallest loss was that due to the formation of CO.

7. With the exception of the air leakage loss, which can be practically eliminated without affecting any of the others, the rest of the losses are to some extent interrelated, and there are certain well defined limits within which the excess air must be maintained in order to reduce the others to a minimum. This is approximately 30 to 40 per cent.

8. The sum of the excess air loss and the loss due to combustible in the ash was nearly constant. It amounted to about 14 per cent, and a decrease in one source of loss could be accomplished only by an increase in the other. Hence the formation of CO was the factor having the greatest bearing on the extent to which excess air could be reduced. The latter could be decreased until CO appeared in excess.

9. The percentage of CO_2 depends to some extent on the attention required by the fire. The CO_2 was low on the tests during which the side door had to be frequently opened in order to allow barring and raking of the fire.

10. The form of baffling shown in Fig. 1 and used on these tests makes the maintenance of high furnace temperatures possible, and reduces the amount of smoke formed.

11. This form of baffling gives trouble due to the difficulty in removing the soot which collects in the corner where the first baffle joins the tile roof of the furnace.

21. *Conclusions Relative to Weathered Coal.*—12. Burning weathered coal is largely a question of correct handling and ignition. Under these circumstances it gives as good results as fresh screenings. It

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must be borne in mind, however, that the conclusions drawn from the weathered coal tests are based solely on the burning of this fuel and do not deal with the deterioration in storage. In making comparisons it must also be noted that from the physical and chemical standpoint the original fuel had deteriorated and that at the time it was used it was on a basis of equality with the Mission Field screenings.

13. Weathered coal requires a little thinner fire and more draft than fresh screenings.

14. When using weathered coal the fuel bed should not approach any nearer to the water-back than from 4 to 6 inches, otherwise trouble with clinker is experienced.

15. Practically as high capacity was obtained with weathered coal as with the other coals used, and, if anything, the fuel bed required less attention.

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APPENDIX.

Formulas and Forms for Calculating Boiler Trials.

Owing to the fact that the adding machine probably is the only type of calculating machine on which more than one mathematical process can be performed without resetting, it has been considered advantageous to arrange the code employed in calculating the results of these tests so that each item contains but one such process, except in the case of additions. A complicated formula may, therefore, be spread out over a number of items in the code. This form is convenient when a large number of tests are to be calculated, in that all like operations are grouped, so that in many cases a saving can be made in the number of resettings required on the machine. The code can also be arranged in columnar form, with the mathematical operation indicated at the head of each column, and the calculations may be made by a clerk having little knowledge of the technical significance of the processes involved. In such cases, however, considerable care must be exercised, and plus and minus signs rigorously observed.

Since, in the main, the methods of calculation recommended in the American Society of Mechanical Engineers' Boiler Code have been followed, it will not be necessary to discuss all of the items. In some cases, however, a departure has been made, and it has been thought advisable to take up these items and discuss the methods and formulas used in their calculation. In the following discussion the numbers refer to the calculating code given on pages 62 to 72.

Item 29. Weight of Combustible Consumed.

It is usual to regard all of the coal as combustible, after deducting the ash and moisture. This being the case, the weight of combustible fired on the grates is $W\left(1-\frac{a}{100}\right)$, in which W is the weight of dry coal and a the percentage of ash taken from the analysis of the dry coal. Of this combustible, part goes through the grate in the form of unconsumed carbon. No essential error is made in regarding all of the combustible in the ash as carbon. Then the actual weight is $\frac{W_1 c}{100}$, where W_1

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is the actual weight of ash and refuse, and c is the percentage of carbon taken from an analysis of the ash and refuse. Therefore:

Item 29 $= W\left(1 - \frac{a}{100}\right) - \frac{W_1 c}{100}$. Item 29 is sometimes simply taken as the difference, $(W - W_1)$. This is in error, however, because all of the ash coming from the coal consumed may not appear in the weighed ash and refuse, for part of it is carried over the bridge wall in the form of fine dust. A larger source of error, however, is due to the fact that the fuel bed itself may contain more or less ash at the close than it did at the start of the test. Owing to this cause it is found that the result obtained by the simple subtraction is sometimes greater and at other times less than that obtained by making use of the analyses as outlined above.

Items 31 to 55. Coal analyses.

Where analyses are made direct on the samples of coal as fired, and both proximate and ultimate analyses reported by the chemist, the calculations included here are unnecessary. In the present case, however, the samples were partly dried before they were sent to the chemist, and the ultimate analyses were calculated from the individual proximate analyses and a composite ash, sulphur, and moisture free analysis.

In order to reduce the analyses as reported by the chemist to terms of coal as fired, each constituent had to be multiplied by the factor,

$$\left(1 - \frac{\text{per cent moisture lost before analysis}}{100}\right).$$

The separate ultimate analyses were then obtained by multiplying each constituent in the composite ash, sulphur, and moisture free analysis by the factor,

$$\left(1 - \frac{\text{per cent ash in dry coal} + \text{per cent sulphur in dry coal}}{100}\right),$$ which was obtained from the individual proximate analyses. The ash and sulphur in the dry coal were obtained by dividing the ash and sulphur in coal as fired by the factor,

$$\left(1 - \frac{\text{per cent moisture in coal as fired}}{100}\right).$$

Item 110. Corrected total weight of water fed to boiler.

The ultimate object of a boiler trial is to find the relation existing between the heat taken up by the boiler, and the heat supplied in the coal. It is, therefore, in reality a heat analysis. Since, however, items are not

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included in the standard code whereby these heat quantities may be separately tabulated, any heat with which the boiler is debited or credited that is not directly included in the weight of coal or water must be reduced to terms of one of these two, and added to or subtracted from the actual weighed amount. In order to put them into shape so that they may be conveniently handled in subsequent calculations, the usual method is to reduce all such quantities to terms of water. The following are some of the items that must be taken account of in this manner: (1) difference in water level at the start from that at the end of a test, (2) difference in pressure at the start from that at the end, (3) leakage from blow-off, etc., (4) heat going into the circulating water from the water-back in case it is not returned to the feed water at a point closer to the boiler than that at which the temperature of the latter is taken. Of these, No. 1 and No. 2 can be combined and treated under two heads.

Let W = weight of water in the boiler.
q = heat of the liquid.
r = latent heat of steam.
x = quality of steam.

Also let the subscript 1 refer to initial, the subscript 2 to the final, and the subscript 0 to average conditions in the boiler, and the subscript 3 to feed water conditions.

If the water level is higher at the close than at the start, the heat that has gone into the excess water, and into the change in the heat of the liquid due to the change in pressure, is the difference between the total heat in the water in the boiler at the close and that at the start. Or, taking all heat quantities above that of the liquid at feed water temperature it is:

$$W_2 (q_2 - q_3) - W_1 (q_1 - q_3).$$

The water that this amount of heat would have evaporated into steam as generated is; $\dfrac{W_2 (q_2 - q_3) - W_1 (q_1 - q_3)}{q_0 + x_0 r_0 - q_3}$.

This should be credited back to the boiler, after the difference between final and initial weights of water has been subtracted; or writing the total correction to automatically take care of signs, and calling the correction W',

$$W' = (W_1 - W_2) + \frac{W_2 (q_2 - q_3) - W_1 (q_1 - q_3)}{q_0 + x_0 r_0 - q_3} \quad \text{I}$$

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The case in which the water level is lower at the close than at the start is somewhat different from the above. In this event, all of the water in the boiler at the close of the test has been changed from a heat state represented by q_1, to one represented by q_2, while a weight corresponding to $(W_1 - W_2)$ has been evaporated. Since we do not know the exact conditions under which the latter took place, about the only assumption warranted is that it took place under the average conditions of the test. The total heat change to be taken into account then is:

$$W_2 (q_2 - q_1) + (W_1 - W_2) (q_0 + x_0 r_0 - q_1).$$

Calling the equivalent amount of water, or the correction, W'',

$$W'' = \frac{W_2 (q_2 - q_1) + (W_1 - W_2) (q_0 + x_0 r_0 - q_1)}{q_0 + x_0 r_0 - q_2} \quad \text{II}$$

On expanding equations I and II, and collecting the terms, it becomes evident that they both reduce to the same expression, which is:

$$W''' = W_1 \frac{(q_0 + x_0 r_0 - q_1) - W_2 (q_0 + x_0 r_0 - q_2)}{q_0 + x_0 r_0 - q_2} \quad \text{III}$$

Equation III has been made use of in the code for the combined corrections for changes in pressure and water levels. In case a blow-off leaks, or hot water is lost from any other point in the boiler, the heat loss may easily be seen to be $W_4 (q_0 - q_2)$, in which W_4 = the weight of water leaking out. The correction then is:

$$W^{\text{IV}} = \left[\frac{W_4 (q_0 - q_2)}{q_0 + x_0 r_0 - q_2} \right] - W_4.$$

If the water back forms part of the circulating system of the boiler, or the water is returned at a point nearer to the boiler than that at which the temperature of the feed water is read, no account need be taken of it. If this is not the case, however, the water back should be regarded as part of the heating surface of the boiler, and the heat going to it credited to the latter.

Let W_5 = weight of water going through the water back, and t_i its initial and t_f its final temperature. Then the correction, W^{V}, is

$$\frac{W_5 (t_f - t_i)}{q_0 + x_0 r_0 - q_2}$$

Designating by W the total amount of water weighed to boiler, corrected for tank levels and items which do not include heat quantities,

$$\text{Item } 110 = W + W''' + W^{\text{IV}} + W^{\text{V}}.$$

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Item 111. Factor of evaporation.

In this code the water evaporated has not been reduced to dry steam and the dry steam then reduced to equivalent evaporation, but the reduction from steam as generated to equivalent evaporation has been made in one step, making use of the "total" or "true" factor of evaporation:

$$F = \frac{q_0 + x_0 r_0 - q_3}{970.4},$$

in which F = factor of evaporation, q_0 = heat of the liquid at boiler pressure, x_0 = quality of steam, r_0 = latent heat at boiler pressure, and q_3 = heat of the liquid at feed water temperature.

Item 126. Heat available for use by the boiler.

The heat available for use by the boiler has been regarded as all of the heat in the gas leaving the furnace, computed above the temperature of the steam in the boiler. This is equal to the heating value per pound of dry coal minus the furnace losses per pound of dry coal, up to the temperature of the steam in the boiler.

Item 133. Efficiency of boiler, furnace, and grates, based on zero air leakage.

This is an hypothetical quantity that permits of no rigid solution, and has been partly discussed on page 21. Of a number of possible assumptions the following seem to be the most tenable. All of the heat supplied to the boiler comes from the heat in the coal (H). A certain portion of this (H_1) is absorbed by the water in the boiler, and the rest goes to losses, of which the air leakage loss (H_2) is one. If there had been no loss due to air leakage, the heat (H_2) would have been available for absorption by the water. Since this process would not have taken place at 100 per cent efficiency, it seems reasonable to assume that it would have done so at the efficiency of the boiler alone. If we designate the latter by (e), then the probable heat absorption would have been:

$$H_1 + eH_2, \text{ and Item } 133 = \frac{H_1 + eH_2}{H}.$$

Item 153. Theoretical weight of dry gas per lb. of dry coal.

Theoretically the gas given off per lb. of dry coal should consist of the sum of the products of combustion of the carbon and sulphur in the coal, plus the nitrogen carried in with the air necessary for the complete combustion of all the combustible constituents, including the hydrogen, plus the nitrogen in the coal. Since some of the oxygen for combustion came from the free O_2 in the coal, the nitrogen equivalent of this should

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be deducted from the above. Letting C = per cent carbon in dry coal, S = per cent sulphur, H_2 = per cent hydrogen, N_2 = per cent nitrogen, and O_2 = per cent oxygen, the corresponding weights of each constituent in the dry coal will be 1/100 of each of the above.

From the stoichiometric relations, the sum of these products of combustion may now be deduced, or

$$\text{Item } 153 = 0.1261\,(C) + 0.0535\,(S) + 0.268\,(H_2) + 0.01\,(N_2) - 0.0335\,(O_2).$$

Item 162. Actual weight of dry gas per lb. of dry coal.

This calculation is based on the fact that all of the carbon in the coal which was actually burned, appears in the gas.

If CO_2, CO, and O_2 represent the parts by volume of the constituents in a given volume of flue gas, the parts by weight may be found by multiplying the above by their corresponding molecular weights. The sum of these relative weights in the flue gas is, $44\,(CO_2) + 28\,(CO + N_2) + 32\,(O_2)$. The relative weight of the C in the same volume is $12\,(CO_2 + CO)$. Hence the weight of gas per lb. of carbon consumed is:

$$\frac{44\,(CO_2) + 28\,(CO + N_2) + 32\,(O_2)}{12\,(CO_2 + CO)} = \frac{11\,(CO_2) + 7\,(CO + N_2) + 8\,(O_2)}{3\,(CO_2 + CO)}.$$

Since N_2 is always determined by difference: $N_2 = 100 - (CO_2 + CO + O_2)$, and the above reduces to: weight of gas per lb. carbon consumed $= \dfrac{4\,(CO_2) + O_2 + 700}{3\,(CO_2 + CO)}$.

Let W_1 = weight of dry coal fired, W_2 = weight of ash and refuse, (a) = per cent carbon in dry coal and (b) per cent carbon in ash. Then the weight of carbon actually consumed per lb. of dry coal =

$$\frac{0.01W_1a - 0.01W_2b}{W_1}, \text{ and Item } 162 =$$

$$\left[\frac{0.01\,(W_1a - W_2b)}{W_1}\right]\left[\frac{4\,(CO_2) + O_2 + 700}{3\,(CO_2 + CO)}\right].$$

This is slightly in error due to the fact that SO_2 is formed from the sulphur in the coal. Part of this is absorbed by the water in the sampling apparatus, while some of it probably goes into the KOH pipette, and thus appears in the analysis as CO_2. It would be rather difficult to determine

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the exact distribution of these two. However, a simple calculation will show that the maximum error due to this source does not exceed 0.2 per cent.

Item 174. Theoretical weight of air per lb. of dry coal.

This can be calculated directly from the stoichiometric relations and the analysis of the coal.

If C, H_2, S, and O_2 represent the percentage by weight of the various constituents in the coal, then the weight of O_2 required is $0.0267\ (C) + 0.08\ (H_2) + 0.01\ (S) - 0.01\ (O_2)$, and,

$$\text{Item } 174 = \frac{0.0267\ (C) + 0.08\ (H_2) + 0.01\ (S) - 0.01\ (O_2)}{0.23} =$$

$$0.116\ (C) + 0.348\ (H_2) + 0.0435\ (S - O_2).$$

Item 177. Actual weight of air per lb. of dry coal.

All of the N_2 appearing in the gas, with the exception of that already in the coal, came from the air taken into the furnace. By a line of reasoning similar to that discussed under Item 162, it may be shown that the weight of N_2 per lb. of carbon consumed $= \dfrac{7\ (N_2)}{3\ (CO_2 + CO)}$.

Then the weight of air per lb. of carbon consumed $= \dfrac{N_2}{0.33\ (CO_2 + CO)}$.

Taking the weight of carbon consumed per lb. of dry coal from the discussion under Item 162,

$$\text{Item } 177 = \left[\frac{0.01\ (W_1 a - W_2 b)}{W_1}\right]\left[\frac{N_2}{0.33\ (CO_2 + CO)}\right].$$

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FORMS FOR CALCULATING BOILER TRIALS.

(1) Test number.
(2) Date.
(3) Duration, hrs.
(4) Grate surface, sq. ft.
(5) Water heating surface, sq. ft.
(6) Barometer reading, inches of Hg.
(7) Atmospheric pressure, lb. per sq. in.; 0.491 (Item 6).
(8) Steam pressure, lb. per sq. in. gage.
(9) Steam pressure, lb. per sq. in. absolute; (Item 8) + (Item 7).
(10) Draft between damper and boiler, in. water.
(11) Draft at end of 3rd pass, in. water.
(12) Draft between 1st and 2nd pass, in. water.
(13) Draft at end of combustion chamber, in. water.
(14) Draft over fire in furnace, in. water.
(15) Draft in ash pit, in. water.
(16) Temperature of outside air, deg. F.
(17) Temperature of fire room, deg. F.
(18) Temperature of steam, deg. F.
(19) Temperature of feed water, at boiler, deg. F.
(20) Temperature of ash, deg. F.
(21) Temperature of gas escaping from boiler, deg. F.
(22) Weight of coal fired, lb.
(23) $\left(1 - \frac{\text{Item 41}}{100}\right)$.
(24) Weight of dry coal fired, lb.; (Item 22) (Item 23).
(25) Total ash and refuse, lb.
(26) $\left(1 - \frac{\text{Item 55}}{100}\right)$.
(27) Weight of combustible fired, lb.; (Item 24) (Item 26).
(28) Weight of carbon in ash, lb.; $\frac{\text{(Item 25) (Item 56)}}{100}$.
(29) Weight of combustible consumed, lb.; (Item 27) − (Item 28).

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(30) Per cent ash and refuse in dry coal; $\frac{(\text{Item } 25)}{(\text{Item } 24)} \times 100$.

PROXIMATE ANALYSIS (COAL AS RECEIVED BY CHEMIST).

(31) Fixed carbon, per cent.
(32) Volatile, per cent.
(33) Moisture, per cent.
(34) Ash, per cent.
(35) Sulphur (separately determined), per cent.
(36) Moisture loss previous to sending to chemist, per cent.
(37) $1 - \frac{(\text{Item } 36)}{100}$.

PROXIMATE ANALYSIS (COAL AS FIRED).

(38) Fixed carbon, per cent; (Item 37) (Item 31).
(39) Volatile, per cent; (Item 37) (Item 32).
(40) Moisture (as received by chemist reduced to coal as fired), per cent; (Item 37) (Item 33).
(41) Total moisture in coal as fired, per cent; (Item 40) + (Item 36).
(42) Ash, per cent; (Item 37) (Item 34).
(43) Sulphur (separately determined), per cent; (Item 37) (Item 35).

ULTIMATE ANALYSIS (ASH, MOISTURE, AND SULPHUR FREE COAL).

(44) Carbon, per cent.
(45) Hydrogen, per cent.
(46) Oxygen, per cent.
(47) Nitrogen, per cent.
(48) (Item 54) + (Item 55).
(49) $1 - \frac{(\text{Item } 48)}{100}$.

ULTIMATE ANALYSIS (DRY COAL).

(50) Carbon (C), per cent; (Item 49) (Item 44).
(51) Hydrogen (H_2), per cent; (Item 49) (Item 45).
(52) Oxygen (O_2), per cent; (Item 49) (Item 46).
(53) Nitrogen (N_2), per cent; (Item 49) (Item 47).
(54) Sulphur (S), per cent; $\frac{(\text{Item } 43)}{(\text{Item } 23)}$.
(55) Ash, per cent; $\frac{(\text{Item } 42)}{(\text{Item } 23)}$.

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ANALYSIS OF ASH (DRY).

(56) Carbon, per cent.

(57) Earthy matter, per cent.

(58) Dry coal fired per hour, lb.; $\frac{\text{(Item 24)}}{\text{(Item 3)}}$.

(59) Combustible consumed per hour, lb.; $\frac{\text{(Item 29)}}{\text{(Item 3)}}$.

(60) Dry coal per sq. ft. of grate surface per hour, lb.; $\frac{\text{(Item 58)}}{\text{(Item 4)}}$.

(61) Combustible per sq. ft. of water heating surface per hour, lb.; $\frac{\text{(Item 59)}}{\text{(Item 5)}}$.

(62) Calorific value per lb. of dry coal, by calorimeter, B. t. u.

(63) Calorific value per lb. of combustible, by calorimeter, B. t. u.; $\frac{\text{(Item 62)}}{\text{(Item 26)}}$.

(64) Temperature in atmospheric side of calorimeter, deg. F.

(65) Superheat in calorimeter, deg. F; (Item 64) — 211.5.

(66) Quality of steam (x_0); from Marks and Davis chart.

(67) Total weight of water, from weighing tanks, lb.

(68) Initial height in feed tank, in.

(69) Final height in feed tank, in.

(70) (Item 68) — (Item 69).

(71) Weight of water per inch of tank, lb.

(72) Feed tank correction; (Item 70) (Item 71).

(73) Initial height of water in gage glass, in.

(74) Final height of water in gage glass, in.

(75) Initial steam pressure, lb. per sq. in. gage.

(76) Initial steam pressure, lb. per sq. in. absolute; (Item 75) + (Item 7).

(77) Final steam pressure, lb. per sq. in. gage.

(78) Final steam pressure, lb. per sq. in. absolute; (Item 77) + (Item 7).

(79) Density of water at initial pressure, lb. per cu. ft.

(80) Density of water at final pressure, lb. per cu. ft.

(81) Initial volume of water in boiler, cu. ft.

(82) Final volume of water in boiler, cu. ft.

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(83) Initial weight of water in boiler (W_1), lb.; (Item 79) (Item 81).

(84) Final weight of water in boiler (W_2), lb.; (Item 80) (Item 82).

(85) Heat of the liquid at initial steam pressure (q_1), B. t. u.

(86) Heat of the liquid at final steam pressure (q_2), B. t. u.

(87) Heat of the liquid at average steam pressure (q_0), B. t. u.

(88) Heat of the liquid at feed water temperature (q_3), B. t. u.

(89) Latent heat at average steam pressure (r_0), B. t. u.

(90) Weight of water to water-back, lb.

(91) Temperature of water entering water-back, deg. F.

(92) Temperature of water leaving water-back, deg. F.

(93) Total weight of hot water leakage from blow-offs, etc., lb.

(94) $x_0 r_0$; (Item 66) (Item 89).

(95) $q_0 + x_0 r_0$; (Item 94) + (Item 87).

(96) $q_0 + x_0 r_0 - q_1$; (Item 95) — (Item 85).

(97) $W_1 (q_0 + x_0 r_0 - q_1)$; (Item 83) (Item 96).

(98) $q_0 + x_0 r_0 - q_2$; (Item 95) — (Item 86).

(99) $W_2 (q_0 + x_0 r_0 - q_2)$; (Item 84) (Item 98).

(100) $W_1 (q_0 + x_0 r_0 - q_1) - W_2 (q_0 + x_0 r_0 - q_2)$; (Item 97) — (Item 99).

(101) $q_0 + x_0 r_0 - q_3$; (Item 95) — (Item 88).

(102) Correction for pressure and boiler level, lb.; $\frac{\text{(Item 100)}}{\text{(Item 101)}}$.

(103) (Item 92) — (Item 91).

(104) (Item 90) (Item 103).

(105) Water-back correction; $\frac{\text{(Item 104)}}{\text{(Item 101)}}$.

(106) (Item 87) — (Item 88).

(107) (Item 106) (Item 93).

(108) Steam equivalent of blow-off leak; $\frac{\text{(Item 107)}}{\text{(Item 101)}}$.

(109) Correction for leakage from blow-off, etc.; (Item 93) — (Item 108).

(110) Total weight of water to boiler, corrected; (Item 67) + (Item 72) + (Item 102) + (Item 105) + (Item 109).

(111) Factor of evaporation; $\frac{\text{(Item 101)}}{970.4}$.

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(112) Equivalent evaporation from and at 212° F., lb.; (Item 110) (Item 111).

(113) Equivalent evaporation per hr., lb.; $\frac{\text{(Item 112)}}{\text{(Item 3)}}$.

(114) Equivalent evaporation per sq. ft. of water heating surface per hr., lb.; $\frac{\text{(Item 113)}}{\text{(Item 5)}}$.

(115) Horsepower developed; $\frac{\text{(Item 113)}}{34.5}$.

(116) Builder's rated horsepower.

(117) Percentage of builder's rating developed, per cent; $\frac{\text{(Item 115)}}{\text{(Item 116)}}$ 100.

(118) Water apparently evaporated per lb. of coal fired, lb.; $\frac{\text{(Item 110)}}{\text{(Item 22)}}$.

(119) Equivalent evaporation per lb. of coal fired, lb.; $\frac{\text{(Item 112)}}{\text{(Item 22)}}$.

(120) Equivalent evaporation per lb. of dry coal, lb.; $\frac{\text{(Item 112)}}{\text{(Item 24)}}$.

(121) Equivalent evaporation per lb. of combustible consumed, lb.; $\frac{\text{(Item 112)}}{\text{(Item 29)}}$.

(122) Heat absorbed by boiler per lb. of combustible consumed, B. t. u.; 970.4 (Item 121).

(123) Efficiency of boiler and furnace excluding grate, per cent; $100 \frac{\text{(Item 122)}}{\text{(Item 63)}}$.

(124) Efficiency of boiler, furnace, and grate, per cent; $100 \frac{\text{(Item 186)}}{\text{(Item 62)}}$.

(125) (Item 189) + (Item 196) + (Item 198) + (Item 204) + (Item 206) + (Item 209) + (Item 211) + (Item 214).

(126) Heat available for use by boiler, B. t. u.; (Item 62) — (Item 125).

(127) Efficiency of furnace and grate, per cent; $100 \frac{\text{(Item 126)}}{\text{(Item 62)}}$.

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(128) Efficiency of boiler, per cent; $100 \frac{(\text{Item } 186)}{(\text{Item } 126)}$.

(129) (Item 62) − (Item 211).

(130) Efficiency of furnace, per cent; $100 \frac{(\text{Item } 126)}{(\text{Item } 129)}$.

(131) $\frac{(\text{Item } 224)\ (\text{Item } 128)}{100}$.

(132) (Item 186) + (Item 131).

(133) Over-all efficiency based on zero air leakage, per cent;

$100 \frac{(\text{Item } 132)}{(\text{Item } 62)}$.

(134) Cost of coal per ton of 2,000 lb., dollars.

(135) Tons of coal per 1,000 lb. of water apparently evaporated;

$\frac{0.5}{(\text{Item } 118)}$.

(136) Cost of coal to evaporate 1,000 lb. of water under observed conditions, dollars; (Item 134) (Item 135).

(137) Tons of coal per 1,000 lb. of water evaporated from and at 212°F.;

$\frac{0.5}{(\text{Item } 119)}$.

(138) Cost of coal to evaporate 1,000 lb. of water from and at 212°F., dollars; (Item 134) (Item 137).

ANALYSIS OF DRY GAS AT FURNACE.

(139) CO_2, per cent.

(140) O_2, per cent.

(141) CO, per cent.

(142) N_2 (by difference), per cent.

ANALYSIS OF DRY GAS AT FLUE.

(143) CO_2, per cent.

(144) O_2, per cent.

(145) CO, per cent.

(146) N_2 (by difference), per cent.

(147) 0.1261 (Item 50).

(148) 0.0535 (Item 54).

(149) 0.268 (Item 51).

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(150) 0.01 (Item 53).

(151) 0.0335 (Item 52).

(152) (Item 147) + (Item 148) + (Item 149) + (Item 150).

(153) Theoretical dry gas for complete combustion per lb. of dry coal, lb.; (Item 152) — (Item 151).

(154) 4 (Item 139).

(155) (Item 154) + (Item 140) + 700.

(156) (Item 139) + (Item 141).

(157) 3 (Item 156).

(158) Actual weight of dry gas per lb. of carbon consumed, at furnace, lb.; $\frac{\text{(Item 155)}}{\text{(Item 157)}}$.

(159) Total weight of carbon fired, lb.; $\frac{\text{(Item 24) (Item 50)}}{100}$.

(160) Weight of carbon consumed, lb.; (Item 159) — (Item 28).

(161) Weight of carbon consumed per lb. of dry coal fired, lb.; $\frac{\text{(Item 160)}}{\text{(Item 24)}}$.

(162) Actual weight of gas per lb. of dry coal, at furnace, lb.; (Item 161) (Item 158).

(163) 4 (Item 143).

(164) (Item 163) + (Item 144) + 700.

(165) (Item 143) + (Item 145).

(166) 3 (Item 165).

(167) Actual weight of gas per lb. of carbon consumed, at flue, lb.; $\frac{\text{(Item 164)}}{\text{(Item 166)}}$.

(168) Actual weight of dry gas per lb. of dry coal, at flue, lb.; (Item 161) (Item 167).

(169) 0.116 (Item 50).

(170) 0.348 (Item 51).

(171) (Item 52) — (Item 54).

(172) 0.0435 (Item 171).

(173) (Item 169) + (Item 170).

(174) Theoretical weight of air per lb. of dry coal, for complete combustion, lb.; (Item 173) — (Item 172).

(175) 0.33 (Item 156).

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(176) Actual weight of air per lb. of carbon consumed, at furnace, lb.; $\frac{\text{(Item 142)}}{\text{(Item 175)}}$.

(177) Actual weight of air per lb. of dry coal, at furnace, lb.; (Item 161) (Item 176).

(178) 0.33 (Item 165).

(179) Actual weight of air per lb. of carbon consumed, at flue, lb.; $\frac{\text{(Item 146)}}{\text{(Item 178)}}$.

(180) Actual weight of air per lb. of dry coal, at flue, lb.; (Item 161) × (Item 179).

(181) Weight of air leakage, per lb. of dry coal, lb.; (Item 180) — (Item 177).

(182) Weight of excess air per lb. of dry coal, at furnace, lb.; (Item 177) — (Item 174).

(183) Per cent excess air, at furnace; $100 \frac{\text{(Item 182)}}{\text{(Item 174)}}$.

(184) Weight of excess air per lb. of dry coal, at flue, lb.; (Item 180) — (Item 174).

(185) Per cent excess air, at flue; $100 \frac{\text{(Item 184)}}{\text{(Item 174)}}$.

HEAT BALANCE.

(186) Heat absorbed by boiler per lb. of dry coal, B. t. u.; 970.4 (Item 120).

(187) (Item 18) — (Item 17).

(188) 0.24 (Item 153).

(189) Loss due to theoretical gas up to temperature of steam, B. t. u.; (Item 188) (Item 187).

(190) 212 — (Item 17).

(191) (Item 18) — 212.

(192) 0.5 (Item 191).

(193) 970.4 + (Item 190) + (Item 192).

(194) 0.01 (Item 22) (Item 41).

(195) $\frac{\text{(Item 194)}}{\text{(Item 24)}}$.

(196) Loss due to moisture in coal, up to steam temperature, B. t. u.; (Item 195) (Item 193).

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(197) 0.09 (Item 51).

(198) Loss due to H_2 in coal, up to steam temperature, B. t. u.; (Item 197) (Item 193).

(199) Wet bulb reading, deg. F.

(200) Dry bulb reading, deg. F.

(201) Grains of moisture per lb. of air (from Carrier's chart).

(202) Lb. moisture per lb. air; $\frac{\text{(Item 201)}}{7{,}000}$.

(203) Moisture from air per lb. dry coal, lb.; (Item 177) (Item 202).

(204) Loss due to moisture in air, up to steam temperature, B. t. u.; (Item 203) (Item 193).

(205) 0.24 (Item 182).

(206) Loss due to excess air, up to steam temperature, B. t. u.; (Item 205) (Item 187).

(207) $\frac{\text{(Item 145)}}{\text{(Item 165)}}$.

(208) 10,140 (Item 207).

(209) Loss due to CO formed, B. t. u.; (Item 208) (Item 161).

(210) $\frac{\text{(Item 28)}}{\text{(Item 24)}}$.

(211) Loss due to combustible in ash, B. t. u.; 14,540 (Item 210).

(212) (Item 20) — (Item 17).

(213) 0.28 (Item 212).

(214) Loss due to temperature of leaving ash and refuse, B. t. u.; 0.01 (Item 30) (Item 213).

(215) (Item 21) — (Item 18).

(216) Loss due to theoretical dry gas, above steam temperature, B. t. u.; (Item 215) (Item 188).

(217) 0.5 (Item 215).

(218) Loss due to moisture in coal, above steam temperature, B. t. u.; (Item 195) (Item 217).

(219) Loss due to H_2 in coal, above steam temperature, B. t. u.; (Item 197) (Item 217).

(220) Loss due to moisture in air, above steam temperature, B. t. u.; (Item 203) (Item 217).

(221) Loss due to excess air, above steam temperature, B. t. u.; (Item 215) (Item 205).

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(222) (Item 21) — (Item 17).

(223) 0.24 (Item 222).

(224) Loss due to air leakage, total, B. t. u.; (Item 181) (Item 223).

(225) (Item 186) + (Item 216) + (Item 218) + (Item 219) + (Item 220) + (Item 221) + (Item 224).

(226) Radiation, conduction, and unaccounted-for loss, B. t. u.; (Item 126) — (Item 225).

(227) Heat absorbed by boiler, per cent; $\frac{100\ (\text{Item } 186)}{(\text{Item } 62)}$.

(228) Loss due to theoretical gas, up to steam temperature, per cent; $\frac{100\ (\text{Item } 189)}{(\text{Item } 62)}$.

(229) Loss due to moisture in coal, up to steam temperature, per cent; $\frac{100\ (\text{Item } 196)}{(\text{Item } 62)}$.

(230) Loss due to H_2 in coal, up to steam temperature, per cent; $\frac{100\ (\text{Item } 198)}{(\text{Item } 62)}$.

(231) Loss due to moisture in air, up to steam temperature, per cent; $\frac{100\ (\text{Item } 204)}{(\text{Item } 62)}$.

(232) Loss due to excess air, up to steam temperature, per cent; $\frac{100\ (\text{Item } 206)}{(\text{Item } 62)}$.

(233) Loss due to CO formed, per cent; $\frac{100\ (\text{Item } 209)}{(\text{Item } 62)}$.

(234) Loss due to combustible in ash, per cent; $\frac{100\ (\text{Item } 211)}{(\text{Item } 62)}$.

(235) Loss due to temperature of ash and refuse, per cent; $\frac{100\ (\text{Item } 214)}{(\text{Item } 62)}$.

(236) Loss due to theoretical gas, above steam temperature, per cent; $\frac{100\ (\text{Item } 216)}{(\text{Item } 62)}$.

(237) Loss due to moisture in coal, above steam temperature, per cent; $\frac{100\ (\text{Item } 218)}{(\text{Item } 62)}$.

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(238) Loss due to H_2 in coal, above steam temperature, per cent; $\frac{100\ (\text{Item } 219)}{(\text{Item } 62)}$.

(239) Loss due to moisture in air, above steam temperature, per cent; $\frac{100\ (\text{Item } 220)}{(\text{Item } 62)}$.

(240) Loss due to excess air, above steam temperature, per cent; $\frac{100\ (\text{Item } 221)}{(\text{Item } 62)}$.

(241) Loss due to air leakage, total, per cent; $\frac{100\ (\text{Item } 224)}{(\text{Item } 62)}$.

(242) Radiation, conduction, and unaccounted-for loss, per cent; $\frac{100\ (\text{Item } 226)}{(\text{Item } 62)}$.

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THE COKING OF COAL AT LOW TEMPERATURES WITH SPECIAL REFERENCE TO THE PROPERTIES AND COMPOSITION OF THE PRODUCTS

BY
S. W. PARR
AND
H. L. OLIN

BULLETIN NO. 79
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ENGINEERING EXPERIMENT STATION

BULLETIN NO. 79 MAY, 1915

THE COKING OF COAL AT LOW TEMPERATURES WITH SPECIAL REFERENCE TO THE PROPERTIES AND COMPOSITION OF THE PRODUCTS

BY S. W. PARR, PROFESSOR OF APPLIED CHEMISTRY, AND H. L. OLIN, INSTRUCTOR IN CHEMISTRY

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THE COKING OF COAL AT LOW TEMPERATURES WITH SPECIAL REFERENCE TO THE PROPERTIES AND COMPOSITION OF THE PRODUCTS

I. Introduction.

1. *Preliminary.*—This report covers a series of studies made during the period from 1911 to 1913 on the coking properties of Illinois coal. It is a continuation of the work described in Bulletin No. 60 of the University of Illinois Engineering Experiment Station.* Its distinctive feature has been the use of an apparatus which would yield the main products of coke, gas, and tar in quantities sufficient for a detailed study of these products, and, to a certain extent, quantities sufficient for a determination of their values by practical tests on a commercial scale.

2. *Resumé of Previous Work.*—In the experiments described in Bulletin No. 60 the apparatus used had a capacity of 6 to 8 pounds of coal at a charge. Notwithstanding this limited capacity, certain fundamental facts were developed as follows:

(a) The formation of coke depends upon the presence of certain constituents having a melting point which is lower than the temperature at which decomposition or carbonization takes place.

(b) Oxidation of these compounds may easily take place and the greatest coking effect is obtained where the opportunity for the minimum amount of oxidation has occurred. The condition prescribed, therefore, is that there shall be the least possible exposure to oxidation either before or during the process of carbonization.

(c) Coals containing an excessive quantity of the coking substance produce a light porous coke. The texture of the product may be modified by use of pressure and by close packing of the charge and

*The Coking of Coal at Low Temperatures, by S. W. Parr and H. L. Olin.

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especially by mixing with material which has already passed through the coking process. Such a mixture provides the physical conditions whereby the gases formed may readily pass out of the mass without carrying along the cementing substances.

(d) By use of temperatures between 400° and 500°C. all of the resulting products are of a type distinctly different from those obtained by the usual high temperature procedure.

3. *Outline of Present Investigation.*—An apparatus was designed to utilize about 100 pounds of coal. Experience in the use of the apparatus indicated also the main principles which should be embodied in a commercial equipment. The coking process was studied, and the mixture for producing the best product determined. It was found that a smokeless fuel may be produced especially well adapted to domestic purposes, including its use in open grates. Its freedom from tar or condensable hydrocarbons makes it easily adapted to generating producer gas, thus affording a good substitute for anthracite coal in suction gas producer practice.

In the study of the composition and properties of the tar, this material was found to have a very low content of free carbon, a relatively high percentage of light boiling distillate, and an unusually high content of tar acids or phenols. The latter fact is of special interest to the wood preserving industry.

II. Experimental Work.

4. *Description of Coking Oven.*—The apparatus used in the experiments is an elaboration of that employed in 1910, described on page 5 of Bulletin 60, and is capable of producing material in greater quantities than was possible with the older type.

The device was manufactured by Burr and Company of Champaign, Illinois, and is illustrated in Fig. 1. As shown in the detailed diagram Fig. 2, it consists of a boiler plate shell A, lined with asbestos to prevent excessive radiation of heat; within this, forming the coking chamber, is a shell B of the same material containing a cone of light sheet iron C, perforated with 3/16 in. holes, designed to confine the coal charge and to allow a free circulation of gases. To obtain the non-oxidizing atmosphere such as was used with the old apparatus, steam was admitted from the high pressure main at E, passed through the coil F where it was superheated by the hot currents ascending from the gas burner and then conducted into the coking chamber. The heat was supplied by a blast ring burner D, connected with the gas and air

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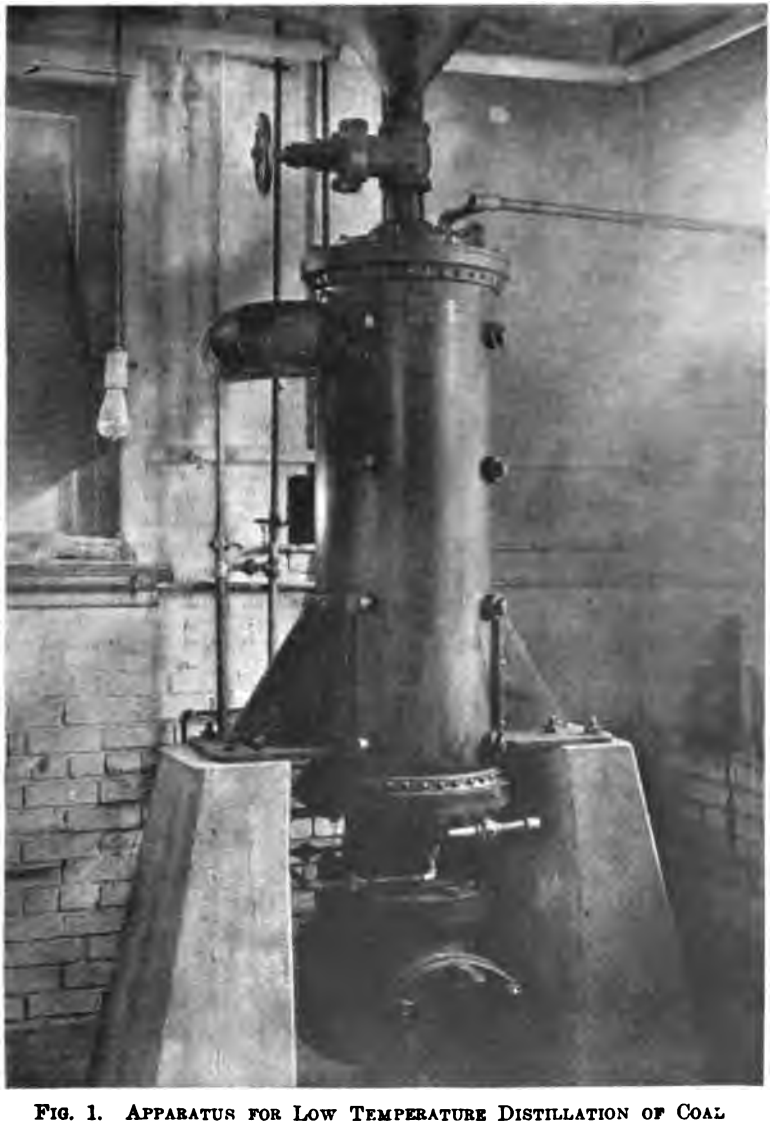

FIG. 1. APPARATUS FOR LOW TEMPERATURE DISTILLATION OF COAL

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mains, and no difficulty was experienced in producing the desired temperatures. The charge of crushed coal was fed into the hopper G and admitted to the retort through a large gate valve. The coked residue was removed, after cooling the apparatus, through the bottom at H. Gases of combustion escaped through the opening J, which was connected with a flue, while the distillates were conducted through an outlet pipe to a condenser consisting of several four-foot lengths of inch pipe connected by return elbows. Cold water was allowed to drip over this gridiron-like contrivance. The tars were passed through a water sealed exit at the bottom, while the gases, fairly clean, were collected in a gasometer.

5. *Coking Tests.*—To study further the coking qualities of Illinois coals at temperatures ranging from 400°C. to 500°C. and to obtain a quantity of the coke residue sufficient in amount for practical tests in order to determine its commercial value, a series of runs was made using the apparatus described. Numerous coals from the different fields of the State were included in this set of experiments. In this report, however, products from representative types only are illustrated and described since the results of tests of different coals of a given field showed little variation. Of particular interest during the process of distillation was the behavior of the coals from the northern districts, especially those from Vermilion County. As the heating progressed, a black pitchy substance dripped from the joints of the containing ovens but it hardened immediately on cooling, forming a brittle mass much resembling asphalt. This was undoubtedly the cementing principle which is instrumental in forming coke. According to Lewes it consists of substances derived from the resins of the original coal sources, which melt at about 300°C. and decompose at slightly higher temperatures yielding on the one hand liquid products which distil out as tar vapors and hydrocarbon gases, and on the other, a pitch residuum, which at 500°C. forms a mass of coke. His general theory that these resinic substances are readily oxidizable and in their oxidized condition have much to do with coke formation is borne out in these experiments by the fact that no weathered coal of any type produced the gummy exudation mentioned, although there was no apparent diminution in the amount of gases given off. Compared with the bituminous coals of the Eastern States, those of Illinois are exceedingly rich in this resinous binding material. Even those from the southern districts of the State while not as "fat" as those from Sangamon and Vermilion Counties, nevertheless, much surpass in this respect coals like the Pocahontas, and those from Ohio and Pennsylvania. Strangely enough the abun-

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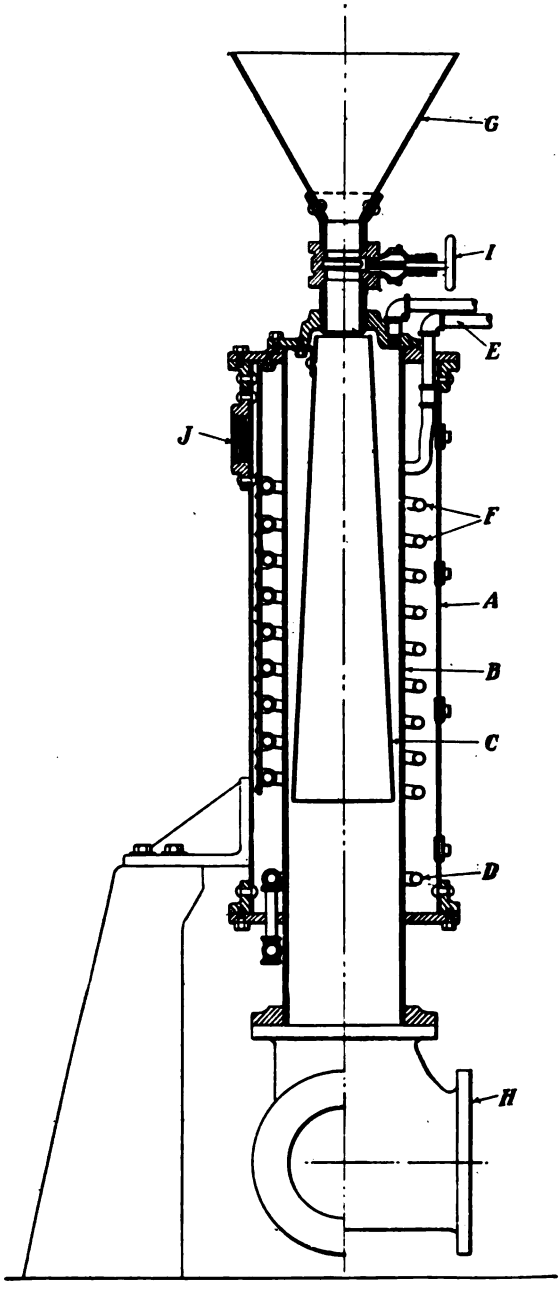

FIG. 2. CROSS SECTION OF APPARATUS FOR LOW TEMPERATURE DISTILLATION OF COAL.

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dance of the coking material which the western types possess is the cause of their inferiority in the matter of making dense, hard coke; for with the decomposition of the resinic bodies and the deposition of cementing carbon there occurs at the same time an evolution of large quantities of gases which inflate the pasty mass and make the resulting coke more or less light and spongy. Indeed, certain Vermilion County coals, after being heated under conditions which allow free expansion, present the appearance of hardened froth, because of the excessive development of cell structure in the coke.

As has been shown in the previous work the porosity of the product may be appreciably reduced by subjecting the contents of the oven during the heating period to a considerable pressure. Another means to the same end which was recognized in the first experiments and which has been applied in the latter series is the addition of inert coke dust to the raw coal as fed into the retort. This material acting in the capacity of a "blotter" reduces the plasticity of the softened mass and allows the gases to escape freely without producing a blowing effect.

The diluting medium in the case of the specimens shown here was a mixture of various semi-cokes which had accumulated in the course of the work. The material was crushed and ground to a fineness of 40 or 50 mesh and thoroughly mixed with the rest of the charge in ball mills.

An analysis of a composite sample of this semi-coke breeze gave the following results:

TABLE 1.

COMPOSITION OF INERT COKE MIXTURE.

	Actual	Dry
Moisture	1.85	
Ash	11.90	12.15
Volatile Matter	19.85	20.22
Sulphur	2.62	2.67
Fixed Carbon	66.40	67.65
Calorific Value (B. t. u.)	11243	11454

The effect of the addition of this foreign substance is exceedingly interesting. Certain types of coal, as for instance those from Vermilion County, produce hard firm coke when mixed with as much as 100 per cent of their own weight of the coke dust; those from the southern districts with higher fixed carbon do not need so great a dilution. Not only is the texture made firmer by this treatment but the density is increased. Attention will be called to the differences in the properties of the cokes in discussing the individual samples.

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Coke was made first from Vermilion County screenings such as were furnished the power plant of the University of Illinois. In order to remove the dust which may have been more or less weathered, the coal was passed over a quarter inch screen.

The resulting coke, shown in Fig. 3, is extremely light and porous, having a specific gravity of only .650. Its texture is firm, notwithstanding the low temperature at which it was formed and it has proved its ability to stand considerable handling without excessive dusting. Moreover, its porosity probably accounts in a large measure for the success obtained in using it in the gas producer test described later, since the carbon is easily accessible to the blasts of hot air and steam.

The coke shown in Fig. 4 was made from Saline County coal and was obtained from a run in which the time and temperature conditions were the same as in that of B-10. It illustrates the superiority of a coal from the southern field for coking purposes. Compared with the Vermilion County sample its texture is hard and firm and it has a density more than 6 per cent higher than the latter. It may be seen by referring to the photographs that the cellular structure of the Saline County sample is relatively close and solid, and bears little resemblance to the other.

One of the best products obtained in this series of experiments was from a coal from Williamson County. This sample, B-16, Fig. 5, shows a fine even grain and has a density of .750, more than 15 per cent higher than B-10. It has remarkable strength, both tensile and compressive, and stands rough handling without appreciable dusting.

Other coals from Williamson County have shown the good coking qualities which characterize the southern types. The particular sample illustrated in Fig. 6 shows an uncarbonized center but the outer portion is consistently dense and hard and has considerable strength.

In the course of the work tests were made of many coals from other localities as well as from different beds in the same locality. Included in the list are samples from Perry, Franklin, Jefferson, and Jackson Counties, but the cokes made from them are so similar in quality to those already described that they are not separately discussed.

TABLE 2.

COMPOSITION OF COKE FROM RAW COALS.

Sample	Moisture	Ash	Volatile Matter	Fixed Carbon	S	Heat Value B. t. u.
B-10	1.85	13.8	13.20	71.10	3.0	11891
B-12	1.20	11.9	10.85	75.95	1.6	12520
B-16	3.25	12.4	11.50	72.85	1.5	12600
B-19	2.25	11.75	12.30	73.70	1.55	12415

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The effect of the addition of a fine inert diluting medium to different types of raw coal is seen in samples B-18 and B-21, Fig. 7 and Fig. 8. The former is the product from the heating of a mixture of equal parts of Williamson County coal and the coke dust described on page 8. It is loose in texture and crumbles easily showing a deficiency in bonding material. On the other hand, B-21, made from equal parts of Vermilion County coal and the coke in question, is quite as hard and firm as the raw coal product and considerably denser. B-22, Fig. 9, is from Vermilion County coal diluted with one-half its weight of inert material and while it resembles B-21 it has a lower specific gravity.

Williamson County coal although unable to cement itself firmly when mixed with an equal weight of non-coking material nevertheless works successfully in a mixture of two parts coal to one part inert material. B-25, Fig. 10, is both coherent and dense proving that the dilution limit has at least not been exceeded in applying this mixing ratio.

Saline County coal likewise produces better coke with the addition of foreign material. B-13, Fig. 11, raw Saline County coal two parts and coke dust one part, is superior in every way to the coke from the raw coal alone.

The composition of this series of samples is given in the following table.

TABLE 3.

COMPOSITION OF COKES FROM MIXTURES OF COAL AND COKE DUST.

Sample	Moisture	Ash	Volatile Matter	Fixed Carbon	S	Heat Value B. t u.
B-21	1.35	14.95	13.7	70.0	3.15	11750
B-18	1.30	12.9	13.90	71.90	2.50	12295
B-22	1.60	14.50	12.70	71.20	2.9	11825
B-13	1.40	14.2	15.22	69.18	2.8	12150
B-25	1.45	15.4	10.50	72.65	3.0	11920

The rise in the density of the coke with the addition of the inert substance is shown by the results of the following specific gravity measurements.

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TABLE 4.

DENSITIES OF COKES.

Description		Density
Saline County Coal	B-12 (unmixed)	.687
	B-24 (raw ¾, dust ¼)	.775
	B-13 (raw ⅔, dust ⅓)	.868
Williamson County Coal	B-16 (unmixed)	.750
	B-25 (raw ⅔, dust ⅓)	.969
Vermilion County Coal	B-10 (unmixed)	.650
	B-22 (raw ⅔, dust ⅓)	.848
	B-21 (raw ½, dust ½)	852

For purposes of comparison it is interesting to note that B-29, Fig. 12, a 16-hour Solvay coke from an Illinois coal has a density of .830; B-30, Fig. 13, a 48-hour compressed charge, .986; while a representative sample of bee-hive Connellsville has a density of about 1.12.

While the methods described in the foregoing may not be applied directly on a commercial scale to operations for making metallurgical coke, still it is hoped that the results obtained may at least help to establish the characteristics of the coals of the State with regard to their adaptability to this use and to suggest methods and principles on which to work in solving the problem of making cokes of good quality from such material. Unquestionably they possess coking properties to a marked degree but need to be treated differently from the eastern coals with their higher percentages of fixed carbon.

6. *Gas Producer Tests.*—Early in the course of this work, the study of the products resulting from the distillation of bituminous coals at low temperatures suggested the possibility of using them as fuels for the gas producer whereby it was hoped to obtain gases sufficiently free from tar to be suitable for use in the internal combustion engine. Previous experiments had shown that the moderate temperatures of the preliminary heating period were effective in expelling the major part of the tar-producing substances and that the residue, distilled a second time at high temperatures, yielded gases remarkably free from heavy condensation products.

In order to give the matter a practical test, arrangements were made with Professor C. R. Richards of the Department of Mechanical Engineering of the University of Illinois for the use of the necessary apparatus of his department and with Mr. A. P. Kratz of the Engineering Experiment Station for his services in conducting the trial

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run. By means of the device already described a quantity of coke was prepared and it was fired in the usual manner.

The fuel used in this test was the semi-coke product obtained in subjecting Vermilion County screenings from the University power plant to low temperature distillation (400°C–550°C). This test material, four or five hundred pounds in weight, consisted of pieces varying in size from three-fourths of an inch to two inches in diameter, but the charge as fired contained some dust. It was light and porous and lay on the fuel bed without showing much tendency to pack. The following tables show its composition.

TABLE 5.

PROXIMATE ANALYSIS OF PRODUCER TEST FUEL.

Moisture	2.28
Ash	15.82
Sulphur	3.13
Volatile Matter	18.00
Fixed Carbon	63.90
Calorific Value (B. t. u.)	11601

TABLE 6.

ULTIMATE ANALYSIS OF PRODUCER TEST FUEL.

Carbon	69.86
Hydrogen	2.76
Oxygen	5.15
Nitrogen (estimated)	1.00
Sulphur	3.13
Ash	15.82
Water	2.28

The producer used was a Number 3 Otto, designed to operate on anthracite pea coal, with a wet scrubber attached. The latter was merely a shell filled with coke through which the gas, admitted at the bottom, passed counter to a stream of water flowing from the top. Using anthracite coal the normal capacity of the producer was 4500 cu. ft. of gas per hour.

In place of the gas engine which under normal working conditions draws the gas from the scrubber, a Schütte-Köerting steam ejector was used. This delivered the mixture of steam and gas to a condenser and thence to a Westinghouse meter of the wet type which had been calibrated just before the test was made.

Inasmuch as the supply of fuel was limited the usual method of

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starting and stopping was varied somewhat. Before beginning the experimental part of the work, the producer was fired with anthracite and thoroughly warmed up. This fire was then drawn and a new one made with a weighed amount of wood and the fuel to be tested, and the operation continued until gas of a good quality was given off, when the test was declared begun. During this preliminary period, as in the rest of the run, the measured gas was sampled by means of the continuous sampler which is a part of the installation.

At the close of the test, the fuel bed was burned as low as was deemed practicable, the ash pit cleaned, and the ash weighed and sampled. The residue on the grates was then drawn out, quenched, weighed, and sampled.

After the test was started all the coal put into the producer was weighed. The total fuel charge used in the test then included the coal equivalent of the wood and coal used to start, plus the coal fired during the test, minus the coal equivalent of the carbon in the gas given off before the formal start, minus the coal equivalent of the residue in the fuel bed at the close. Putting this into a formula:

$$W = W_1 + W_2 - \frac{(W_3 + W_5)\ 14560 + W_4 \times 62000}{H}$$

W = Total weight of equivalent coal fired during test.
W_1 = Total weight of equivalent coal in producer at start.
W_2 = Total weight of coal fired during test.
W_3 = Total weight of carbon appearing in gas before starting.
W_4 = Total weight of hydrogen appearing in gas before starting.
W_5 = Total weight of carbon within fuel bed at close of test.
H = Heating value of the coal, B. t. u.

Since the first sample contained practically no hydrogen the last term in the above formula becomes zero.

Gas samples were drawn from a point beyond the ejector where the gas was under pressure, and were taken over mercury. Coal and ash samples were taken in the usual manner.

Water fed into the vaporizer was weighed in a tank on scales. The weight of the overflow from the vaporizer was obtained in the same manner, and the weight of the water going into the fuel bed from this source was the difference between these two.

All temperatures were taken with mercury thermometers with the exception of that of the gas at the producer outlet which was obtained by means of a Hoskins pyrometer.

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An attempt was made to get tar samples just as the gas left the producer but it was found that the amount of tar formed was not appreciable and the sampling was discontinued.

The Junker calorimeter was used to obtain the heating value of the gas, about one sample an hour being taken.

The form for recording the results of this test is abridged from the one given in Bulletin* No. 50 of the University of Illinois Engineering Experiment Station, where details of methods of computation may also be found.

The firing sheet for this test showed that the producer ran successfully and gave little trouble. Because of the small diameter of the fuel bed, resulting in considerable friction, the lightness of the material used, and its slight tendency to coke and arch, the fuel above the bed proper did not feed down as rapidly as it was burned. Hence it was necessary about once an hour to poke it down with a slice bar and in this respect it probably required a little more attention than a charge of anthracite.

It may be noted, however, that it was necessary to poke and clean the grates from the bottom but once during the six hours of the trial. With so small a producer this indicates a remarkable freedom from ash and clinker trouble. On cleaning the fire small pieces of clinker were found in the ash, but there was none at all sticking to the sides, where it usually collects. The high grate efficiency, 98.2 per cent, also shows that the fires required little attention, since poking and cleaning invariably force much unburned carbon out into the ash.

The fact that the fire required so little attention has an added significance in connection with a producer having a small fuel bed, such as the one used in this test. Usually, the inrush of air, when the doors are opened for cleaning, dilutes the gases sufficiently to make them too lean to burn, but with the fuel in question less trouble was experienced in this respect than is usually the case with anthracite, for at no time did the pilot flame go out and there was no great variation in the calorimeter readings.

A study of the temperature of the gas leaving the producer throws considerable light on the condition of the fuel bed. If the bed becomes clinkered, or if holes form in the fire, air goes through without coming into intimate contact with the carbon. This chimney effect causes the

*Tests of a Suction Gas Producer. Garland and Kratz. 1912.

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gas to burn at the surface of the bed and the temperature to rise. Sometimes it is very difficult to keep the temperature down and cleaning and poking have little effect. In the case under discussion, however, the temperature never rose above 420°C except during the last twenty minutes of the test, when the fire was so low that it was essentially a furnace and not a producer.

No trouble was experienced with tar. There was little evidence of its presence in the fuel bed, and an attempt to get a sample of it from the gas leaving the producer showed that it was present in negligible quantities only. On cleaning the ejector at the close of the run a small amount of tar was noticed but most of the deposit was of the nature of scale. It is to be remembered that before going to the ejector the gas had first passed through the scrubber.

The efficiencies (hot gas 74.4 per cent and cold gas 64.9 per cent) compare very favorably with those obtained in using anthracite coal of the same ash content.

Data and results are given in the following table.

TABLE 7.

RESULTS OF GAS PRODUCER TRIAL.

Made by A. P. Kratz and H. L. Olin.	
Mechanical Engineering Laboratory, University of Illinois.	
Make of Producer, Otto.	
Kind of Fuel—Semi-coked Vermilion County Coal.	
Type of Producer—Suction for Anthracite.	
Duration of Trial—hours	6.33

DIMENSIONS AND PROPORTIONS

Great area, sq. ft.	1.666
Mean diameter of fuel bed, ft.	1.545
Depth of fuel bed, ft.	2.21
Area of fuel bed, sq. ft.	1.877
Approximate width of air spaces in grate, inches	0.5
Area of air space, sq. ft.	0.722
Ratio of air space to whole grate	0.433
Area of discharge pipe, sq. ft.	0.165
Water heating surface in vaporizer, sq. ft.	2.7

AVERAGE PRESSURES

Average corrected barometer reading, inches mercury	29.52
Draft in ash pit, inches water	0.45
Suction at producer outlet, inches water	0.88
Pressure at meters, inches water	3.69
Absolute pressure at meters, inches mercury	29.69
Vapor pressure at meters, inches mercury	1.9
Dry gas pressure at meters, inches water	27.79

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TABLE 7. (CONTINUED)

AVERAGE TEMPERATURES (Degrees Centigrade)

At barometer,	28.3
Of fire room,	33.3
Of feed water entering vaporizer,	23.4
Overflow from vaporizer,	84.8
Of water entering scrubber,	14.8
Of water leaving scrubber,	45.7
Of gases leaving producer	378.0
Of gases leaving first scrubber,	34.5
Of gases entering meters,	38.2
Of gases entering meters, (absolute)	311.2

FUEL

Weight of wood used in starting, lb.	9
Volume of gas given off before start of test, cu. ft.	2238
Volume of standard gas given off before start of test, cu. ft.	2010
Weight of equivalent coal as fired, lb.	381
Percentage of moisture in coal	2.28
Total weight of dry coal fired, lb.	372
Total ash and refuse, lb.	37
Total weight of residual, lb.	49
Total weight of combustible, lb.	280.6

COMPOSITION OF PRODUCER FUEL

Fixed carbon, per cent	70.77
Volatile matter, per cent	18.00
Moisture, per cent	2.28
Ash, per cent	15.82
Sulphur, separately determined, per cent	3.13

ULTIMATE COMPOSITION OF PRODUCER FUEL

Carbon, per cent	71.50
Hydrogen, per cent	2.82
Oxygen, per cent	5.26
Nitrogen, per cent	1.02
Sulphur, per cent	3.20
Ash, per cent	16.20
Moisture in sample coal as received, per cent	2.28

COMPOSITION OF DRY ASH AND REFUSE

Carbon, per cent	14.48
Earthy matter, per cent	85.52

COMPOSITION OF RESIDUAL FUEL ON GRATE

Carbon	36.46
Volatile matter	7.10
Moisture	1.18
Ash	55.30

FUEL CONSUMPTION PER HOUR

Dry coal per hour, lb.	58.8
Dry coal per sq. ft. of grate area per hr., lb.	35.3
Dry coal per sq. ft. of fuel bed per hr., lb.	31.3

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TABLE 7. (CONTINUED)

CALORIFIC VALUE OF FUEL

Calorific value by oxygen calorimeter, per lb. dry coal, B. t. u.	11601

WATER

Total weight fed to vaporizer, lb.	440.0
Total weight of overflow, lb.	256.0
Water actually evaporated in vaporizer, lb.	184.0
Weight of water fed to producer, lb.	
(a) From vaporizer	184.0
(b) In air (estimated)	6.0
(c) In coal	8.7
Total	199
Total weight of water decomposed as used in calculations, lb.	91.
Total weight of moisture in gas leaving producer, lb.	108.
Weight of water decomposed per lb. of gas generated, lb.	0.0588
Weight of water decomposed per lb. of dry coal fired, lb.	2.42
Total weight of scrubber water, lb.	5880

GAS

Calorific value per cu. ft. of standard gas from calorimeter, B. t. u. (high value)	121
Specific weight of standard gas, lb. per cu. ft.	0.0670
Specific heat of dry gas leaving producer (estimated)	0.260
Total volume of gas from meters, cu. ft.	26927
Total volume of standard gas at 16.5°C. and 30 in., cu. ft.	23150
Volume of standard gas per hr. cu. ft.	3660
Volume of standard gas per lb. of dry coal cu. ft.	62.3
Total weight of standard gas, lb.	1552.
Weight of standard gas per lb. of dry coal, lb.	4.17

COMPOSITION OF GAS BY VOLUME

Carbon dioxide, CO_2	4.15
Carbon monoxide, CO	21.10
Oxygen, O_2	0.30
Hydrogen, H_2 (estimated from calorific value)	11.80
Marsh gas, CH_4	1.50
Nitrogen, N_2 by difference	61.00

COMPOSITION OF GAS BY WEIGHT

Carbon dioxide, CO_2	7.19
Carbon monoxide, CO	23.17
Oxygen, O_2	0.38
Hydrogen, H_2	0.93
Marsh gas, CH_4	0.96
Nitrogen, N_2 by difference	67.37

EFFICIENCY

Grate efficiency, per cent	98.2
Hot gas efficiency, based on high heating value, per cent	74.4
Cold gas efficiency, based on low heating value, per cent	64.9

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7. *Adaptation of the Coke to Domestic Appliances.*—A limited amount of the coke was available for testing its adaptability to house-heating purposes. The ordinary household appliance is especially well adapted for distilling off the hydrocarbons of coal and sending them into the air unburned or partly burned and accompanied by large volumes of smoke.

In the coke product here described the heavy hydrocarbons have been removed. The coke itself, therefore, is clean both in handling and in burning. However, there remains approximately 20 per cent of volatile matter which enters into the process of combustion after the coke has attained a temperature at or beyond the point to which it had been subjected in the coking process. These conditions result in a free burning substance yet one whose combustible constituents may not produce smoke in burning.

An open grate was selected as furnishing the best opportunity for observing the behavior of the material. The results are summed up as follows: the coke ignites readily, it retains its shape through the process of combustion, a bed of glowing coals quickly results, the very indifferent provision for draft as found in an open grate is sufficient for keeping the combustion lively, there is no smoke produced, and fire is retained over a long period of time, because the interior of the larger pieces holds the fire and continues the combustion until all of the carbonaceous matter is consumed. While the temperature commonly attained by a grate fire would not furnish positive evidence as to the formation of clinker, the indications so far as they went, were altogether favorable. A small amount of coke was tested by burning in a hot air furnace. Not enough material was at hand for a complete test, but so far as observations could be made, it was as favorable as the test in the open grate.

8. *The Sources and Uses of Coal Tar.*—The installation of the first American by-product recovery coke plant in 1893 marks the beginning of a notable rise in the production and use of coal tar in this country.

Previous to that time the isolated illuminating-gas plant was the only source of supply and, indeed, in the smaller places at least, the tar was looked upon as a troublesome waste product rather than as a thing of value.

For making metallurgical coke, types of beehive ovens of varying degrees of efficiency were employed and these, of course, allowed the volatile constituents of the coal, both gaseous and liquid, to be wasted.

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In 1912, not twenty years after the introduction of the new type of oven, 165,000,000 gallons of tar were produced in the United States, and two thirds of this quantity, according to Perry,* came from the by-product coke plant. With the steady increase in by-product oven construction the tar refining industry will assume a proportionate importance.

Of the various fractions obtained in distilling the crude tar the most important are;—benzol and its homologues, used in the color industry and as paint and fat solvents; the carbolic oils, much in demand for disinfectants of various kinds; and the creosote and anthracene oils, of great importance in the preserving of wood. The pitch residue is used in roofing, paving, and road-building, but in these fields it meets strong competition with the mineral bitumens and consequently does not find so ready a market as the other constituents named. But pitch makes up the greatest part of the crude tar aggregate, being in most cases considerably more than half. The bulk of the raw material therefore, is in the form of high temperature condensation products and free carbon of relatively small value, while the supply of the lighter fractions is insufficient to meet the demand. This is especially true of those compounds of the tar which have bactericidal properties. With the rapid depletion of the forests and the cutting off of lumber supplies, wood preserving treatment has become imperative in many industries—particularly that of the railroads—and the lack of available material for this process is causing much concern. American supplies are inadequate for home consumption and nearly 45,000,000 gallons of creosote oils were imported from Europe in 1911. Any modification of the coking process, therefore, that will increase the yield of the light tars by preventing their condensation to compounds of high molecular weight will effect a considerable economy. Considerations of this kind have lent interest to the study of the tars obtained in the course of these experiments, in which temperatures were kept much below those of the gas retort and the commercial coke oven.

9. *Methods for Testing and Analyzing Tars.*—Of the methods proposed and outlined for testing tar, perhaps the best are those published by S. R. Church in a paper on "Methods for Testing Coal Tar and Refined Tars, Oils and Pitches Derived Therefrom."† These tests, as

*Eighth Int. Cong. of App. Chem., 10, 233.

†The Gas Age, 32, 103. Jour. of Ind. and Eng. Chem., 3, 227.

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he says in the introduction, were not put forward as methods for the scientific examination of, or research into, the products of coal tar, but rather as an attempt, in cooperation with other chemists of the company with which he was connected, to revise and standardize the everyday tests applied to the raw materials and products of the American tar distiller.

In this article he takes up the determination of water, free carbon, fixed carbon, and ash, and the measuring of the specific gravity and viscosity of the raw tar, distilled tars, and pitch. He then outlines the chemical investigation of the light oils, carbolic oils, benzols and creosotes, including the estimation of napthalene. In a later paper[1] he gives some supplementary methods.

Prevost Hubbard's "Methods for the Examination of Bituminous Road Materials"[2] approaches the matter solely from the standpoint of the road-builder, and takes up the physical tests necessary to determine the fitness of the material for this special purpose. In its field it is, unquestionably, authoritative.

Among the publications devoted particularly to methods for testing wood preservatives may be mentioned those of the American Railway Engineering Association,[3] the National Electric Light Association,[4] and the Forest Service of the Department of Agriculture.[5]

In addition to the foregoing, the following works take up to a greater or less extent the subject of coal tar analysis:

Sadtler, Industrial Organic Chemistry, 3rd. ed. J. B. Lippincott Co.

Rogers and Aubert, Industrial Chemistry, 1913, Van Nostrand Co.

Lunge, Coal Tar and Ammonia, 4th ed. 1909. Van Nostrand Co.

Allen, Commercial Organic Analysis. Vol. II, part 2, 3rd. ed. 1909. Blakiston's Son and Co., Philadelphia.

Mulliken, Identification of Pure Organic Compounds. 1904. Wiley and Sons.

10. *Properties of Tar Products From Low Temperature Coal Distillation.*—The tar used in this investigation was the product obtained in a separate series of runs made with Vermilion County coal.

[1]Jour. of Ind. and Eng. Chem., 5, 195.
[2]U. S. Dept. of Agriculture, Office of Public Roads, Bulletin 38, 1911.
[3]Bulletin 65, American Railway Eng. Assoc.
[4]Report of Committee on Preservative Treatment of Poles and Crossarms, 1911.
[5]Dean and Bateman, Circular 112, "Analysis and Grading of Creosotes."

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To prevent oxidation and evaporation the tar was protected as soon as formed with a water seal, and kept in air-tight cans until used for analysis.

The tar is fluid at temperatures considerably below room conditions. In appearance it is black to rich brown with varying thickness of layer, and it possesses an exceedingly disagreeable odor even when judged by coal tar standards.

The specific gravity measured with a pyknometer at 20° was 1.069. Its low density makes its separation from water by gravity somewhat difficult and there is a tendency, moreover, for high fractions to float on the surface of the water. Its viscosity is low and in this respect it resembles a mineral oil rather than a high temperature coal tar.

The percentage of free carbon, or more accurately, of substances insoluble in toluene and benzene, was found by treating a comparatively large sample of the dry tar, (about 36 grams) with toluene, and heating the mixture on the steam bath. The solution was decanted through two S and S filter cones (33 mm. and 26 mm.) one within the other. After three treatments the whole mass was transferred to the cones and extracted with benzene in a Soxhlet apparatus until the filtrate was colorless. Check results showed a free carbon content of 1.35 per cent.

Fixed or combined carbon may be determined by a method based on the report of the Committee on Coal Analysis of the American Chemical Society.* The cracking of an oil or tar is, however, so closely dependent upon time, temperature, and pressure conditions that a simple laboratory test of the kind outlined by the Society would have little significance.

11. *Distillation of Tar.*—In order to dry the tar a preliminary distillation 125° was made and the light oil runnings after being separated from the water were returned to the retort. The apparatus used was an ordinary Jena distilling flask attached to a Liebig condenser.

In the earlier attempts to make this separation much trouble was caused by the tendency of the water in the crude material to produce a succession of "bumpings." These explosions were often so sudden and violent as to throw a considerable part of the charge out of the retort. This annoying feature was prevented, however, by running a slow stream of air through a tube extending nearly to the bottom of the vessel, and later, in order to prevent oxidation which seemed

*Jour. Am. Chem. Soc., 21, 1116.

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to take place at the higher temperatures under these conditions, carbon dioxide, which had been previously washed and dried, was substituted. This method was used in all subsequent distillations of the crude, wet tar and served the double purpose of providing an inert atmosphere in which to carry on the heating of the substances making up the material, many of which show remarkable chemical activity, and of making the boiling proceed smoothly and quietly. It is to be noted, however, that the current of gas sent through the apparatus carries over with it a considerable quantity of material whose boiling point is at a temperature above that maintained in the retort. In redistilling the dry tar to fractionate the sample this precautionary measure was unnecessary, and therefore was not taken.

The escape of the first light oil runnings was prevented by surrounding the receiving vessel with a freezing mixture. To condense the heavier part of the second fraction, only the inner tube of the condenser was used so that heat might be applied to facilitate the flow of the viscous fluid.

To insure greater accuracy a subsequent redistillation of the light oil fraction was made with the use of a Lebel-Henninger tube, with the thermometer so adjusted that the top of the bulb was level with the side neck. The Liebig condenser was used as before. With this apparatus it was possible to secure fractions with sharply defined boiling points, an important prerequisite to identifying individual compounds.

TABLE 8.

RESULTS OF PRELIMINARY DISTILLATION OF TAR.

Fraction	Temperature Range	Percentage
Light oil	below 210° C	17.2
Heavy oil	210°–325°	52.7
Pitch	above 325°	30.1

12. *Examination of Light Oil.*—The low boiling fraction purified as indicated above is a clear amber colored liquid with a disagreeable odor. It is very susceptible to the action of light and air and finally turns dark red even when sealed in an atmosphere of carbon dioxide.

The separation and identification of all the individual constituents of a substance like coal tar is a great task. Lunge, in his work "Coal Tar and Ammonia" has described more than two hundred distinct compounds which occur in tars of different qualities and his list is probably not complete. We are concerned in this work merely with a

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FIG. 3. B-10. VERMILION COUNTY COKE

FIG. 4. B-12. SALINE COUNTY COKE

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FIG. 5. B-16. WILLIAMSON COUNTY COKE

FIG. 6. B-19. WILLIAMSON COUNTY WASHED NUT COKE

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FIG. 7. B-18. COKE FROM WILLIAMSON COUNTY COAL MIXED WITH COKE DUST

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FIG. 8. B-21. COKE FROM EQUAL PARTS OF VERMILION COUNTY COAL AND COKE DUST

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FIG. 9. B-22. COKE FROM VERMILION COUNTY COAL AND COKE DUST

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FIG. 10. B-25. COKE FROM WILLIAMSON COUNTY COAL AND COKE DUST

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FIG. 11. B-13. COKE FROM SALINE COUNTY COAL AND COKE DUST

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FIG. 12. B-29. 16-HOUR SOLVAY COKE

FIG 13. B-30. 48-HOUR COMPRESSED SOLVAY COKE

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few of the most important substances which are of interest because of their practical value and not with those which are of interest from the scientific standpoint only.

The phenols and other acid substances of the light oil were separated from the neutral and basic constituents by shaking the fraction in a separatory funnel with half its volume of 10 per cent caustic soda solution. After this treatment had been repeated until no further reduction in volume of the residue took place, this caustic solution was drawn off and acidified with sulphuric acid and the whole extracted with ether. The ether extract, when evaporated down, yielded crude phenol and its homologues.

The amines were removed from the oily residue after the phenol extraction by shaking it with dilute sulphuric acid, of sp. gr. 1.15, with warming. The solution was separated as before, neutralized with caustic soda, and the bases extracted with ether.

The method for separating paraffins and benzenes is based upon the familiar principle that the hydrocarbons of the paraffin series are practically inert toward sulphuric acid, while those of the aromatic series react with comparative ease, forming sulphonic acids, in which sulpho groups replace hydrogen of the benzene ring. These acids are very soluble in water and are therefore easily separated from the insoluble oily paraffin residues.

The process used was that adopted by the Forest Products Laboratory.* Ten cubic centimeters of the fraction to be tested were measured into a Babcock milk bottle. To this was added 40 cc. of 37 N sulphuric acid (made by adding the calculated amount of fuming sulphuric acid to the ordinary acid, sp. gr. 1.84) 10 cc. at a time. The mixture after being kept at 100° C for an hour was cooled and then whirled for five minutes in a Babcock separator. The volume of the unsulphonated or paraffin residue was read off directly. Equally good results are obtained, however, by pouring the mixture into a large volume of water and separating by means of the funnel.

As a step toward identifying those constituents which do not react with acids and bases, a sample of the oil from which the phenols and pyridines had been removed but which had not been sulphonated, was subjected to a second redistillation, with the measurement of the volumes going over between comparatively narrow limits of temperature.

A summary of the results obtained from the various tests and analyses follows:

*Bateman, E. Modification of the Sulphonation Test for Cresote. Forest Service, Circular 191, U. S. Dept. of Agriculture.

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TABLE 9.

LIGHT OIL FRACTION (TO 210°C).

Specific gravity .966

	Percentage on Basis of Crude Tar	Percentage on Basis of Light Oil Fraction
Light oil fraction	17.2	100.0
Phenols (and other acid substances	5.7	33.0
Amines (and other bases)	0.9	5.3
Paraffins	3.12	18.1

The results of the refractionation of the light oil from which the acids and bases had been removed are given in the following table:

TABLE 10.

DISTRIBUTION OF NEUTRAL SUBSTANCES IN LIGHT OIL.

Fraction	Percentage on Basis of Light Oil Fraction	Percentage on Basis of Crude Tar
20°–75° low boiling bodies	5.26	.25
75°–95° crude benzol (90%)	2.10	.38
95°–125° crude toluol	7.36	1.33
125°–170° crude solvent naptha	26.30	4.77
170°–200° crude heavy naptha	14.73	2.67
200°–210° crude heavy naptha	7.30	1.32

The classification as made in Table 10 is based on the outline for the examination of light oil as given by F. E. Dodge in his article on "Coal Tar and Its Distillation Products."* It is understood that this is a generalization only, and that it shows composition by classes of compounds rather than by individuals. However, since the different homologues of a class within rather close limits of the boiling point usually differ little in character, a fair idea of the composition of the mixture may be gained without further separation.

Fraction No. 1, besides some benzene, contains also pentanes and hexanes of the paraffin series, ranging in boiling point from 31° C to 64° C. No. 2 includes some of the heptanes boiling in the nineties; No. 3, according to tables complied from data† obtained by distilling known mixtures, consists of benzene and toluene, the latter predominating, while fraction No. 4 includes the xylenes boiling from 138° C to 143° C, with perhaps some mesitylene, b.p. 164°C. Likewise the part reported as phenols contains besides, one or all of the three cresols, although the close range of boiling points (190°, 201°, and 202°) made a quantitative fractionation of the crude extract impracticable.

*Industrial Chemistry. Rogers and Aubert, 1912. p. 492.

†G. E. Davis. Industrial Chemistry Rogers and Aubert. p. 499.

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Finally, the rapid darkening of both the neutral and the active fractions on even short time exposures to air points to the presence of various unsaturated substances, difficult to isolate and probably of no great practical importance.

13. *Examination of Heavy Oil Fraction, 210°C to 325°C.*—The heavy oil obtained from the first distillation to which was added the residue boiling above 210°, from the redistillation of the light oil, is a thick viscous liquid of a rich brown color. After standing for a time a non-crystalline sediment is formed.

Tar acids were determined by the methods used in the examination of the light oils except that after the addition of the caustic soda it was necessary to warm the mixture in order to facilitate the separation, while the paraffin and aromatic content of substances was found by sulphonation as before. Likewise a sample from which the acid constituents had been removed was refractionated and the volumes of distillate given off between close temperature limits were noted.

The napthalene content was determined by cooling, with a freezing mixture, the heavy oil sample from which the acids had been extracted, in order to crystallize out any of this substance which might be present. The test gave zero results; the conclusion is that no napthalene was present. This result is confirmed by observations made when the original tar was distilled from the coal, for at no time did the tar condenser become clogged as it would if napthalene were going over even in small quantities, nor did the pungent odor of its vapors ever become noticeable.

The method for the quantitative assay of anthracene as given by Allen* was used for this material. The sample, dissolved in boiling glacial acetic acid, was treated with chromic acid slowly dropped through a reflux condenser to oxidize any anthracene present to anthraquinone. The results showed that the amount present was negligible.

TABLE 11.

COMPOSITION OF HEAVY OIL (210°C–325°C).

Specific gravity 1.032

Fraction	Percentage on Basis of Crude Tar	Percentage on Basis of Heavy Oil Fraction
Heavy oil fraction	52.7	100.0
Tar acids	22.2	42.13
Paraffins	6.2	32.66
Napthalene	.0	.0
Anthracene	.0	.0

*Commercial Organic Analysis. Vol. II, part 2, p. 229.

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Distillation of the acid free samples gave the following results.

TABLE 12.

DISTRIBUTION OF NEUTRAL SUBSTANCES IN HEAVY OIL.

Fraction	Percentage on Basis of Crude Tar	Percentage on Basis of Heavy Oil Fraction
210° to 250°	2.87	10.3
250° to 270°	13.55	48.5
270° to pitch	11.58	41.2

In attempting to identify the various substances of the heavy oil fraction the worker finds the problem even more difficult than in the case of the light oil because the higher boiling points and molecular weights of the compounds make them more difficult to separate. A partial purification of the acid bearing extract of this part of the tar showed by the boiling points that a high percentage of creosols was present. The higher boiling members were not identified. It is possible that they are polyhydric phenols, such as Lewes mentions as being present in the tar from coalite and which he says form resinous masses difficult to investigate.

Fraction No. 2 is the neutral part of the cut in tar distilling known as creosote oils. Emmet and Reingruber* say of this class of substances that after removing basic, oxygenated, and crystallizable bodies, there remain several isomeric dimethylnapthalenes, the separation of which has proved unmanageable, and which constitute the major portion of the fraction.

14. *Pitch.*—The residuum from the original distillation of the crude tar is a hard, black substance, rather brittle, breaking with a bright fracture. As indicated by the table of its properties it is a hard pitch, as contrasted with soft pitch having a melting point of about 75° C.

TABLE 13.

PITCH FRACTION (ABOVE 325° C).

Per cent on basis of crude tar	30.1
Melting point	110° C
Specific gravity	1.27
Free carbon	12%

*Annalen 211, 365.

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15. *Oxygen Absorbing Power of Tar.*—In the publication of the results of the preliminary studies of low-temperature tar,* attention was called to the fact that both the light and the heavy fractions were readily oxidizable and the question was raised whether these oils might not serve as drying bodies or paint vehicles by forming coatings or films on oxidation. In an attempt to approximate a quantitative measurement of this capacity for absorbing oxygen, a series of iodine absorption determinations was made, with rather variable results.

Further investigations of the kind have proved what was conceded as a probability at the time, that other reactions than those of simple saturation take place, chief among which is substitution with the formation of hydriodic acid. While fairly consistent results have been obtained in the supplementary work under strictly standard conditions, slight changes in the concentration of the iodine solution, in the time of digestion and particularly in the temperatures of the reacting substances so varied the values found that the method is deemed unreliable. Bromine is even more uncertain. Among the substances present in the tar from which hydrogen is easily displaced by the halogens are the phenols, the reaction taking place almost immediately even in the cold.

The only reliable criterion of its value as a paint drier is perhaps the actual test of its behavior when exposed to air. The light oil fraction forms a thin film when spread on a glass plate inclined to 45 degrees. Using good linseed oil as a standard for reference, it is found that the tar oil films are decidedly thinner, i.e., the oil flows more before thickening, and that the time of maximum drying is from two to three times that observed in the case of the linseed oil.

III. Appendix of Supplementary Results.

At the request of S. R. Church, head of the Research Department of the Barrett Manufacturing Company of New York, a small sample of the crude tar remaining after the conclusion of the work described, was sent to the New York Laboratories for examination. Mr. Church, in a communication commenting on it briefly, says, "This tar in its characteristics, resembles somewhat the Scotch blast furnace tars although it is higher in tar acid content than the Scotch tars, and apparently not quite so high in paraffin-like bodies. The most interesting feature to us is its exceptionally high content of oxygenized compounds of phenoloid character."

*Parr and Olin. Bulletin No. 60, University of Illinois Eng. Exp. Sta. p. 13.

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"Should tar of this nature become a commercial product, it would undoubtedly have a certain value to the tar distillers although it would, of course, have to be handled in an entirely different manner from the ordinary coal tars."

The outline of his results is given in the following table.

TABLE 14.

TESTS OF TAR DERIVED FROM THE DISTILLATION OF COAL FROM VERMILION CO., ILLINOIS. (S. R. CHURCH.)

Water, per cent	21.7
Tests on Dry Tar:	
Specific gravity at 15.5°C	1.072
Free carbon, per cent	1.3
Light Oil (to 210°) per cent by vol.	15.2
Heavy Oil (to pitch) per cent by vol.	40.2
Light Oil:	
Tar acids, per cent	30.0
Sulphonation residue, per cent	15.0
Heavy Oil:	
Standard retort distillation:	
Total per cent to 170°C	0.5
200°	2.0
210°	6.6
235°	33.6
270°	61.0
315°	65.8
355°	94.9
Tar acids in total distillation	50.0
Sulphonation residues on fractions	4 to 5 per cent
Pitch	
Melting point,	76°C
Free carbon, per cent	9.0

Of equal interest is the supplementary report furnished by Mr. E. B. Fulks of the American Creosoting Company of Louisville, Kentucky, who visited the laboratory early in the year 1913 and obtained a sample of a tar similar to the one that has been discussed. He writes: "This tar is quite different from ordinary coal tar, in that it is thinner, has a lower specific gravity and much smaller percentage of pitch. The principal difference however, is in the high percentage of tar acids and in the presence of considerable quantities of paraffinoid bodies. These differences probably would make it necessary to work this tar somewhat differently from the method employed for ordinary tars but undoubtedly it would have considerable commercial value."

"The quantity at our disposal was so small that we were unable to separate an amount large enough to test the preservative qualities of that proportion which would be used for this purpose. Apparently, however, this tar would produce from 30 to 40 per cent of oil of fair preservative value. The high percentage of tar acids would make it a

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valuable source of material for the manufacture of antiseptic solutions, sheep dip, etc. The pitch contains a small proportion of free carbon and probably could be used for roofing purposes and paving filler."

The report of his analysis is given in the following table:

TABLE 15.

OIL FROM LOW TEMPERATURE COKING EXPERIMENTS
UNIVERSITY OF ILLINOIS.

Description: Thick, dark brown oil, having a very disagreeable odor resembling Pintsch gas; liquid at ordinary temperature.

Specific gravity at 38° C.			1.041
Water			Trace
Tar acids by volume			27.0%
Fractions			
210°		10.6% Liquid	
235°		8.9% "	
270°		12.5% "	
315°		13.1% "	
355°		14.1% "	
Res.		40.4	
Hard, black, brittle, bright fracture.			

Paraffin Oils:

	Fractions	Per cent of Fraction	Per cent of Straight Oil
210° C	9.7	25.0	2.4
235°	9.0	10.0	0.9
270°	12.3	10.0	1.2
315°	13.7	10.0	1.4
355°	15.0	15.0	2.3
	59.8		8.2

Total per cent paraffin in fractions below 355° C. 13.7%
Total per cent paraffin in fractions below 355° C. based on whole oil 8.2%

IV. SUMMARY.

1. Coke of good density and hardness may be made by mixtures of semi-coke and raw coal if both are finely divided and evenly mixed. A variation is noticeable in the quantity of such non-coking material which may be incorporated with different coals. For example, fresh coal from Vermilion County will carry such an addition of 100 per cent of its weight to advantage. Coals from Saline and Williamson Counties give coke of the highest density when mixed in the proportion of 50 per cent of their weight with semi-coke.

2. The coke resulting from the low temperature process has from 18 to 22 per cent of volatile matter remaining, but since it has been heated above 400° there should be none of the tar constituents remaining. The most convincing test on this point as also the best method of arriving at a conclusion as to its adaptability for such work was to try out the material in a suction gas producer. The results indicated that

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no clogging effect whatever results, thus showing the absence of tar bodies. The physical operation of the producer as well as the grade of the gas produced was fully equal if not superior to the performance of the outfit when anthracite was used.

3. The semi-coke has such an amount of volatile matter remaining, together with the right degree of coherence as to make it especially well adapted to household use. It is clean to handle, free from dust, and burns without smoke or the formation of soot. Especially to be noted in this connection is its ability to retain a fire without undue attention as to drafts, etc.

4. The average specific gravity of the tar is 1.069. It is rich in low boiling distillate passing over at 210°. This product averages 18 per cent of the total. The pitch residue amounts to approximately 30 per cent and is remarkably free from precipitated carbon.

5. The adaptability of the tar for wood preservation processes seems to be indicated by the high percentage of tar acids. These constituents make up from 28 to 30 per cent of the crude material. The larger part, about 22 per cent is found in the second distillate (210°–325°), only about 7 per cent coming over below 210°.

6. Approximately 10 per cent of the crude tar is found to be low boiling distillate free from the tar acids and suitable for use in internal combustion engines.

7. Naphthalene is absent. The free carbon in the crude tar is less than 2 per cent and the residual product after the light distillate and heavy oils are removed would be classed as hard pitch.

8. A principal feature results from this study of these various substances, namely, that all three of the general divisions of coke, tar, and gas have specific properties of an especially valuable sort which would indicate that the process of coking at low temperatures could be established successfully on a commercial basis.

V. Review of Literature.

In the effort to solve the smoke problem which arises wherever bituminous coal is extensively used, numerous attempts have been made to modify the raw fuel in various ways in order to obtain a product which should be more or less free from smoke producing constituents. Among the first to make the attempt was Col. Scott-Moncrieff,* who proposed to subject coal to the coking process in the ordinary gas re-

*Jour. of Gas Lighting, 101, 823.

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torts at the usual temperature until one-half of the volume of gas usually obtained was driven off. The charge was then drawn and quenched.

The scheme resulted in failure for very obvious reasons. On account of the porosity of coke it is a very poor heat conductor and so the interior of the charge received an insufficient amount of heat to drive off the tar and smoke producing substances while on the other hand the exterior was heated to such a degree that it became essentially hard gas-coke which ignited with difficulty and burned slowly. A mass consisting of a soft uncarbonized interior and a hard shell was the result, a type of fuel which possessed no good qualities and which gained no popularity. At the same time the manufacturer lost one-half of the gas and much of the ammonia that would be produced in ordinary practice.

*In 1907 a radical change in the method of heating was made by W. Parker and the product obtained was extensively advertised under the trade name of "Coalite." This was the coke made by the partial distillation of slack coal in vertical oblong cast iron boxes or stills about ten or twelve feet high and ten by forty-eight inches in cross section. Carbonization was carried on at temperatures near 450° C, yielding a coke having about 10 per cent of volatile matter and 80 per cent of fixed carbon.

The following table shows the composition of the gas obtained in the distillation.

TABLE 16.

COMPOSITION OF GAS OBTAINED IN COALITE MANUFACTURE.

H_2S	1.0	H_2	14.30
CO_2	3.45	CH_4	61.00
O_2	.72	N_2	9.28
Illum.	4.19	CO	6.06

Coalite has not proved to be the commercial success that its promoters hoped to make it, but experimental work is still being carried on with that end in view.

The Premier Tarless Fuels Company† of Battersea, England, have lately installed a plant for the production of smokeless fuel by low temperature distillation. The distinctive features are the use of annular retorts which allow the thickness of the charge to be reduced to 2½ inches and the coking time to be shortened proportionately, and

*Gas World, June 8, 1907. p. 715.

†Jour. of Gas Light & Water Supply. 122, 514.

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the maintainance of a reduced pressure of 27 inches in the retorts. Temperatures of about 500° C are used and the charge is drawn at the end of four or five hours.

A sample yield from a Lancashire coal is given below.

TABLE 17.

YIELD OF PRODUCTS FROM VACUUM PROCESS.

Tarless fuel	78.0 per cent
Tar (per ton of coal)	20.19 gal.
Ammonium sulphate (per ton)	45 lb.
Approximate Composition of Fuel	
Fixed carbon	92.86
Volatile matter	3.86
Ash	3.28

This fuel is said to work well in the gas producer. Another sample with 8.26 per cent of volatile matter gives good results when used for domestic purposes, burning with a free flame, giving out intense heat and little smoke.

In considering the subject of tars and gases and their relationship to the matter of economy in coal carbonization no attempt will be made to review the great mass of literature relating to them, but merely to call attention to results which show causes for variation in the quality and yield, chief of which are range of distillation temperatures and differences in the sizes and shapes of the retorts.

The results obtained by Lewis P. Wright,* published twenty-five years ago, show the effect of increase in heat intensity in a striking manner. He does not state definitely the temperatures at which the different tars were produced but the following table compiled from his data indicates clearly that the gradations were marked.

TABLE 18.

RESULTS OF FOUR RUNS (2cwt. charges).

Sp. Gr. Tar	Duration of Distillation	Gas Yield per Ton of Coal, Cu. Ft.	Free Carbon in Tar, Per Cent
(1) 1.084	8 hrs.	6600	8.69
(2) 1.103	7 hrs.	7200	11.92
(3) 1.149	6 hrs.	8900	15.53
(4) 1.204	5 hrs.	11700	24.67

His analyses of the tars obtained show the characteristic increase in volatile constituents with rise in distillation temperature.

*Jour. Soc. Chem. Ind. 7, 59.

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TABLE 19.

PERCENTAGE COMPOSITION BY WEIGHT OF TARS.

	No. 1	No. 2	No. 3	No. 4
Crude naptha	9.17	9.05	3.73	.99
Light oil	10.50	7.46	4.47	.57
Creosote oil	26.45	25.83	27.29	19.44
Anthracene oil	20.32	15.57	18.13	12.28
Pitch	28.89	36.80	41.80	64.08
Paraffin contents of crude naptha (by volume)	5.0	4.0	1.5	1.0

There is, likewise, a consistent fall in the percentages of the acid in the liquid fraction as the following table shows.

TABLE 20.

PERCENTAGE OF TAR ACIDS.

	In Crude Naptha	In Light Oil	In Creosote
No. 1	13	34	35
No. 2	9	35	29
No. 3	8	29	28
No. 4	6	22	20

Napthalene and anthracene become prominent in the heavier tars according to his investigations, the former appearing in considerable quantity in No. 4 and the latter being at its maximum in No. 3.

As proof that the tendency of increased temperature in the distillation of coal is to destroy, preferably, the phenol bearing light oils intermediate between the crude naptha and the creosote, he cites the case of a tar of specific gravity 1.33, distilled at high temperatures, which showed practically no light oils but which yielded solid napthalene immediately after the naptha had come over. Watson Smith, in an article on "Variation in the Products of the Destructive Distillation of Coals,"* confirms these results, by stating that tars which were produced at very high temperatures in Simon-Carves ovens and examined by him, showed a conspicuous lack of the middle oil fraction which contains the phenols and that the creosote and anthracene oils were semi-solid, the former being thick with napthalene.

He calls attention to a theory of Schulze† that the primary products of the dry distillation of coals are phenols. These phenols are

*Jour. Soc. Chem. Ind. 8, 952.

†Annalen, 227, 143.

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then at the higher temperatures of the retorts split up so as to yield water and high boiling hydrocarbons, or finally, with more entire decomposition, into illuminating gas.

An interesting table showing the effect of change of temperature on the yield of gas and tar is given by V. B. Lewes.*

TABLE 21.

VARIATION OF GAS AND TAR YIELD WITH TEMPERATURE RANGE.

Temp. (C)	Gas yield per ton, cu. ft.	Tar gallons	Sp. gr. of Tar
900°	11 000	9	1.200
800°	10 000	12	1.170
700°	9 000	15	1.140
600°	7 750	18	1.115
500°	6 400	21	1.087
400°	5 000	28	1.060

Lewes points out that tars produced at temperatures between 400° C and 500° C contain relatively low percentages of aromatic substances and high percentages of the paraffin series which are adapted to use as motor fuels. The middle oils are free from napthalene and yield excellent enriching oils. The pitch having practically no free carbon, he says, forms an ideal electric insulating material.

Variations in the quality and composition of the gas are shown in the following table.

TABLE 22.

COMPOSITION OF GASES PRODUCED AT DIFFERENT TEMPERATURES.

	400°	500°	600°	700°	800°	900°
Hydrogen	21.2	28.3	33.8	41.6	48.2	54.5
Saturated hydrocarbons	60.1	56.2	30.7	45.0	39.1	34.2
Unsaturated hydrocarbons	6.3	5.8	5.0	4.4	3.8	3.5

The gas produced at the lowest temperatures quoted has a calorific value of 750 B.t.u. and measures 20 candle power.

Dr. H. W. Jayne, in a paper† before the Fifth International Congress of Applied Chemistry at Berlin in 1903, says in regard to coal tars:

*Jour. of Gas Light. 101, 823.

†The Coal Tar Industry in the U. S. Report of the Fifth International Congress of Applied Chemistry, Section IVa, Volume II, p. 721.

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"The influence of the temperature in carbonizing is strikingly shown by the test of two tars, both from the same coal, and made in the same kind of ovens. One plant was producing gas as its main object. The tar from this source had a gravity of 1.21 and tested 17.5 per cent of free carbon; the light oil fraction was 2.2 per cent of a gravity 0.979, testing 23 per cent to 170 deg.; the pressed napthalene yield was 7.4 per cent. In the second tar, in which coke was the main object, evidently much lower heats were used, the tar having a gravity of 1.137, and testing 3.2 per cent of free carbon; the light oil amounted to 11.9 per cent, and had a gravity of .970, testing 28 per cent to 170 deg., or six times more crude naptha than in the first; the total tar acids were 12.48 per cent, while the pressed napthalene fell to 1.2 per cent." It is evident, he thinks, that in the first tar the light hydrocarbons and tar acids had been destroyed by the temperature employed, with formation of napthalene.

R. P. Perry* states that "Depending upon the coal used and more particularly upon the size and shape of retorts, the travel which the gases make over the hot surfaces, and the temperatures to which the gas is subjected, the coal tars vary within wider limits. In general with the vapors subjected to the high temperatures usually characteristic of gas works as compared with coke works, there is an increase in specific gravity and free carbon. For example the tar from many gas works will average about 1.24 specific gravity at 15.5°C. and about 25 per cent to 30 per cent free carbon by weight, whereas from the by-product coke ovens the specific gravity would average more nearly 1.19 and the free carbon would vary about 5 per cent to 18 per cent, the average being less than 10 per cent."

This so-called free carbon represents a finely divided inert material, largely carbon, which is the portion of tar insoluble in benzol and toluol. Perhaps a very small part is carried over mechanically in the coking process, but for the most part it is due to the cracking of the hydrocarbons by exposure to heat, and the higher percentage usually found in gas works tars is from this cause."

S. R. Church of the Barrett Manufacturing Company, in an interesting article on "Tar and Its By-Products,"† gives a table of results of analyses which were made in his laboratory, of the typical coke-oven tars produced in the United States. It is to be noted that the different

*R. P. Perry. Tar Distillation in the U. S. Eighth International Congress of Applied Chemistry. 10, 283.

†"Tar and Its By-Products," Gas Age, May 15, 1913.

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gas retorts and ovens used vary in size and shape and that the former are, of course, heated to a much higher temperature than the latter.

TABLE 23.

ANALYSES OF TYPICAL TARS; DRY TAR.

	Gas Retort				Coke Oven			
	Horizontal		Inclined	Vertical	United Otto	Semet-Solvay	Koppers	Water gas Eastern
Specific gravity at 15.5° C.	1.266	1.293	1.288	1.158	1.207	1.188	1.186	1.083
Free carbon (insoluble in C_6H_6)	28.8	37.5	24.3		13.3	10.7	6.8	0.1
Specific viscosity (Engler at 100°C)	21.8	30	14.9	2.1	3.4	3	2.1	1.6
Oil to soft pitch 60°C. (Per cent by volume)	13.2		14.3	28.8	21.2	21.8	35.3	43.1
Pitch residue 60°C. mp (Per cent by volume)	86.8		85.7	71.2	78.8	78.2	64.7	56.9
Refractive indices at 60° C. (taken on oil)	1.5932		1.5807	1.5755	1.5987	1.6122	1.6139	1.5678
Sulphonation residue (Per cent taken in oil)	0.4		2.4	4.3				8
Tar acids (Per cent taken in oil)	14.		21.0	29.	12	4		

The figures given here are especially significant when compared with those of low temperature tars.

Vivian B. Lewes* in the series of articles mentioned earlier says of the tar produced in the manufacture of coalite, distilled at 400° to 500° C.: "The low-temperature tar is distinctive in its characteristics. It has a specific gravity of 1.075, is very liquid, and contains an abundance of light solvent oils, very low aromatic hydrocarbons, very little phenol but large quantities of cresol, no napthalene, and very little anthracene, while the free carbon is as a rule below two per cent."

The very low percentage of benzene in the light oils, is made up for by the presence of paraffins, such as hexane, heptane and octane, while there are also present considerable quantities of that curious group of hydrocarbons known as napthenes or hexahydro-benzenes, which play so important a part in Russian petroleum.

As before mentioned, carbolic acid occurs in small quantities, but its higher homologues, such as cresylic acid, etc., occur in much larger quantities than in coal tar, and there are also present quantities of polydydric phenols, or other esters of the type met with in coal tar which form resinous masses difficult to analyze. He states that coals rich in oxygen (10 to 11 per cent) and hydrogen (above 5 per

*The Carbonization of Coal. Lecture IV. Jour. Roy. Soc. Arts. 60, 216.

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cent) and which, therefore, have large percentages of resinic bodies, on being carbonized at low temperatures yield tars rich in phenol and cresol. These results are confirmed by Bornstein. The pitch left as a residue amounts to about 40 per cent of the tar, and is of fine quality owing to the practical absence of free carbon. The ammonium sulphate amounted to only 12 lb. per ton of coal, the temperature being too low for large production.

The tar yield varies with the coal used and in most cases averages twenty gallons per ton of coal.

TABLE 24.

COMPOSITION OF TAR FROM COALITE.

Specific Gravity	1.078				
Distillation on 2,274 cc.	(0.5 gallon)				
	Temp.	By volume on tar	Sp. gr.	Volume of hydrocarbons	Tar acids
Water		2 64			
Light Oil	170°	3.10	.844	3.10	
Carbolic Oil	170°–225°	13.72	.959	8.62	4.80
Creosote Oil	225°–240°	8.35	.988	4.64	3.10
Creosote Oil	240°–270°	8.35	.992	5.45	2.55
Anthracene Oil	270°–300°	8.80	1.029	6.60	1.76
Anthracene Oil	300°–320°	12.31	1.033	8.80	3.10
	Pitch by weight on volume			41.71%	
	Bases			1.32%	

	Light Oils Percentage distilling below	Calculated on Tar
100° C	15.6 by vol.	.55
120°	31.2	1.09
140°	54.7	1.91
170°	82.8	2.90
over 170° and loss	17.2	.20

Prevost Hubbard* has lately published a similar table of his own results obtained from the analyses of tars from twenty-six of the thirty-one by-product coke ovens operating in this country in 1910. His figures may be considered as showing authoritatively the character of the 67,000,000 gallons of tar produced from these sources in that year.

The specific gravities of the samples ranged from 1.133 to 1.214, the majority being lower than 1.200 and indicating low percentages of free carbon. The minimum percentage of free carbon was 2.73, the maximum 16.80, and the average for the 26 samples was 8.38. Eighteen samples contained less than 10 per cent of free carbon, and eight samples more than 10 per cent. About two-thirds of these products

*Coke-Oven Tars of the United States, Circular No. 97. Office of Public Roads, U. S. Dept. of Agriculture.

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might, therefore, be considered as low-carbon tars and the other third, as medium carbon tars.

In accordance with the type of oven in which these products are made, there is a considerable variation in composition, even though temperatures differ but little. It seems probable, therefore, that high pitch and free carbon contents are not simple functions of temperature alone but that other factors complicate the problem.

TABLE 25.

COMPARISON OF TARS FROM VARIOUS TYPES OF OVENS.

Kind of Oven	Maximum Temperature of Retorts (C)	Per Cent Free Carbon	Per Cent Pitch Residue
Koppers	1100°–1444°	3.38	70
Semet-Solvay	1050°–1450°	6.74	63–79
United Otto	1220°–1660°	9.00	69–78
Otto Hoffman	1000°–1100°	12.16	60–78
Otto Hoffman and United Otto (mixed)	1000°–1200°	12.51	76
United Otto and Rothenberg (mixed)	1000°	17.17	76

Furthermore, the tars from different ovens of the same type show marked inconsistencies in many respects. For instance, twelve tars from as many different Semet-Solvay plants in which retort firing temperatures were in each case 1050°–1450° C, and maximum temperatures to which coal was brought, 950°–1150° C, show amounts of free carbon ranging from 4 per cent to 9 per cent; of pitch, from 63 per cent to 79 per cent; and of anthracene oils, from 5.5 per cent to 11 per cent.

Lunge,* the authority on coal tars, discusses only very briefly the effect of temperature range on the quality of tar. Among others, he quotes Behrens,† who found that the tar obtained in the distillation of coal in the ordinary fire-clay gas-retorts was much richer in benzene, tolene, and napthalene than that made from the same kind of coal in coke ovens operated at a lower temperature.

In considering and comparing the foregoing reports it should be borne in mind that as yet no standard or official methods for tar analysis and testing have been adopted by the industry as a whole. It is true, that while the manufacturers of special products such as creosote and road material, each in his own particular field has defined, more or less, his standards, the results obtained are not always comparable, and this is especially true when dealing with such complex hydrocar-

*Lunge, Coal Tars and Ammonia, 4th ed. 1909.

†Dingler's Polytech. Jour. 208, 362.

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bons as coal tars. In spite of this difficulty, however, there is sufficient unanimity among the authorities to establish the fact fully confirmed by the worker in organic chemistry that rise of temperatures tend to produce polymerization with the formation of more complex substances of higher molecular weight, and that comparatively valueless products such as naphthalene and heavy pitch are produced in increasing quantities. In other words, the lighter and more valuable hydrocarbons are being destroyed in favor of the heavier ones which find little industrial use.

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WIND STRESSES IN THE STEEL FRAMES OF OFFICE BUILDINGS

BY

W. M. WILSON

AND

G. A. MANEY

BULLETIN No. 80

ENGINEERING EXPERIMENT STATION

PUBLISHED BY THE UNIVERSITY OF ILLINOIS, URBANA

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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN No. 80 JUNE, 1915

WIND STRESSES IN THE STEEL FRAMES OF OFFICE BUILDINGS.

BY W. M. WILSON, ASSISTANT PROFESSOR OF STRUCTURAL ENGINEERING, AND G. A. MANEY, FORMERLY RESEARCH FELLOW IN DEPARTMENT OF THEORETICAL AND APPLIED MECHANICS.*

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*While the bulletin is the result of the joint efforts of the two writers, certain parts are so distinctly the work of one that it seems desirable to make a statement relative to the part which should be credited to each. The method of making the analysis involving the use of the slope-deflection equations should be credited to G. A. Maney. The conception of the bulletin as a whole, the method of presenting the results, and the authorship of the text should be credited to W. M. Wilson. The numerical calculations of Tables 11 to 22, inclusive, were made by both writers.—The Editor.

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WIND STRESSES IN THE STEEL FRAMES OF OFFICE BUILDINGS

I. Introduction.

1. *Preliminary.*—The increase in the price of land in large cities has made it necessary to build high buildings in order to get a large rentable floor space on a small parcel of land. The type of building generally used is known as the steel-skeleton building. In this type of building the live and dead loads, including the weight of the walls, are carried by a system of beams and girders to columns and are carried by the columns to the footings.

In high buildings the horizontal shear due to the wind load is very large; and, since it is usually impracticable to put diagonal braces between the columns, it is customary to make the steel frame rigid enough to resist the horizontal shear by virtue of the stiffness of the columns and girders. The exact determination of the stresses in a steel frame due to a horizontal shear is one of the problems of structural engineering which remains to be solved. While the writers realize that the method of determining these stresses presented in this bulletin is based upon assumptions which are not exactly true, they believe that the method is more accurate than the methods ordinarily used.

2. *Acknowledgment.*—Messrs. Anderson, Becker, Gomez, and Richart, graduate students in the College of Engineering of the University of Illinois, calculated the moments in Table 23, and the moments as determined by methods I, II, and III in the keys to the diagrams in Figs. 10, 11, and 12, and assisted with the calculations necessary to determine the curves shown in these figures. Professor Ira O. Baker and Professor C. A. Ellis rendered valuable assistance, criticizing the bulletin during its prepartion. The writers gratefully acknowledge indebtedness to these men.

II. Present Methods of Calculating Wind Stresses in Office Buildings.

3. *Classification of Methods.*—Methods of calculating wind stresses in the steel frames of office buildings may be divided into two classes, viz.: (1), those used in the actual design of buildings and (2), those

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which have resulted from an effort to make an exact analysis of the stresses. In the methods of the first class, accuracy has been sacrificed to shorten and simplify the calculations. In the methods of the second class, the aim has been to make an exact analysis rather than an analysis that can be used in the actual design of a building.

For the sake of convenience in reference, methods of the first class are designated as *approximate methods,* and those of the second class are designated as *exact methods.*

4. *Approximate Methods.*—(a) Fleming's Methods. Mr. R. Fleming, in an article in *Engineering News,* describes three methods, which are in current use.* These methods are designated as methods I, II, and III. The three methods, as applied to a building in which all columns of a story have the same section, are based upon the following assumptions:

Assumptions in Method I.

1. A bent of a frame acts as a cantilever.

2. The point of contra-flexure of each column is at mid-height of the story.

3. The point of contra-flexure of each girder is at its mid-length.

4. The direct stress in a column is directly proportional to the distance from the column to the neutral axis of the bent.

Assumptions in Method II.

1. A bent of a frame acts as a series of portals.

2. The point of contra-flexure of each column is at mid-height of the story.

3. The shear is the same on all columns of a story.

4. Each pair of adjacent columns of a bent acts as a portal, and each interior column is a member of two adjacent portals. The direct stress in an interior column, when the column is considered as a member of the portal on one side, is of opposite sign from the direct stress in the same column when considered as a member of the portal on the opposite side and the resultant direct stress is equal to zero.

*Wind Bracing without Diagonals for Steel-Frame Office Buildings, Engineering News, March 13, 1913.

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Assumptions in Method III.

1. A bent of a frame acts as a continuous portal.

2. The point of contra-flexure of each column is at mid-height of the story.

3. The direct stress in a column is directly proportional to the distance from the column to the neutral axis of the bent.

4. The shear is the same on all columns of a story.

(b) Smith's Methods. Professor Albert Smith, in a paper before the Western Society of Engineers, describes a method which he has used in his classes in Structural Engineering at Purdue University.‡ This method is here designated as Method IV.

Assumptions in Method IV.

1. The point of contra-flexure of each column is at mid-height of the story.

2. The point of contra-flexure of each girder is at its mid-length.

3. The shears on the internal columns are equal and the shear on each external column is equal to one-half of the shear on an interior column.

If all of the assumptions of any of these methods are accepted, the stresses in a frame may be determined by applying the fundamental equations of static equilibrium. It is apparent from the assumptions that the results obtained by these methods are radically different.

5. *Exact Methods.*—(a) Melick's Method. Dr. Cyrus A. Melick* used a method which takes into account the form of the elastic curves and the deflections and changes in length of the members. The method is so long that, when applied to a building only four stories high, the amount of work required is almost prohibitive. To apply it to a building twenty stories high would be impracticable if not, in fact, impossible.

(b) Jonson's Method. Mr. Ernest F. Jonson suggested a method which takes into account the deflections of the columns and the changes in the slopes of the tangents to the elastic curves of the columns and girders at the points where they intersect.†

If the method which he suggests were used, it would give the stresses with a fair degree of accuracy; but his method involves so many unknowns that its use would not be practicable in the actual design of buildings.

‡Wind Stresses in the Frames of Office Buildings, by Albert Smith, Journal Western Society of Engineers, Vol. XX, No. 4, p. 341.

*Stresses in Tall Buildings, by Cyrus A. Melick. Bulletin No. 8, College of Engineering, University of Ohio.

†The Theory of Frameworks with Rectangular Panels and Its Application to Buildings which Have to Resist Wind, by Ernst F. Jonson, Tran. Am. Soc. C. E., Vol. 55, p. 413.

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(c) Method of Least Work. Professor Albert Smith has determined the wind stresses in symmetrical bents having two, three, and four spans by the method of least work.* This method is exact, but, of course, is extremely long.

III. Outline of the Proposed Analysis.

6. *Outline of the Method.*—In making an analysis of the stresses, the writers made certain assumptions and applied certain fundamental principles of mechanics and obtained equations from which the stresses in a frame can be determined. The assumptions which have been made are stated in Section IV and the derivation of the equations is given in Sections V and VI.

It can be proven that the moment at an end of a member of a frame is a function of the changes in the slopes of the tangents to the elastic curve of the member at its ends and of the deflection of one end of the member relative to the other end (see equation A, page 13).

In the strained position, all the columns and girders which intersect at one point have been subjected to the same change in slope (see assumption 1, Section IV); the vertical deflections of the ends of all girders are equal to zero; and the horizontal deflections of the tops of all columns of a story are equal (see assumption 2, Section IV).

Consider any story of a bent. Take the point of intersection of the neutral axes of a column and a girder as a free body. It is in equilibrium under the action of the moments at the extremities of the columns and girders which intersect at the point. Each of the moments may be expressed in terms of the changes in the slopes at the extremities of the member, and the deflection of one end of the member relative to the other. A moment equation can therefore be written for each point where the columns and girders intersect, and the only unknown quantities will be the changes in the slopes at the extremities of the columns and the horizontal deflections of the columns in a story.

If all the columns of a story be taken together as a free body, the sum of the moments at the two extremities of all the columns will be balanced by a couple whose moment is equal to the total shear on the story multiplied by the story height. The shear and the height of the story are known, and the moments in the columns can be expressed in terms of the slopes and the deflections at their extremities the same as in the previous equations. It is therefore possible to write as many equations for each story as there are columns in the story, plus one. As the only

*Wind Stresses in the Frames of Office Buildings, by Albert Smith, Journal Western Society of Engineers, Vol. XX, No. 4, p. 341.

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unknown quantities in these equations are the changes in the slopes at the extremities of the columns and the deflection in a story common to all columns, there are as many equations per story as there are unknowns. By solving these equations the slopes and the deflections can be determined. Knowing the slopes and the deflections, the moments can be computed.

The product of the shear on a member and the length of the member is equal to the algebraic sum of the moments at the extremities of the member. Since the moments and the length of the member are known the shear can be computed.

With the shears in the girders known, the direct stress in any column can be determined by taking the column as a free body and equating the sum of the vertical forces to zero.

The direct stress in a girder may be determined in a similar manner.

The method just described is based upon the proposition in mechanics used by Mr. Jonson, but the method which the writers have developed differs from the one used by him in that the changes in the slopes and the deflections have been used as the unknown quantities instead of the direct stresses and the moments. Four members, two columns and two girders, intersect in a point. Each member is subjected to a different direct stress and a different moment, whereas all of the members are subjected to the same change in slope, and all of the columns in a story are subjected to the same deflection. It is therefore apparent that there are fewer unknown slopes and deflections than moments and direct stresses. The large reduction in the number of unknowns very much simplifies the solution of the equations.

IV. Assumptions Upon Which the Analysis Is Based.

7. *Statement of Assumptions.*—The proposed analysis is based upon the following assumptions:

1. The connections between the columns and girders are perfectly rigid.

2. The change in the length of a member due to the direct stress is equal to zero.

3. The length of a girder is the distance between the neutral axes of the columns which it connects and the length of a column is the distance between the neutral axes of the girders which it connects.

4. The deflection of a member due to the internal shearing stresses is equal to zero.

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5. The wind load is resisted entirely by the steel frame.

These assumptions will be discussed in Section XI.

V. Fundamental Equation.

8. *Fundamental Proposition.*—The fundamental equation used in this analysis is based upon the following proposition:

When a member is subjected to flexure, the deflection of any point in the neutral axis from the tangent to the elastic curve at any other point is equal to the moment of the area of the $\frac{M}{EI}$ *diagram for the portion of the member between the two points, about the point where the deflection is measured.*

9. *Proof of the Proposition.*—The line AB, Fig. 1, represents the neutral axis of a member subjected to flexure. The deflection of the member is very much exaggerated in the figure. The actual deflection is so small that the length of the curve may be considered equal to its horizontal projection. It is required to prove that the deflection of any point P from the tangent to the line AB at any other point Q is equal to

$$\int_Q^P \frac{M}{EI}\, dx\, x.$$

Extend the tangents at the extremities of an element of the curve until they intersect the vertical line through P. The intercept on this vertical line between two consecutive tangents is equal to $x\,d\theta$. The total deflection of P from the tangent at Q is equal to the algebraic sum of these intercepts for the elements of the elastic curve between the points Q and P. That is, $y = \int_Q^P x\,d\theta$. The equation of the elastic curve may be written $\frac{d\theta}{dx} = \frac{M}{EI}$. Substituting the value of $d\theta$ from this equation in the expression for y, gives $y = \int_Q^P \frac{M}{EI}\, dx\, x$. The quantity $\frac{M}{EI}\, dx$ can be considered as an elementary area of the $\frac{M}{EI}$ diagram. The quantity $\int_Q^P \frac{M}{EI}\, dx\, x$ can therefore be considered as the moment of the $\frac{M}{EI}$ diagram about the point P.

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10. *Derivation of Fundamental Equation.*—Consider a member which is not acted upon by any external forces or couples except at the ends. The line AB in Fig. 2 represents the neutral axis of such a member. The moment at A is represented by M_{AB} and at B by M_{BA}. The change in the slope of the elastic curve at A due to the external forces is represented by θ_A, and at B by θ_B. The deflection of A from its original position A', is d. The deflection of A from the tangent at B is represented by $(d - l \cdot \theta_B)$; and the deflection of B from the tangent

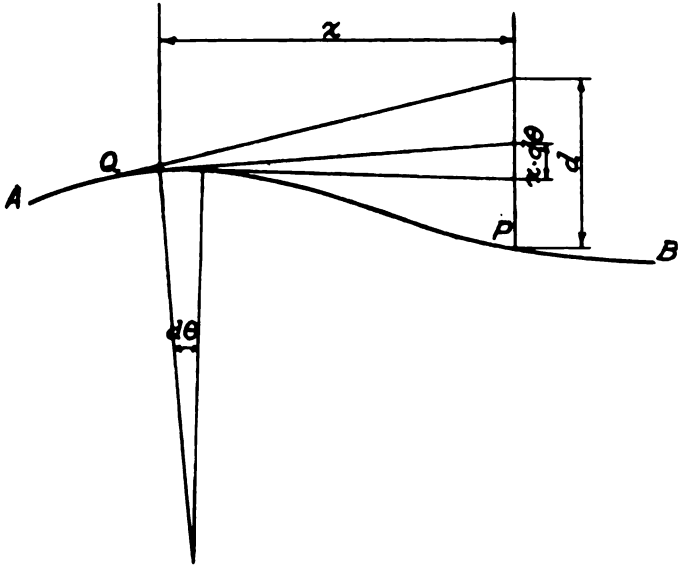

FIG. 1. PROOF OF FUNDAMENTAL EQUATION.

at A is represented by $(d - l \cdot \theta_A)$. The $\frac{M}{EI}$ diagram is shown in Fig. 3.

The signs of the quantities are determined by the following rules:

The change in the slope, or the angular deformation, is positive (+) when the tangent to the elastic curve of the member is turned in a clockwise direction.

Distances and deflections are positive when they are measured in the same direction from the base line as are positive slopes.

The moment acting upon a member at the section where the deflection is measured is positive (+) when it produces a clockwise rotation.

Substituting $(d - l \cdot \theta_B)$, the deflection of A from the tangent at B,

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for y in the equation $y = \int \frac{M}{EI} dx \cdot x$, gives

$$(d - l \cdot \theta_B) = \int_A^B \frac{M}{EI} dx \cdot x.$$

Substituting the value of M from the equation

$$M = M_{AB} + \left[\frac{M_{BA} - M_{AB}}{l}\right] x, \text{ gives}$$

$$(d - l \cdot \theta_B) = \int_A^B \frac{M_{AB}}{EI} x dx + \left[\frac{M_{BA} - M_{AB}}{EI \cdot l}\right] x^2 \cdot dx.$$

If the material is homogeneous and the section uniform, E and I are constants. Performing the indicated integration, gives

$$(d - l \cdot \theta_B) = \frac{M_{AB}}{EI} \frac{l^2}{2} + \frac{M_{BA}}{EI} \frac{l^2}{3} - \frac{M_{AB}}{EI} \frac{l^2}{3}, \text{ or}$$

$$d = l \cdot \theta_B + \frac{l^2}{6EI} \left[2M_{BA} + M_{AB}\right] \ldots\ldots\ldots\ldots\ldots (1)$$

Substituting $(d - l \cdot \theta_A)$, the deflection of B from the tangent at A, for y in the equation, $y = \int \frac{M}{EI} dx \cdot x$, gives

$$(d - l \cdot \theta_A) = \int_B^A \frac{M}{EI} dx \cdot x.$$

The moment at any section in the member, when considering the deflection at B, is of opposite sign from the moment at the same section when considering the deflection at A. (See preceding rule for determining the sign of the moment.)

Substituting the value of M from the equation

$$M = -M_{BA} - \left[\frac{M_{AB} - M_{BA}}{l}\right] x, \text{ gives}$$

$$(d - l \cdot \theta_A) = \int - \frac{M_{BA}}{EI} x dx - \left[\frac{M_{AB} - M_{BA}}{l}\right] x^2 \, dx \text{ or}$$

$$d = l \cdot \theta_A - \frac{M_{BA} l^2}{EI \; 2} - \frac{M_{AB} l^2}{EI \; 3} + \frac{M_{BA} l^2}{EI \; 3}$$

$$d = l \cdot \theta_A + \frac{l^2}{6EI} \left[-M_{BA} - 2M_{AB}\right] \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Multiplying equation (2) by 2, gives

$$2d = 2l \cdot \theta_A + \frac{l^2}{6EI} \left[-2M_{BA} - 4M_{AB}\right] \ldots\ldots\ldots\ldots\ldots (3)$$

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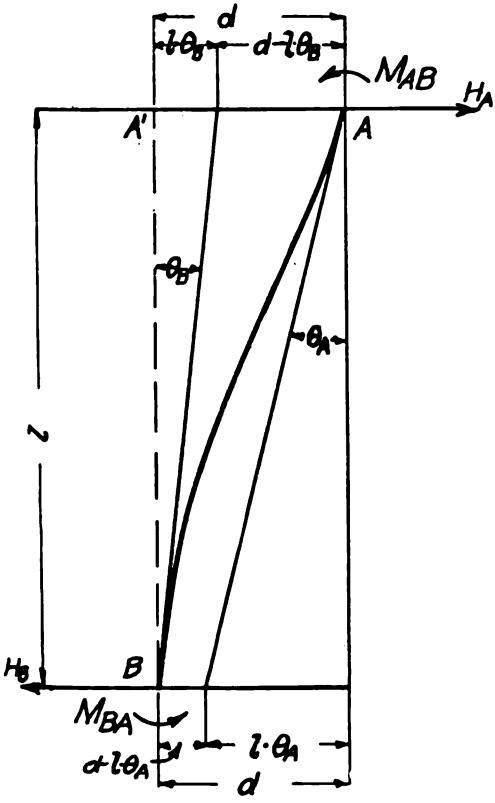

FIG. 2. DERIVATION OF FUNDAMENTAL EQUATION.

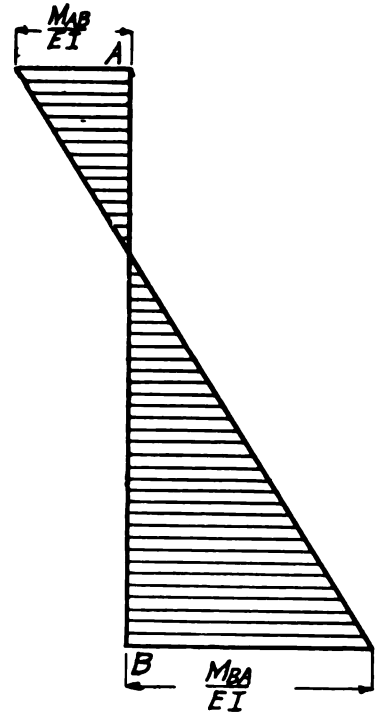

FIG. 3. $\frac{M}{EI}$ DIAGRAM.

Adding equations (1) and (3) gives

$$3d = 2l\cdot\theta_A + l\cdot\theta_B + \frac{l^2}{6EI}\left[-3M_{AB}\right].$$

Substituting K for $\frac{I}{l}$ and R for $\frac{d}{l}$ and solving for M_{AB}, gives,

$$M_{AB} = 2\,EK\ \ [2\,\theta_A + \theta_B - 3R] \ldots\ldots\ldots\ldots\ldots (A)$$

When $d = 0$, equation (A) becomes

$$M_{AB} = 2\,EK\ \ (2\,\theta_A + \theta_B) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (B)$$

Equation (A) is general and may be applied to any length of any member in bending provided the length considered has no intermediate external force applied to it. That is, one or more of the quantities

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θ_A, θ_B, and d may be negative and equation (A) will still give the moment at the point A in both magnitude and sign. Equation (A) is the fundamental equation upon which the analysis which follows is based. Equation (B) is a special form of equation (A). Equation (A) may be expressed as follows:

The moment at the end of any member is equal to 2EK times the quantity: Two times the change in the slope at the near end plus the change in the slope at the far end minus three times the deflection divided by the length. E is the modulus of elasticity of the material and K is the ratio of the moment of inertia to the length of the member.

VI. Derivation of General Equations.

11. *Notation.*—The following notation has been used:

A, B, C, etc. = the columns of a bent, beginning at the right and reading toward the left.

a, b, c, etc.= the bays of a bent, beginning at the right and reading toward the left. The girder in bay a is designated as girder a, in bay b as girder b, in bay c as girder c, etc.

A_1, A_2, A_3, etc. = the intersections of the neutral axes of the girders at the tops of the first, second, third, etc., stories with the neutral axis of column A.

B_1, B_2, B_3, etc. = the intersections of the neutral axes of the girders at the tops of the first, second, third, etc., stories with the neutral axis of column B.

d = deflection of the columns in a story height.

E = modulus of elasticity of the material.

h = length of a column measured from neutral axis to neutral axis of the girders.

I = moment of inertia of the girder and column sections.

$J = 2\sum\left(\frac{I}{l}+\frac{I}{h}\right)$ for all columns and girders which intersect at a point.

$K = \frac{I}{l}$ for girders and $\frac{I}{h}$ for columns.

l = length of a girder measured from neutral axis to neutral axis of the columns.

M = bending moment.

$N = 2\sum\left(\frac{I}{h}\right)$ for all columns in a story.

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$R = \frac{d}{h}$

W = total horizontal shear in a bent at any story.

w = increment of the horizontal shear in a bent in a story height.

θ = change in the slope of the tangent to the elastic curve.

Subscripts are added to the letters to indicate the particular part of a bent to which a given symbol applies. For example, referring to Fig. 4, girder b_1 is the girder in bay b at the top of the first story, A_1 is the intersection of the girder at the top of the first story with

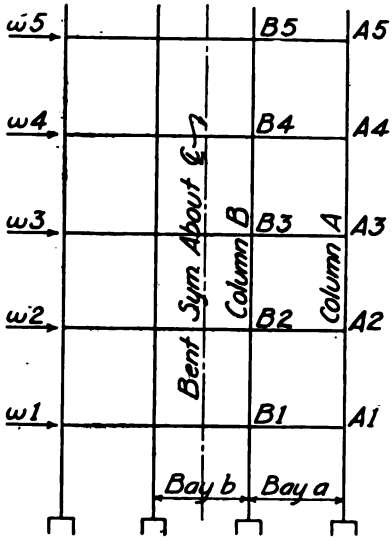

FIG. 4. BOTTOM FIVE STORIES OF A SYMMETRICAL THREE-SPAN BENT.

column A, θ_{A_1} is the slope of the tangent to the elastic curve at the point A_1; d_3 is the deflection of the columns in the third story; J_{B_4} is the J at the point B_4; K_{A_2} is the K of column A in the second story. The moment at the right end of the girder in bay a at the top of the second story is designated as M_2^{AB}, and the moment at the left end of the same girder is designated as M_2^{BA}. The moment at the top of the column B in the third story is designated as M_B^{32}, and the moment at the bottom of the same column is designated as M_B^{23}.

12. *Derivation of Equations.*—Fig. 4 shows the bottom five stories of a symmetrical three-span bent. It is required to find the stresses in all of the members.

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Consider the columns of the third story acting together as a free body. (Any other story could have been used.) The algebraic sum of all of the moments at the tops and the bottoms of all the columns plus the product of the total shear in the story and the story height, is equal to zero. That is, $2\,(M_A^{32} + M_A^{23} + M_B^{32} + M_B^{23}) + W_3 h_3 = 0$

Substituting the values of the moments as given by equations A and B of Section V gives

$$2\,[\,2EK_{A3}\,(2\theta_{A3} + \theta_{A2} - 3R_3) + 2EK_{A3}\,(2\theta_{A2} + \theta_{A3} - 3R_3) + 2EK_{B3}\,(2\theta_{B3} + \theta_{B2} - 3R_3) + 2EK_{B3}\,(2\theta_{B2} + \theta_{B3} - 3R_3)\,] + W_3 h_3 = 0.$$

Letting $N = 2\sum\left(\frac{I}{h}\right)$ for all of the columns in a story, and collecting, gives

$$2K_{A3}\,\theta_{A2} + 2K_{B3}\,\theta_{B2} - N_3 R_3 + 2K_{A3}\,\theta_{A3} + 2K_{B3}\,\theta_{B3} = -\frac{W_3 h_3}{6E} \ldots\ldots (1)$$

Consider the point A_3 as a free body. Taking $\sum M = 0$, gives $M_A^{34} + M_A^{32} + M_3^{AB} = 0$. Substituting the values of these moments as given by equations A and B gives,

$$2EK_{A4}\,(2\theta_{A3} + \theta_{A4} - 3R_4) + 2EK_{A3}\,(2\theta_{A3} + \theta_{A2} - 3R_3) + 2EK_{a3}\,(2\theta_{A3} + \theta_{B3}) = 0.$$

Combining the coefficients of the unknowns and cancelling the common factor $2E$ gives,

$$\theta_{A2}K_{A3} + \theta_{A3}\,(2K_{A4} + 2K_{A3} + 2K_{a3}) + \theta_{A4}\,K_{A4} + \theta_{B3}K_{a3} - R_3 3K_{A3} - R_4 3K_{A4} = 0.$$

Substituting J_{A3} for $2\sum\left(\frac{I}{h} + \frac{I}{l}\right)$ of all the members that intersect at A_3, the equation becomes:

$$K_{A3}\,\theta_{A2} - 3K_{A3}\,R_3 + J_{A3}\,\theta_{A3} + K_{a3}\,\theta_{B3} - 3K_{A4}R_4 + K_{A4}\theta_{A4} = 0 \ldots (2)$$

The point B_3 is in equilibrium under the action of the four moments M_3^{BA}, M_3^{BB}, M_B^{34}, and M_B^{32}. Equating the sum of these four moments to zero gives,

$$2EK_{a3}\,(2\theta_{B3} + \theta_{A3}) + 2EK_{b3}\,(2\theta_{B3} + \theta_{B3}) + 2EK_{B4}\,(2\theta_{B3} + \theta_{B4} - 3R_4) + 2EK_{B3}\,(2\theta_{B3} + \theta_{B2} - 3R_3) = 0.$$

Combining the coefficients for the unknowns and cancelling the common factor $2E$, gives,

$$2\theta_{B3}\,(K_{a3} + K_{b3} + K_{B4} + K_{B3}) + \theta_{A3}K_{a3} + \theta_{B3}K_{b3} + \theta_{B4}K_{B4} + \theta_{B2}K_{B3} - 3R_4K_{B4} - 3R_3K_{B3} = 0.$$

Substituting J_{B3} for $2\sum\left(\frac{I}{h} + \frac{I}{l}\right)$ of all the members intersecting at B_3, the equation becomes,

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$$K_{B_3}\theta_{B_2} - 3K_{B_3}R_3 + K_{a_3}\theta_{A_3} + (K_{b_3} + J_{B_3})\theta_{B_3} - 3K_{B_4}R_4 + K_{B_4}\theta_{B_4} = 0 \quad \ldots\ldots (3)$$

The three equations, 1, 2, and 3, can be written for any story by making the proper changes in the subscripts. As there are only three unknowns, θ_A, θ_B, and R for each story, as many equations can be written for a bent as there are unknowns to be determined. It is possible to solve these equations for the unknown quantities algebraically, but the large number of equations involved makes the work very difficult. It is simpler to substitute the numerical values of the coefficients in the equations and solve for the numerical values of the unknown quantities by the process of elimination explained in Section VII.

For convenience in the application of this process of elimination, the equations are written in tabular form as shown in Table 3 in which the unknown changes in the slopes and the ratios of deflection to story height are written at the tops of the columns and the coefficients of these unknowns are written below. For example, in writing equation A of Table 3, which is $-N_1R_1 + 2K_{A_1}\theta_{A_1} + 2K_{B_1}\theta_{B_1} = -\frac{W_1h_1}{6E}$ $-N_1$, the coefficient of R_1, is placed in the column under R_1; $2K_{A_1}$, the coefficient of θ_{A_1}, is placed in the column under θ_{A_1}; $2K_{B_1}$, the coefficient of θ_{B_1}, is placed in the column under θ_{B_1}; and $-\frac{W_1h_1}{6E}$ is placed in the column headed "Right-Hand Member of Equation." Having the equations written in this form, it is unnecessary to repeat the unknown quantities when eliminating them from the equations by the method used in Table 14.

Table 3 contains the general equations to be used in determining the slopes and the deflections in a symmetrical three-span bent any number of stories high. The subscripts 1, 2, and 3 refer to the first, second, and third stories respectively, and the subscripts x, y, and z refer to the second from the top, the next to the top, and the top stories respectively. The equations for the intervening stories are of the same form, but have different subscripts.

By using the general method outlined above, that is, by writing an equation similar to equation 1 for each story and an equation similar to equations 2 and 3 for each intersection of a column with a girder for each story, as many equations as there are unknowns can be written for a bent containing any number of spans and any number of stories. General equations for *symmetrical* bents of from one to five spans are given in Tables 1 to 5 and similar equations for *unsymmetrical* bents of from one to five spans are given in Tables 6 to 10.

In order to check the equations of Tables 1 to 10, a model of a

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Table 1.

General Equations for a Symmetrical Single-Span Bent Any Number of Stories High.

Equation	Left-Hand Member of Equation													Right-Hand Member of Equation
	Story No. 1		Story No. 2		Story No. 3		Intervening Stories	Story No. X (Second from Top)		Story No. Y (Next to Top)		Story No. Z (Top)		
	R_1	θ_{A1}	R_2	θ_{A2}	R_3	θ_{A3}		R_X	θ_{AX}	R_Y	θ_{AY}	R_Z	θ_{AZ}	
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height													
A	$-N_1$	$2K_{A1}$												$-W_1h_1+6E$
B	$-3K_{A1}$	$J_{A1}+K_{a1}$	$-3K_{A2}$	K_{A2}										0
C		$2K_{A2}$	$-N_2$	$2K_{A2}$										$-W_2h_2+6E$
D		K_{A2}	$-3K_{A2}$	$J_{A2}+K_{a2}$	$-3K_{A3}$	K_{A3}								0
Similar equations for intervening stories														
W									$2K_{AY}$	$-N_Y$	$2K_{AY}$			$-W_Yh_Y+6E$
X									K_{AY}	$-3K_{AY}$	$J_{AY}+K_{aY}$	$-3K_{AZ}$	K_{AZ}	0
Y											$2K_{AZ}$	$-N_Z$	$2K_{AZ}$	$-W_Zh_Z+6E$
Z											K_{AZ}	$-3K_{AZ}$	$J_{AZ}+K_{aZ}$	0

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VO-SPAN BENT

I.

Equation	Story No. 1 (Top)			Story No. Y (Next to Top)			Story No. Z (Top)			Right-Hand Member of Equation
	R_1	θ_{A1}	θ_{AY}	R_Y	θ_{AY}	θ_{BY}	R_Z	θ_{AZ}	θ_{BZ}	
	ry Height									
A	$-N_1$	$2K_{A1}$								$-W_1h_1 \div 6E$
B	$-3K_{A1}$	J_{A1}								O
C	$-3K_{B1}$	$2K_{a1}$								O
D		$2K_{A2}$								$-W_2h_2 \div 6E$
E		K_{A2}								O
F										O
U			K_{BY}	$-N_Y$	$2K_{AY}$	K_{BY}				$-W_Yh_Y \div 6E$
V				$-3K_{AY}$	J_{AY}	K_{aY}	$-3K_{AZ}$	K_{AZ}		O
W			K_{BY}	$-3K_{BY}$	$2K_{aY}$	J_{BY}	$-3K_{BZ}$		K_{BZ}	O
X					$2K_{AZ}$	K_{BZ}	$-N_Z$	$2K_{AZ}$	K_{BZ}	$-W_Zh_Z \div 6E$
Y					K_{AZ}		$-3K_{AZ}$	J_{AZ}	K_{aZ}	O
Z						K_{BZ}	$-3K_{BZ}$	$2K_{aZ}$	J_{BZ}	O

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3.

ETRICAL THREE-SPAN BENT
STORIES HIGH.

uation									Right-Hand Member of Equation
Story No. X (Second from Top)			Story No. Y (Next to Top)			Story No. Z (Top)			
R_X	θ_{AX}	θ_{BX}	R_Y	θ_{AY}	θ_{BY}	R_Z	θ_{AZ}	θ_{BZ}	
Deflection to Story Height									
									$-W_1 h_1+6E$
									0
									0
									$-W_2 h_2+6E$
									0
									0
rvening stories									
	$2K_{AY}$	$2K_{BY}$	$-N_Y$	$2K_{AY}$	$2K_{BY}$				$-W_Y h_Y+6E$
	K_{AY}		$-3K_{AY}$	J_{AY}	K_{aY}	$-3K_{AZ}$	K_{AZ}		0
		K_{BY}	$-3K_{BY}$	K_{aY}	$J_{BY}+K_{bY}$	$-3K_{BZ}$		K_{BZ}	0
				$2K_{AZ}$	$2K_{BZ}$	$-N_Z$	$2K_{AZ}$	$2K_{BZ}$	$-W_Z h_Z+6E$
				K_{AZ}		$-3K_{AZ}$	J_{AZ}	K_{aZ}	0
					K_{BZ}	$-3K_{BZ}$	K_{aZ}	$J_{BZ}+K_{bZ}$	0

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R-Span Bent

Equation	Story No. X (Top)		Story No. Y (Next to Top)				Story No. Z (Top)				Right-Hand Member of Equation
	R_X	θ_{CX}	R_Y	θ_{AY}	θ_{BY}	θ_{CY}	R_Z	θ_{AZ}	θ_{BZ}	θ_{CZ}	
	Height										
A											$-W_1 h_1 + 6E$
B											0
C											0
D											0
E											$-W_2 h_2 + 6E$
F											0
G											0
H											0
S	J_Y	K_{CY}	$-N_Y$	$2K_{AY}$	$2K_{BY}$	K_{CY}					$-W_Y h_Y + 6E$
T			$-3K_{AY}$	J_{AY}	K_{aY}		$-3K_{AZ}$	K_{AZ}			0
U	J_Y		$-3K_{BY}$	K_{aY}	J_{BY}	K_{bY}	$-3K_{BZ}$		K_{BZ}		0
V		K_{CY}	$-3K_{CY}$		$2K_{bY}$	J_{CY}	$-3K_{CZ}$			K_{CZ}	0
W				$2K_{AZ}$	$2K_{BZ}$	K_{CZ}	$-N_Z$	$2K_{ZV}$	$2K_{BZ}$	K_{CZ}	$-W_Z h_Z + 6E$
X				K_{AZ}			$-3K_{AZ}$	J_{AZ}	K_{aZ}		0
Y					K_{BZ}		$-3K_{BZ}$	K_{aZ}	J_{BZ}	K_{bZ}	0
Z						K_{CZ}	$-3K_{CZ}$		$2K_{bZ}$	J_{CZ}	0

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Equation	Story No. Y (Next to Top)				Story No. Z (Top)				Right-Hand Member of Equation
	R_Y	θ_{AY}	θ_{BY}	θ_{CY}	R_Z	θ_{AZ}	θ_{BZ}	θ_{CZ}	
A	$-N$								$-W_1 h_1+6E$
B	$-3K$								0
C	$-3K$								0
D	$-3K$								0
E									$-W_2 h_2+6E$
F									0
G									0
H									0
S	Y	$2K_{AY}$	$2K_{BY}$	$2K_{CY}$					$-W_Y h_Y+6E$
T	K_{AY}	J_{AY}	K_{aY}		$-3K_{AZ}$	K_{AZ}			0
U	K_{BY}	K_{aY}	J_{BY}	K_{bY}	$-3K_{BZ}$		K_{BZ}		0
V	K_{CY}		K_{bY}	$J_{CY}+K_{cY}$	$-3K_{CZ}$			K_{CZ}	0
W		$2K_{AZ}$	$2K_{BZ}$	$2K_{CZ}$	$-N_Z$	$2K_{AZ}$	$2K_{BZ}$	$2K_{CZ}$	$-W_Z h_Z+6E$
X		K_{AZ}			$-3K_{AZ}$	J_{AZ}	K_{aZ}		0
Y			K_{BZ}		$-3K_{BZ}$	K_{aZ}	J_{BZ}	K_{bZ}	0
Z				K_{CZ}	$-3K_{CZ}$		K_{bZ}	$J_{CZ}+K_{cZ}$	0

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:TRICAL SINGLE-SPAN BENT
RIES HIGH.

Equation	ion									Right-Hand Member of Equation
	Story No. X (Second from Top)			Story No. Y (Next to Top)			Story No. Z (Top)			
	R_X	θ_{AX}	θ_{BX}	R_Y	θ_{AY}	θ_{BY}	R_Z	θ_{AZ}	θ_{BZ}	
	Flection to Story Height									
A	$-N_1$									$-W_1h_1 \div 6E$
B	$-3K_1$									O
C	$-3K_1$									O
D										$-W_2h_2 \div 6E$
E										O
F										O
	ming stories									
U		K_{AY}	K_{BY}	$-N_Y$	K_{AY}	K_{BY}				$-W_Yh_Y \div 6E$
V		K_{AY}		$-3K_{AY}$	J_{AY}	K_{aY}	$-3K_{AZ}$	K_{AZ}		O
W			K_{BY}	$-3K_{BY}$	K_{aY}	J_{BY}	$-3K_{BZ}$		K_{BZ}	O
X					K_{AZ}	K_{BZ}	$-N_Z$	K_{AZ}	K_{BZ}	$-W_Zh_Z \div 6E$
Y					K_{AZ}		$-3K_{AZ}$	J_{AZ}	K_{aZ}	O
Z						K_{BZ}	$-3K_{BZ}$	K_{aZ}	J_{BZ}	O

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SPAN BENT

Equation	R_{CX}	Story No. Y (Next to Top) R_Y	θ_{AY}	θ_{BY}	θ_{CY}	Story No Z (Top) R_Z	θ_{AZ}	θ_{BZ}	θ_{CZ}	Right-Hand Member of Equation
	y Height									
A	$-K$									$-W_1 h_1+6E$
B	$-3K$									O
C	$-3K$									O
D	$-3K$									O
E										$-W_2 h_2+6E$
F										O
G										O
H										O
S	$_{CY}$	$-N_Y$	K_{AY}	K_{BY}	K_{CY}					$-W_Y h_Y+6E$
T		$-3K_{AY}$	J_{AY}	K_{aY}		$-3K_{AZ}$	K_{AZ}			O
U		$-3K_{BY}$	K_{aY}	K_{BY}	K_{bY}	$-3K_{BZ}$		K_{BZ}		O
V	$_{CY}$	$-3K_{CY}$		K_{bY}	J_{CY}	$-3K_{CZ}$			K_{CZ}	O
W			K_{AZ}	K_{BZ}	K_{CZ}	$-N_Z$	K_{AZ}	K_{BZ}	K_{CZ}	$-W_Z h_Z+6E$
X			K_{AZ}			$-3K_{AZ}$	J_{AZ}	K_{aZ}		O
Y				K_{BZ}		$-3K_{BZ}$	K_{aZ}	J_{BZ}	K_{bZ}	O
Z					K_{CZ}	$-3K_{CZ}$		K_{bZ}	J_{CZ}	O

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Equation	Story No. Y (Next to Top)				Story No. Z (Top)					Right-Hand Member of Equation
	θ_{AY}	θ_{BY}	θ_{CY}	θ_{DY}	R_Z	θ_{AZ}	θ_{BZ}	θ_{CZ}	θ_{DZ}	
A										$-W_1 h_1+6E$
B										0
C										0
D										0
E										0
F										$-W_2 h_2+6E$
G										0
H										0
I										0
J										0
Q	K_{AY}	K_{BY}	K_{CY}	K_{DY}						$-W_Y h_Y+6E$
R	J_{AY}	K_{aY}			$-3K_{AZ}$	K_{AZ}				0
S	K_{aY}	J_{BY}	K_{bY}		$-3K_{BZ}$		K_{BZ}			0
T		K_{bY}	J_{CY}	K_{cY}	$-3K_{CZ}$			K_{CZ}		0
U			K_{cY}	J_{DY}	$-3K_{DZ}$				K_{DZ}	0
V	K_{AZ}	K_{BZ}	K_{CZ}	K_{DZ}	$-N_Z$	K_{AZ}	K_{BZ}	K_{CZ}	K_{DZ}	$-W_Z h_Z+6E$
W	K_{AZ}				$-3K_{AZ}$	J_{AZ}	K_{aZ}			0
X		K_{BZ}			$-3K_{BZ}$	K_{aZ}	J_{BZ}	K_{bZ}		0
Y			K_{CZ}		$-3K_{CZ}$		K_{bZ}	J_{CZ}	K_{cZ}	0
Z				K_{DZ}	$-3K_{DZ}$			K_{cZ}	J_{DZ}	0

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DNS
NY

Y (Top)			Story No. Z (Top)						Right-Hand Member of Equation
θ_{EY} θ_{CY}	θ_{DY}	θ_{EY}	R_Z	θ_{AZ}	θ_{BZ}	θ_{CZ}	θ_{DZ}	θ_{EZ}	
own S									
									$-W_1 h_1 + 6E$
									0
									0
									0
									0
									0
									$-W_2 h_2 + 6E$
									0
									0
									0
									0
K_{E1}									0
Simila									
CY	K_{DY}	K_{EY}							$-W_Y h_Y + 6E$
			$-3K_{AZ}$	K_{AZ}					0
pY			$-3K_{BZ}$		K_{BZ}				0
CY	K_{cY}		$-3K_{CZ}$			K_{CZ}			0
cY	J_{DY}	K_{dY}	$-3K_{DZ}$				K_{DZ}		0
	K_{dY}	J_{EY}	$-3K_{EZ}$					K_{EZ}	0
CZ	K_{DZ}	K_{EZ}	$-N_Z$	K_{AZ}	K_{BZ}	K_{CZ}	K_{DZ}	K_{EZ}	$-W_Z h_Z \div 6E$
			$-3K_{AZ}$	J_{AZ}	K_{aZ}				0
			$-3K_{BZ}$	K_{aZ}	J_{BZ}	K_{bZ}			0
CZ			$-3K_{CZ}$		K_{bZ}	J_{CZ}	K_{cZ}		0
	K_{DZ}		$-3K_{DZ}$			K_{cZ}	J_{DZ}	K_{dZ}	0
		K_{EZ}	$-3K_{EZ}$				K_{dZ}	J_{EZ}	0

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Equation	Left Hand		Story No. Z (Top)								Right-Hand Member of Equation
		θ_{F3}	θ_{FY}	R_Z	θ_{AZ}	θ_{BZ}	θ_{CZ}	$\theta_{1\cdot Z}$	θ_{EZ}	θ_{FZ}	
	known Slopes										
A											$-W_1 h_1 \div 6E$
B	3										0
C	3										0
D	3										0
E	3										0
F	3										0
G	3										0
H											$-W_2 h_2 \div 6E$
I											0
J											0
K											0
L											0
M											0
N		K_{F3}									0
	nilar equatio										
M			K_{FY}								$-W_Y h_Y \div 6E$
N				$-3K_{AZ}$	K_{AZ}						0
O				$-3K_{BZ}$		K_{BZ}					0
P				$-3K_{CZ}$			K_{CZ}				0
Q				$-3K_{DZ}$				K_{DZ}			0
R			K_{eY}	$-3K_{EZ}$					K_{EZ}		0
S			J_{FY}	$-3K_{FZ}$						K_{FZ}	0
T			K_{FZ}	$-N_Z$	K_{AZ}	K_{BZ}	K_{CZ}	K_{DZ}	K_{EZ}	K_{FZ}	$-W_Z h_Z \div 6E$
U				$-3K_{AZ}$	J_{AZ}	K_{aZ}					0
V				$-3K_{BZ}$	K_{aZ}	J_{BZ}	K_{bZ}				0
W				$-3K_{CZ}$		K_{bZ}	J_{CZ}	K_{cZ}			0
X				$-3K_{DZ}$			K_{cZ}	J_{DZ}	K_{dZ}		0
Y				$-3K_{EZ}$				K_{dZ}	J_{EZ}	K_{eZ}	0
Z			K_{FZ}	$-3K_{FZ}$					K_{eZ}	J_{FZ}	0

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bent was tested and the measured deflections and changes in the slopes were compared with the same quantities calculated by the above equations. As the entire model was cut from a sheet of celluloid the joints were perfectly rigid. The results of the tests are given in Fig. 15. The fact that the measured and computed deflections and changes in the slopes agree very closely indicates that the above analysis is correct.

A solution of a numerical problem illustrating the use of the equations in Table 3 is given in Section VII.

VII. Numerical Problem.

13. *Determination of the Stresses in a Symmetrical Three-Span Twenty-Story Bent.*—Fig. 5 shows a symmetrical three-span bent twenty stories high. The bent resists a horizontal wind load of 30 lb. per sq. ft. on a vertical strip one foot wide. It is required to find the moment and the shear in the columns and girders and the direct stress in the columns.

The properties of the girder and column sections are shown in Table 11, page 48. The equations of Table 3 are applicable. The numerical values of the constants in these equations are given in Table 12. Table 13 contains the equations of Table 3 with the substitution of the numerical values of the constants given in Table 12. The figures given in the right-hand column are coefficients of .0001 as indicated at the head of the column; that is, the right-hand members of the equations are equal to .0001 times the numbers in the right-hand column. This method of writing the right-hand members of the equations obviates the necessity of repeating a large number of ciphers.

The unknown quantities in the equations of Table 13 are eliminated in Table 14. The quantity R_1 appears in the first three equations of Table 13 only. Dividing each of these equations by the coefficient of R_1 gives the three equations A, B, and C of Table 14, in all of which the coefficient of R_1 is equal to plus unity. Combining these latter equations as indicated, that is, subtracting equation B from A to get equation (A — B) and subtracting equation C from B to get equation (B — C), gives two new equations from which R_1 has been eliminated. Reducing the coefficients of the left-hand terms of equations (A — B) and (B — C) to plus unity gives equations 1 and 2 of Table 14. Equations D and E of Table 13 have also been reduced so that the coefficients of the left-hand term of each are equal to plus unity. They are rewritten in their new form as equations D and E in Table 14. Combining the four equations 1, 2, D, and E, as indicated in Table 14, eliminates

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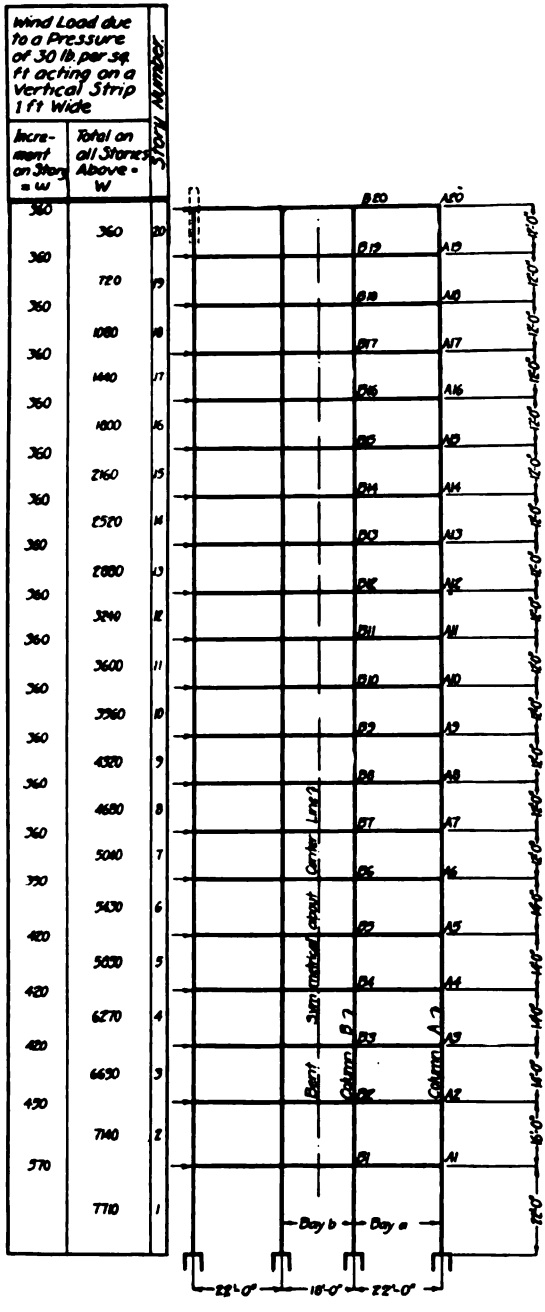

FIG. 5. SYMMETRICAL THREE-SPAN BENT, TWENTY STORIES HIGH.

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the left-hand term, θ_{A1}. By continuing this process all the unknown quantities are eliminated except the last, or θ_{B20}. Its value thus becomes known and is found to be .0338 × .0001 radians. With the value of θ_{B20} known, the value of θ_{A20} can be determined from equation 153. With θ_{B20} and θ_{A20} both known, the value of R_{20} can be determined from equation 150. The other values of θ and R can be determined in a similar manner.

The process of determining the values of θ and R is given in Table 15. The equations used are taken from Table 14.

As in Tables 13 and 14, the right-hand member is equal to the number in the column at the right multiplied by .0001. To illustrate, equation 150 is

$$R_{20} - .2702\,\theta_{A20} - .2993\,\theta_{B20} = .0735 \times .0001.$$

The left-hand term of the left-hand member of each equation is the unknown which is to be determined. The first line in each group is the algebraic form of the equation; and the successive lines are the numerical values of the corresponding terms. For example, equation 155 is

$$\theta_{B20} = .0338 \times .0001 \text{ or } .00000338.$$

Again, equation 153 is:

$$\theta_{A20} - .0443\,\theta_{B20} = .0502 \times .0001.$$

From the preceding equation

$$.0443\,\theta_{B20} = .0443 \times .00000338 = .0015 \times .0001.$$

The sign of this quantity as a term of the left-hand member was negative, and the term will, therefore, be positive after it has been transferred.

Equation 153 now becomes

$$\theta_{A20} = .0502 \times .0001 + .0015 \times .0001 = .0517 \times .0001,$$

as indicated in the last line of the group containing equation 153. Equation 150 contains R_{20}, θ_{A20}, and θ_{B20}; but θ_{A20} and θ_{B20} have been determined by equations 153 and 155, respectively, therefore R_{20} can be determined by equation 150. Similarly, equation 147, containing θ_{B19} and the three known quantities, R_{20}, θ_{A20}, and θ_{B20} can be used to determine θ_{B19}. Equation 147 differs from the preceding equations in that it contains both positive and negative terms in the left-hand member. The second term of the left-hand member, being negative, when transferred, is added to the right-hand member. The sum of the two quantities is .0890 × .0001. Since the third and fourth terms of the left-hand member are positive, their sum, when transferred, is

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subtracted from the quantity above. The final form of the equation is: $\theta_{B_{19}} = .0835 \times .0001$. In a similar manner the other slopes, θ, and the ratios of the deflection to the story height, R, can be determined. θ is expressed in radians, and R is an abstract quantity.

The tabulation of the calculations as shown in Tables 14 and 15 facilitates the solution of a large number of equations. There are however a number of practical considerations which should be borne in mind when solving a problem by this method.

An error at any point affects all of the calculations which follow. It is therefore desirable to have two computers carry on the work simultaneously and compare results at frequent intervals in order to avoid the loss of time due to errors.

A number of combinations can be made from each group of equations. It is desirable to combine the equations so as to make the coefficient of the left-hand term of the left-hand member of the resulting equation as large as possible compared with the coefficients of the other terms in the equation. To illustrate this point, consider equations 25, 26, and M of Table 14. If equation M had been subtracted from equation 26, the coefficient of θ_{B_4} in the resulting equation would have been .1463 and the coefficient of R_5 would have been 2.9027. The latter is about twenty times as great as the former. Any small error in the actual value of the former coefficient would be a large percentage of error. This same percentage of error would be introduced into the latter coefficient and produce a large actual error. With the combination of equations 25 and M as used, the coefficient of θ_{B_4} is 1.0695 and of R_5, 3.4313. The latter coefficient is only about three times as large as the former and hence the effect of an inaccuracy in the value of the former upon the latter is correspondingly smaller than in the first case.

It is possible in some cases to combine the equations in such a way as to give equations which are algebraically independent, but which are numerically nearly identical. Combining such equations gives results which are likely to be inaccurate. Any tendency of the equations to become identities can be avoided by changing the order in which they are combined.

In getting the values of the remaining unknown quantities after one has been determined, each unknown is expressed in terms of the known quantities. By referring to Table 14 it is seen that in the first equation of a group of equations, any one of which could be used to get the value of an unknown, the coefficient of the quantity which is to be determined is larger than the coefficients of the known quantities. If

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this equation is used, any inaccuracies in the known quantities will affect the accuracy of the results less than they would if an equation had been used in which the coefficient of the unknown is less than the coefficient of the known quantities. The various equations available should be examined and the one used which causes the least accumulation of inaccuracies.

In order to determine the extent to which inaccuracies in the calculations accumulate, two independent sets of calculations were made; one set was made on a 20-inch slide rule and the other on a Fuller's cylindrical slide rule. The two sets of calculations were compared at frequent intervals, and mistakes were corrected; but no adjustment of inaccuracies was made. The maximum variation in the slopes and deflections as determined in the two sets of calculations was very small. This indicates that the calculations can be made with a slide rule without greater inaccuracies than are permissible.

The moments at the ends of the columns and girders can be determined by substituting the values of the deflections and the slopes, given in Table 15, in equations A and B of Section V. To facilitate this work, the quantities which occur in these equations are given in Tables 16 and 17. The values of R and θ taken from Table 15 are given in the second, third, and fourth columns of Table 16; and the functions of the values of R and θ which occur in the equations are given in the remaining columns of the same table. The values of K taken from Table 12 are given in the second, third, fourth, and fifth columns of Table 17; and the functions of the values of K which occur in the equations are given in the remaining columns of the table. The moments at the ends of the columns and girders given by the fundamental equation, $M_{AB} = 2EK\ (2\theta_A + \theta_B - 3R)$, are the products of two factors, one of which is given in Table 16 and the other in Table 17. The values of the moments are given in Table 18.

The shear in any member is equal to the algebraic sum of the moments at the two ends of the member divided by its length. The direct stress at any section in column A is equal to the algebraic sum of the shears in the girders in bay a above the section. The direct stress at any section in column B is equal to the algebraic sum of the shears in the girders in bay a plus the algebraic sum of the shears in the girders in bay b above the section. The shears in the columns and girders and the direct stresses in the columns are given in Table 19.

The sum of all of the moments at the top and the bottom of all the columns in a story is equal to the total shear on the story multiplied

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by the story height. The algebraic sum of the moments in the girders on the two sides of a column is equal to the algebraic sum of the moments in the columns on the two sides of the girder.

The total shear on a story multiplied by the story height and the algebraic sum of all of the moments at the top and the bottom of all columns of a story, as given in Table 18, are given in the second and third columns of Table 20. The algebraic sum of the moments in column *A* at sections immediately above and below the girders and the moments in the girders in bay *a* at sections adjacent to column *A*, are given in the fourth and fifth columns of Table 20. The algebraic sum of the moments in column *B* at sections immediately above and below the girders and the algebraic sum of the moments in the girders in bays *a* and *b* at sections adjacent to column *B*, are given in the sixth and seventh columns of Table 20. The accuracy of the computations can be checked by comparing the two values for the same quantity as given in adjacent columns of Table 20. To illustrate, for the sixth story, the total shear in the story multiplied by the story height is 913,000 inch-pounds, and the sum of the moments at the tops and bottoms of all columns in the story is 914,400 inch-pounds. For a perfect check, the two quantities would be equal. An inspection of the table shows that the values check very closely.

The moments given in Table 18 are due to a horizontal wind load of 30 lbs. per sq. ft. acting on a vertical strip one foot wide. To obtain the moments due to the wind load on any portion of the building, multiply the moments in Table 18 by the width in feet of the portion of the building on which the wind load acts.

VIII. Approximate Methods.

14. *Nomenclature of Methods.*—The method used in Section VII to determine the stresses in a symmetrical three-span twenty-story bent can be used in the actual design of a building, but a shorter method is desirable. The writers propose an approximate method which is much shorter than the method used in Section VII and which they believe is sufficiently accurate to be used in the actual design of buildings. For the sake of convenience in reference, this approximate method will be designated as the "Proposed Approximate Method" as distinguished from the method used in Section VII, which will be designated as the "Slope-Deflection Method." The slope-deflection method can be modified to advantage under certain special conditions. Such modifications

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will be designated as "Modifications of the Slope-Deflection Method."

15. *Proposed Approximate Method.*—The proposed approximate method is based upon the following assumptions in addition to those used in the slope-deflection method.

1. The changes in the slope at the top of a column in the story above and in the story below the one in which the stresses are to be determined, are equal to the change in the slope at the top of the corresponding column in the latter story.

2. The ratio of the deflection to the length of the columns in the story above the one in which the stresses are to be determined, is equal to the ratio of the deflection to the length of the columns in the latter story.

In other words, in determining the stresses in the second story, θ_{A1} and θ_{A3} are assumed to be equal to θ_{A2}; θ_{B1} and θ_{B3} are assumed to be equal to θ_{B2}; and R_3 is assumed to be equal to R_2. Also, in figuring the stresses in the third story θ_{A2} and θ_{A4} are assumed to be equal to θ_{A3}; θ_{B2} and θ_{B4} are assumed to be equal to θ_{B3}; and R_4 is assumed to be equal to R_3. This does not mean, however, that the values of θ_{A2}, θ_{B2}, and R_2 used in determining the stresses in the second story are equal to the values of θ_{A3}, θ_{B3}, and R_3 respectively, used in determining the stresses in the third story.

An examination of the equations in Table 3 shows that, if assumptions 1 and 2 were true, three equations containing only three unknown quantities could be written for each story of the bent. To illustrate, consider the equations for the second story. In accordance with assumptions 1 and 2,

$$\theta_{A1} = \theta_{A2} = \theta_{A3}$$
$$\theta_{B1} = \theta_{B2} = \theta_{B3}$$
$$R_2 = R_3$$

Substituting θ_{A2} for θ_{A1} and θ_{A3}, θ_{B2} for θ_{B1} and θ_{B3}, and R_2 for R_3 in equations D, E, and F of Table 3 gives:

$$-N_2R_2 + 4K_{A2}\theta_{A2} + 4K_{B2}\theta_{B2} = -\frac{W_2h_2}{6E} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$-3(K_{A2} + K_{A3})R_2 + (K_{A2} + J_{A2} + K_{A3})\theta_{A2} + K_{a2}\theta_{B2} = 0 \ldots\ldots\ldots (2)$$

$$-3(K_{B2} + K_{B3})R_2 + K_{a2}\theta_{A2} + (K_{B2} + J_{B2} + K_{b2} + K_{B3})\theta_{B2} = 0 \ldots (3)$$

These equations have been written for the second story. Similar equations can be written for any other story by making the proper changes in the subscripts.

Equations 1, 2, and 3, obtained from the equations in Table 3, can be used in determining the stresses in a symmetrical three-span bent

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any number of stories high. If similar changes are made in the equations of Tables 1 to 10 inclusive, groups of equations can be obtained, one for each bent, which can be used to determine the stresses in any story of *symmetrical* bents of from one to five spans and also of *unsymmetrical* bents of from one to five spans.

The sum of the moments at the tops and bottoms of all columns of a story is equal to the total shear in the story multiplied by the story height. The distribution of this moment to the ends of the columns depends upon: first, the ratio of the K of column A to the K of column B; second, the ratio of the K of column A to the K of girder a; and third, the ratio of the K of girder a to the K of girder b.

The distribution of the moment was determined in a number of bents for which the ratios of the K of column A to the K of column B, the K of column A to the K of girder a, and the K of girder a to the K of girder b had different values. Diagrams showing the distribution of the moment in these bents also give the distribution of the moment in other bents. The curves in Figs. 6, 7, and 8 show the distribution of the moment in symmetrical three-span bents. In Fig. 6, the curves in group I show the moment at the top and the bottom of column A in bents in which the K of column A and the K of column B are equal. The abscissae represent the ratios of the K of girder a to the K of girder b; and the ordinates represent the moment at the top and the bottom of column A in per cent of $W \times h$. Beginning at the top and reading down, the curves are for bents for which the ratio of the K of column A to the K of girder a equals 0.5, 1, 2, and 4 respectively. The moment in column A of a bent for which the ratio of the K of column A to the K of girder a has any intermediate value can be determined by interpolation.

The curves in groups II, III, IV, and V, of Fig. 6, when used with the curves in group I, give the moment at the top and the bottom of column A in bents in which the K of column A and the K of column B are not equal. The curves in groups II, III, IV, and V are for bents for which the ratio of the K for column A to the K for girder a equals 4, 2, 1, and 0.5, respectively. Beginning at the left of each group and reading to the right, the curves are for bents for which the ratio of the K for column A to the K for column B equals 0.5, 1, and 2, respectively.

The moment at the top and the bottom of column A can be obtained from the diagram in Fig. 6 in the following manner:

First consider the moment in a bent in which the K of column A is

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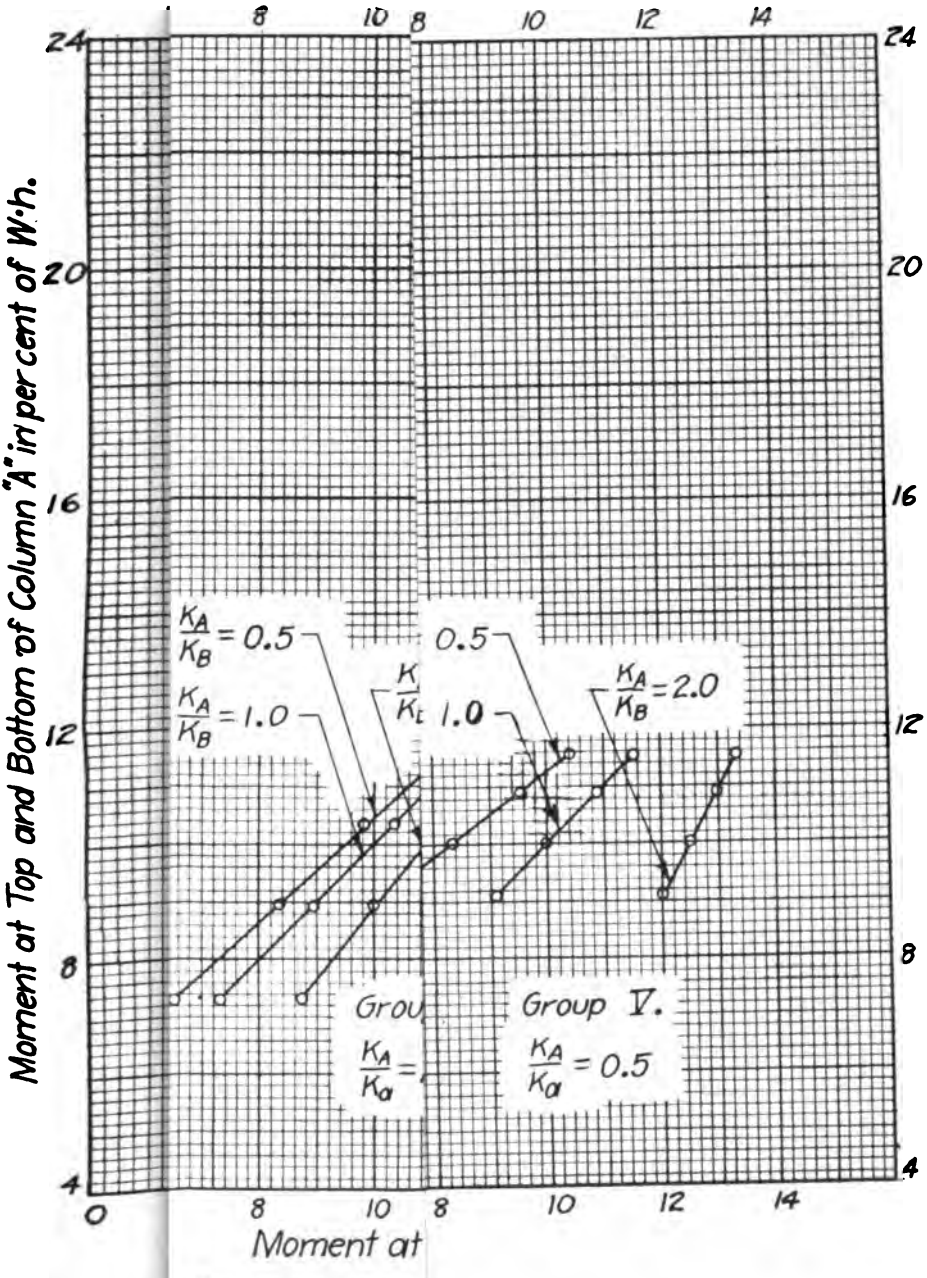

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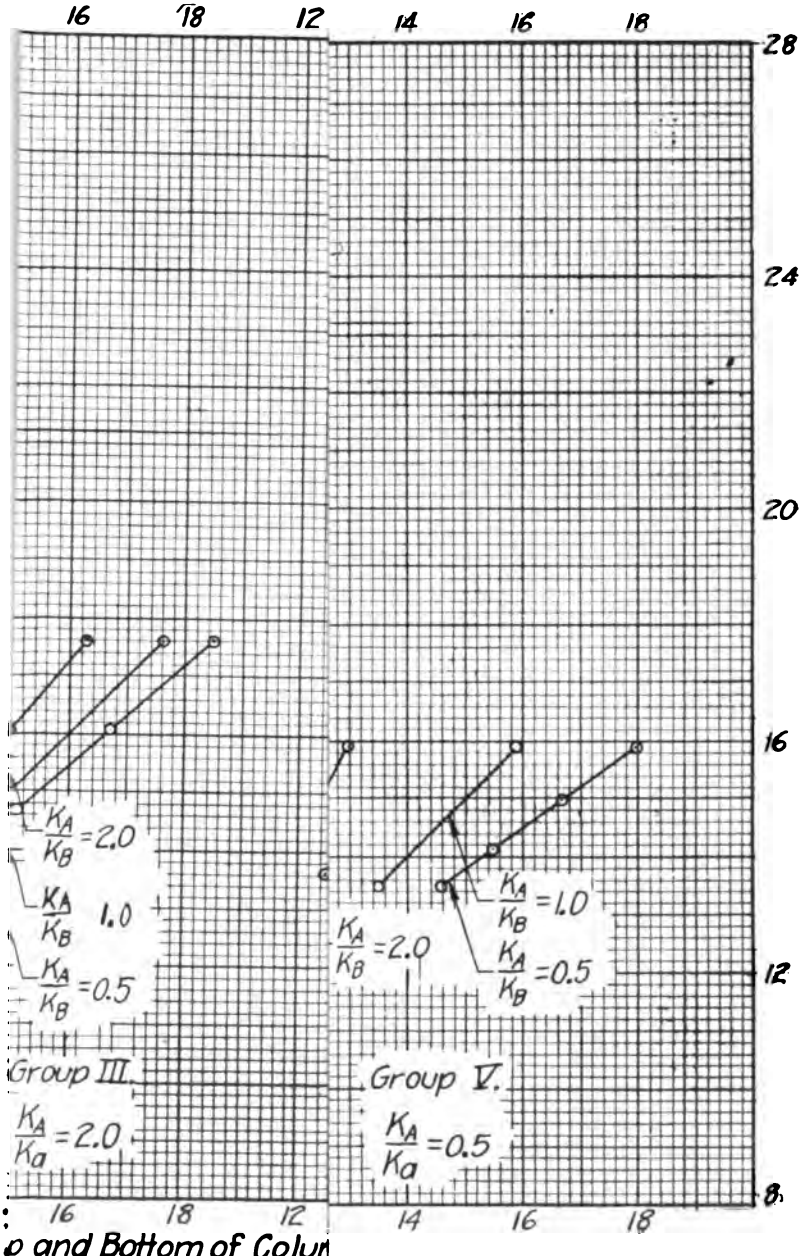

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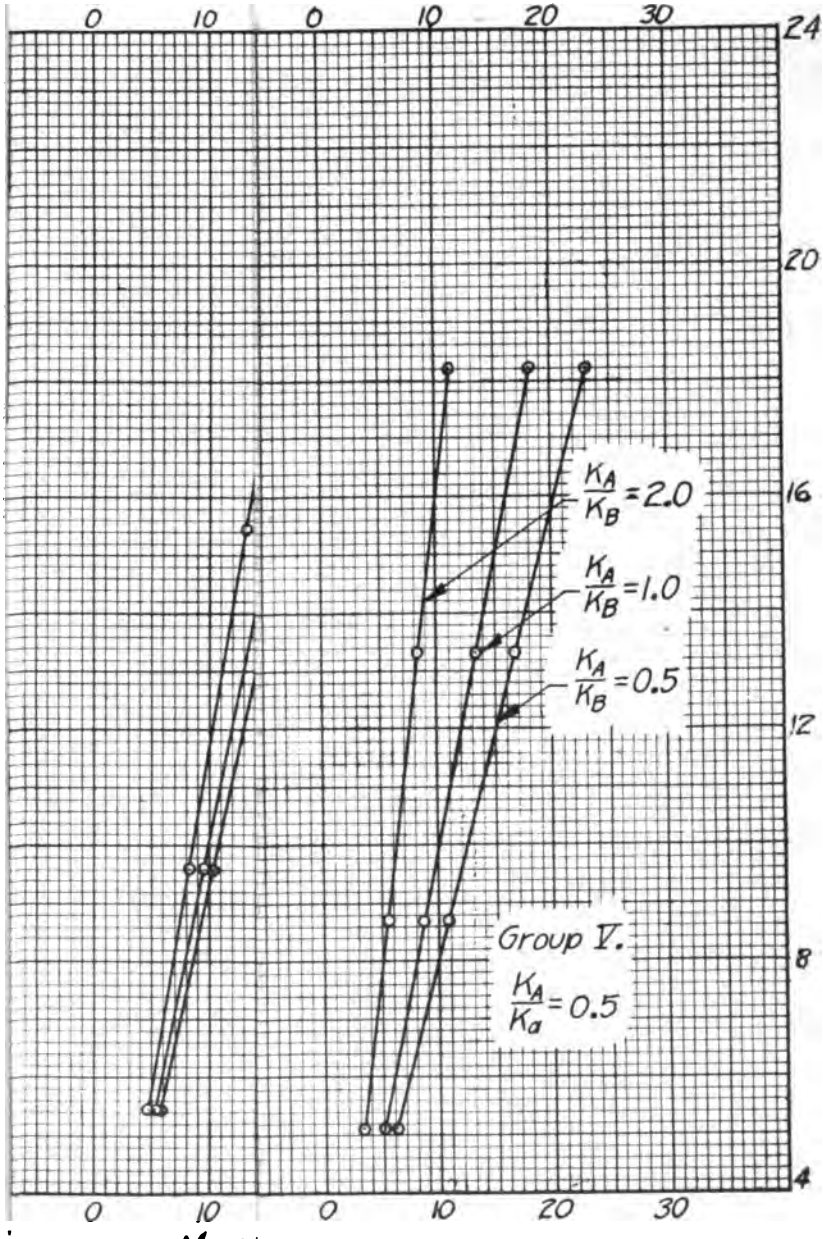

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equal to the K of column B. Determine the value of $\frac{K_a}{K_b}$ and of $\frac{K_A}{K_a}$. Use the curves in group I. Trace a vertical line whose abscissa equals $\frac{K_a}{K_b}$ until it intersects the curve corresponding to the value of $\frac{K_A}{K_a}$. Project the point of intersection horizontally to the left and read the moment in per cent of $W \times h$ from the vertical scale. As W and h are known quantities the moment is determined.

Next consider the moment in a bent in which the K of column A and the K of column B are not equal. Determine the value of $\frac{K_a}{K_b}$, $\frac{K_A}{K_a}$, and $\frac{K_A}{K_B}$. First use the curves of group I. Trace a vertical line whose abscissa equals $\frac{K_a}{K_b}$ until it intersects the curve corresponding to the value of $\frac{K_A}{K_a}$. Project the point of intersection horizontally to the right to the group of curves corresponding to the value of $\frac{K_A}{K_a}$ until it intersects the particular curve of this group corresponding to the value of $\frac{K_A}{K_B}$. Project this intersection point vertically downward and read the moment in per cent of $W \times h$ from the horizontal scale.

Similarly, the moments at the top and the bottom of column B can be obtained from Fig. 7, and the moment at the end of girder b can be obtained from Fig. 8. It should be noted, however, that the moment in girder b depends equally upon the $W \times h$ in the story above and in the story below the girder. This being true, in getting the moment in girder b, the average of the $W \times h$ for the two stories should be used.

The moment at the right end of girder a balances the sum of the moments in column A just above and below the girder and is equal to their algebraic sum. The moment at the left end of girder a balances the sum of the moments at the right end of girder b together with the moments in column B just above and below girder a and is equal to their algebraic sum. It is therefore possible to obtain the moment at the ends of all members in a bent from the curves in Figs. 6, 7, and 8, subject, of course, to the error due to the use of assumptions 1 and 2 of this Section.

The curves in Figs. 6, 7, and 8 show that a large change in the ratio of the K of one member to the K of another member causes a

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relatively small change in the distribution of the moments in the bent.

16. *Numerical Problem.*—To illustrate the use of these curves the solution of a problem is presented.

The seventh story of a symmetrical three-span bent is 20 ft. high, and is subjected to a shear of 3,000 lbs. The eighth story of the same bent is 20 ft. high, and is subjected to a shear of 2,500 lbs. The properties of the members in the seventh and eighth stories are as follows:

$K_A = 30$ in.^{3}; $K_B = 40$ in.^{3}; $K_a = 20$ in.^{3}; and $K_b = 16$ in.^{3}. It is required to find the moments at the ends of all members in the seventh story.

$W \times h = 3{,}000 \times 20 \times 12 = 720{,}000$ in. lb. in the seventh story.

$W \times h = 2{,}500 \times 20 \times 12 = 600{,}000$ in. lb. in the eighth story.

$$\frac{K_A}{K_B} = \frac{30}{40} = .75$$

$$\frac{K_A}{K_a} = \frac{30}{20} = 1.5$$

$$\frac{K_a}{K_b} = \frac{20}{16} = 1.25$$

To get the moment in column A use Fig. 6. At the left of the figure trace the ordinate whose abscissa is 1.25 to a point half way between the two middle curves ($\frac{K_a}{K_A} = 1.5$, which is half way between 1 and 2), and project this point horizontally to a point half way ($\frac{K_A}{K_B} =$ .75, which is half way between .5 and 1.0) between the two left-hand curves of Group II, and also to a point half way between the two left-hand curves of Group III. The abscissa of the former point is 9.35 per cent, and of the latter point 9.15 per cent. As $\frac{K_A}{K_a}$ is equal to 1.5 or the average of 1 and 2, the moment at the top and the bottom of column A in the seventh story, M_{A_7}, is the average of 9.35 per cent and 9.15 per cent or 9.25 per cent of $W \times h$, that is, $M_{A_7} = .0925 \times 720{,}000 = 66{,}700$ in. lb. The moment, M_{A_8}, at the top and the bottom of column A in the eighth story is $.0925 \times 600{,}000$, or $M_{A_8} = 55{,}500$ in. lb. Similarly, the moment at the top and bottom of column B, M_{B_7}, is 15.75 per cent of $W \times h$, that is, $M_{B_7} = .1575 \times 720{,}000 = 113{,}500$ in. lb. in the seventh story; and $M_{B_8} = .1575 \times 600{,}000 = 94{,}400$ in. lb. in the eighth story. The moment M_{b_7}, at the end of girder b at the top of the seventh story is 13.85 per cent of $W \times h$, that is,

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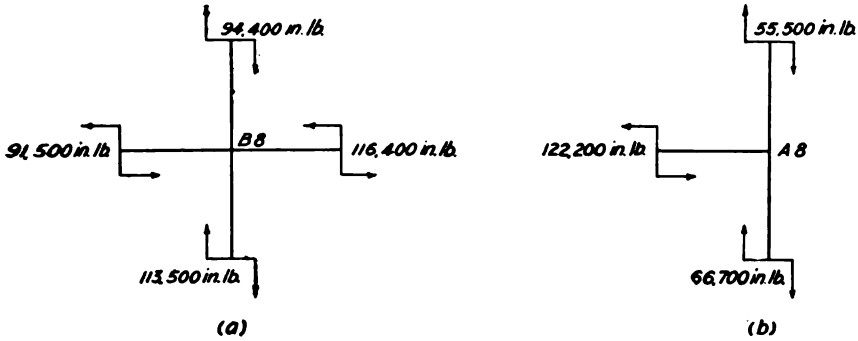

FIG. 9. MOMENTS ACTING AT POINTS *A8* AND *B8* OF A SYMMETRICAL THREE-SPAN BENT.

$$M_{b_7} = .1385 \left[\frac{720{,}000 + 600{,}000}{2}\right] = 91{,}500 \text{ in. lb.}$$

The moment at the right end of girder *a* at the top of the seventh story is equal to the sum of 66,700 in. lb., the moment at the top of column *A* in the seventh story, and 55,500 in. lb., the moment at the bottom of column *A* in the eighth story, or 122,200 in. lb. (see Fig. 9b). The moment at the left end of girder *a* is equal to the sum of 113,500 in. lb., the moment at the top of column *B* in the seventh story, and 94,400 in. lb., the moment at the bottom of column *B* in the eighth story, less 91,500 in. lb., the moment in the end of girder *b;* that is, the moment at the left end of girder *a* is 116,400 in. lb. (see Fig. 9a).

A comparison of the moments in a bent as obtained by the proposed approximate method and by the slope-deflection method is given in Tables 23 to 26 inclusive.

17. *Modifications of the Slope-Deflection Method.*—The writers made a study to determine the effect of a change in the section of one member of a bent upon the moment in the other members. The moments were determined in a number of bents in which $K = 1$ for all members except the one whose effect upon the distribution of the moment was to be studied. The quantity $\frac{Wh}{6E}$ was taken equal to 1 for all stories and for all bents. The K of column B was given successively the values 0.5, 1, 2, and 4; and the corresponding moment at the ends of columns A and B and at the ends of girders a and b was calculated

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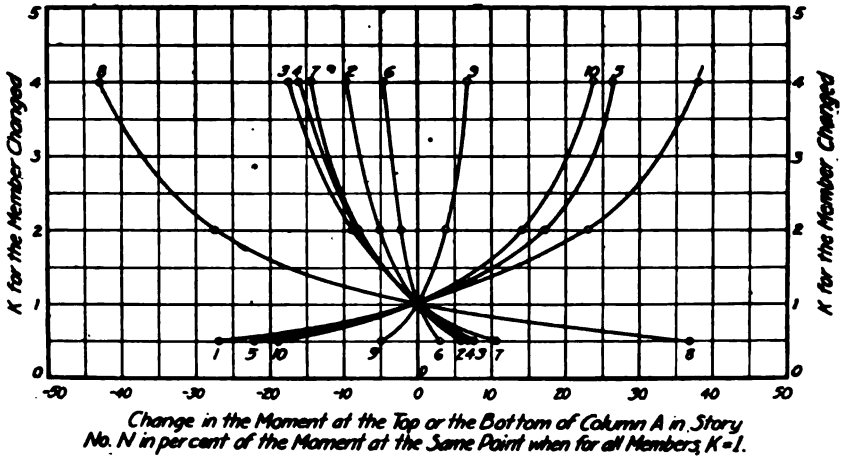

FIG. 10. DIAGRAM SHOWING CHANGE IN THE MOMENT AT THE TOP AND THE BOTTOM OF COLUMN *A* OF A SYMMETRICAL THREE-SPAN BENT DUE TO A CHANGE OF *K* IN THE OTHER MEMBERS.

KEY TO THE DIAGRAM IN FIG. 10.

Member Changed	Designation of the Curve Showing the Change in the Moment at	
	Top of Column *A*	Bottom of Column *A*
Girder *a* in Story No. N	1	2
" " No. (N—1)	2	1
Girder *b* " " No. N	3	4
" " No. (N—1)	4	3
Column *A* " " No. N	5	5
" " No. (N—1)	6	7
" " No. (N+1)	7	6
Column *B* " " No. N	8	8
" " No. (N—1)	9	10
" " No. (N+1)	10	9

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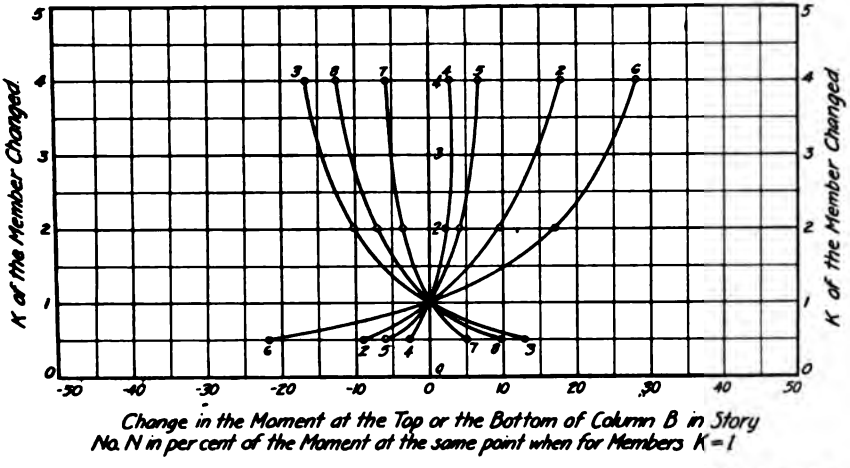

FIG. 11. DIAGRAM SHOWING CHANGE IN THE MOMENT AT THE TOP AND THE BOTTOM OF COLUMN *B* OF A SYMMETRICAL THREE-SPAN BENT DUE TO A CHANGE OF *K* IN THE OTHER MEMBERS.

KEY TO THE DIAGRAM IN FIG. 11.

Member Changed	Designation of the Curve Showing the Change in the Moment at	
	Top of Column *B*	Bottom of Column *B*
Girder *a* in Story No. N	—	1
" " No. (N—1)	1	—
Girder *b* " " No. N	2	—
" " No. (N—1)	—	2
Coulmn *A* " " No. N	3	3
" " No. (N—1)	4	5
" " No. (N+1)	5	4
Column *B* " " No. N	6	6
" " No. (N—1)	7	8
" " No. (N+1)	8	7

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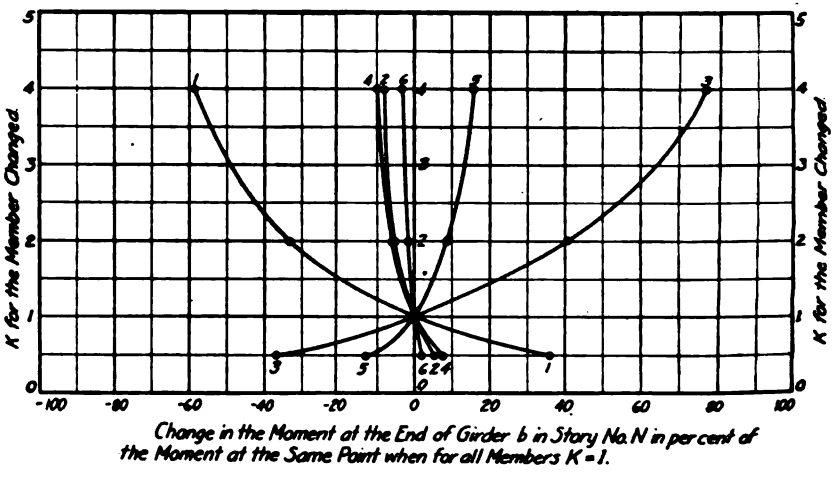

FIG. 12. DIAGRAM SHOWING CHANGE IN THE MOMENT AT THE END OF GIRDER *b* OF A SYMMETRICAL THREE-SPAN BENT DUE TO CHANGE OF *K* IN THE OTHER MEMBERS.

KEY TO THE DIAGRAM IN FIG. 12.

Member Changed	Designation of the Curve Showing the Change in the Moment at the End of Girder *b*
Girder *a* in Story No. N	1
" " No. (N—1)	2
" " No. (N+1)	2
Girder *b* " " No. N	3
Column *A* " " No. N	4
" " No. (N+1)	4
Column *B* " " No. N	
" " No. (N—1)	6
" " No. (N+1)	5

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in the story in which the K of column B was changed, and also in the stories immediately above and below that story. Similarly, the K of column A, the K of girder a and the K of girder b were given successively the values 0.5, 1, 2, and 4, and the corresponding values of the moment at the ends of the girders and the columns were determined in the particular story and also in the stories immediately above and below that story. The effect of the changes in the values of the K of the members upon the moment is given in Figs. 10, 11, and 12.

Fig. 10 shows the changes in the moment at the top and the bottom of column A. The abscissae are changes in the moment expressed in per cent of the moment in a bent for which the K of all members is equal to 1. Increases in the moment are laid off to the right of the origin, and decreases are laid off to the left. The ordinates represent the values of the K in the member changed. The story in which the moment was calculated is designated as story No. N, the story above is designated as story No. $(N+1)$, and the story below No. $(N-1)$. Each curve shows the change in the moment at the top or the bottom of column A due to a change in the value of the K of some member. The number of the curve which shows the change in the moment in column A due to a change in the K of any particular member can be obtained from the key to the diagram in Fig. 10. For example, curve No. 3 shows the change in the moment at the top of column A due to a change in the K of girder b in story No. N. If the K of girder b is made equal to 2, the moment at the top of column A is 8.9 per cent less than when the K of girder b is equal to 1. Curve No. 3 also shows the change in the moment at the bottom of column A due to a change in the K of girder b in story No. $(N-1)$. If the K for girder b in story No. $(N-1)$ is equal to 2, the moment at the bottom of column A is 8.9 per cent less than when the K for girder b is equal to 1.

Similarly, the changes in the moment in column B due to changes in the K of the members of the bent are given in Fig. 11 and the changes in the moment in girder b are given in Fig. 12.

Fig. 10 shows that the moment in column A of any story not only depends upon the value of the K of the members in the same story but depends also upon the value of the K of the members in the adjacent stories. Figs. 11 and 12 show that the same statement is true relative to the moment in column B and girder b. This being true, any approximate method which considers only the members in the story in which the stresses are to be determined can not give accurate results except

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in bents which have no very sudden changes in the sections of the columns and girders.

Where there are sudden changes in the column and girder sections, the proposed approximate method is not accurate. In such cases it is sometimes possible to use a modification of the slope-deflection method and obtain quite accurate results with a comparatively small expenditure of labor. This modification of the slope-deflection method may be made in either of two ways, as follows: (a), the bent may be divided by a horizontal plane between any two adjacent floors, and the lower part be treated independently of the upper part; or (b), the bent may be divided by two horizontal planes any number of stories apart, and the portion between these planes may be treated independently of the other portions of the building. These modifications will be considered in detail.

(a) The wind stresses in the top stories of a bent are comparatively small and their exact determination is not important. An examination of the equations of Table 15 shows that the coefficient of the quantity to be determined is greater than the coefficients of the quantities in which the unknown is expressed. This being true, errors in the slopes and the deflection of one story have but little effect upon the slopes and deflections of the succeeding stories. Further examination of the same table shows that there is a comparatively small difference between the change in the slope at the top of a column in one story and the change in the slope at the top of the same column in the story below. If the two changes in slope are assumed to be equal, the error introduced is small. Bearing these facts in mind, consider again the twenty-story bent of Section VII. If θ_{A12} and θ_{B12} are assumed equal to θ_{A11} and θ_{B11} respectively in equations f, g, and h of Table 13, the first 34 equations will contain 34 unknown quantities which can be determined by the method used in Tables 14 and 15. That is, assuming that the changes in the slopes of the columns in one story are equal to the changes in the slopes of the same columns in the story below, is equivalent to dividing the bent into two parts; and the equations for the lower part can be solved independently of those of the upper part. If the slopes which are assumed equal are not really equal, the calculated slopes and deflections in the top story of the lower part will not be exact, but the results of a number of calculations show that the error in the slopes and the deflection in the next to the top story is so small that it may be neglected. This being true, if, in the actual design of the twenty-story bent of Section VII, it is con-

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sidered unnecessary to calculate the wind stresses in the top ten stories, the bottom eleven stories may be treated as a complete bent and the stresses determined by the method outlined above, thereby decreasing the work. The stresses in the bottom ten stories will be quite exact. The bent could have been divided at any other story without affecting the accuracy of the results.

If there are sudden changes in the members the bent can be divided above the story in which the change occurs, and the stresses in the lower part can be determined by the method outlined above.

Table 23 shows that the largest errors in the proposed approximate method are in the first story. The moments can be determined in the first story by the modification of the slope-deflection method as follows:

Assume that the slopes in the third story are equal to the corresponding slopes in the second story. The first seven equations of Table 13 will then contain only seven unknown quantities. The solution of these equations is given in Tables 21 and 22. The changes in the slopes and the deflection for the first story as given in Table 22 agree very closely with the values for the same quantities given in Table 15, and the moments, being functions of the slopes and deflection, will also agree very closely with the moments given in Table 18.

(b) The moments in any particular story may be determined if there is some story below the one in question in which it can be seen by inspection that the changes in the slopes at the tops and the bottoms of the columns are equal. Suppose the moments are to be determined in the tenth story of the bent shown in Fig. 5, and that it is apparent from inspection that $\theta_{A8} = \theta_{A9}$, and $\theta_{B8} = \theta_{B9}$. Then assume that $\theta_{A12} = \theta_{A11}$, and $\theta_{B12} = \theta_{B11}$. The ten equations, Y to h inclusive of Table 13, will contain ten unknown quantities. If the slopes in the eighth and ninth stories, which have been assumed to be equal, are equal then the slopes and the deflection, and therefore also the moments, in the tenth story will be quite accurate, but if the slopes *below* the story in question which are assumed to be equal, are not really equal, the required moments will not be exact. That is, a difference between the slopes *below* the story in question affects the results, whereas a difference between the slopes *above* the story does not materially affect the results.

18. *Application of the Proposed Approximate Method and Modification of the Slope-Deflection Method.*—To obtain the moment in a bent in which there are sudden changes in the sections, the following com-

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bination of methods can sometimes be used to advantage. Use the proposed approximate method for obtaining the moment in the portion of the bent in which there are no sudden changes in the sections of the members. Use a modification of the slope-deflection method to get the moments in the bottom story and in any intermediate stories in which sudden changes in the members occur. The results thus obtained will be sufficiently accurate to be used in the actual designs of buildings, and the amount of work required will not be excessive.

IX. Comparison of the Approximate Methods With the Slope-Deflection Method.

19. *Symmetrical Three-Span Bent with Short Middle Span.*—For comparison, the moments in the symmetrical three-span twenty-story bent shown in Fig. 5 were calculated by the five following methods:

1. The slope-deflection method.
2. The proposed approximate method.
3. The three methods described by Mr. Fleming and known as methods I, II, and III.

The moments as determined by these methods are given in Tables 23 and 24. For each story the moments in the upper line are in 1,000 in. lb., and those in the lower line are in per cent of the moments as determined by the slope-deflection method. Tables 23 and 24 show in the case of this bent: first, that the moments determined by methods II and III are very seriously in error; second, that the moment determined by method I and by the proposed approximate method agree very closely with the moment determined by the slope-deflection method except at points where there are sudden changes in the members of the bent; third, that the errors in the moment determined by the proposed approximate method are less for the girders than for the columns.

20. *Symmetrical Three-Span Bent with Long Middle Span.*—The distribution of the moment determined by the slope-deflection method is affected by the ratio of the K of girder a to the K of girder b. As this ratio does not affect the distribution of the moment determined by methods I, II, and III, the accuracy of the latter methods will depend upon the relative values of K for the two girders.

In the bent shown in Fig. 5, the K is less for girder a than for girder b, since the girders have substantially the same sections and girder a is longer than girder b. In order to determine the effect of the relative values of the K of girders a and b upon the accuracy of the approximate methods, the moment in the bottom four stories of the

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bent shown in Fig. 13, a bent like the one shown in Fig. 5 except that the long and short spans have been interchanged, was determined by each of the five methods mentioned in article 19. The results are given in Tables 25 and 26. The moments determined by the proposed approximate method are as accurate as those given in Tables 23 and 24; whereas the moments determined by method I are inaccurate, and those determined by methods II and III are very inaccurate.

21. *Effect of the Proportions of a Bent upon the Accuracy of Method I.*—In order to determine more fully the effect of the proportions of a bent upon the accuracy of method I, the moments were determined in a number of bents having different proportions, by the slope-deflection method and by method I. These moments are given in Table 27.

In bents 1 to 9 of this table all girders and columns have the same section. The ratio of the moment of inertia of the columns to the moment of inertia of the girders and also the ratio of the moment of inertia of the girder in bay *a* to the moment of inertia of the girder in bay *b*, affects the moment determined by the slope-deflection method but does not affect the moment as determined by method I. If the sections of the columns and girders are not the same, the difference between the moments determined by the two methods might vary even more than Table 27 indicates. Any errors in the moment due to sudden changes in the sections of the bent are in addition to the errors indicated in Table 27.

The difference between the two methods is due to the fact that in method I the direct unit stress in a column is proportional to the distance of the column from the neutral axis of the bent; whereas in the slope-deflection method the stress in a column depends upon the shears in the girders, and the shears in the girders depend upon the changes in the slopes at the ends of the girders and upon the moment of inertia of the girder sections.

22. *Accuracy of the Approximate Methods when the Moment of Inertia of the Girders is Proportional to the Bending Moment.*—In the comparison of the approximate methods with the slope-deflection method given in Tables 23 to 27, the sections of the girders were chosen without reference to the moment to which they were subjected, except the girders in the bent shown in Fig. 5. The girders in this bent were designed to resist the bending moment determined by method I. Another investigation was made to determine the accuracy of the approxi-

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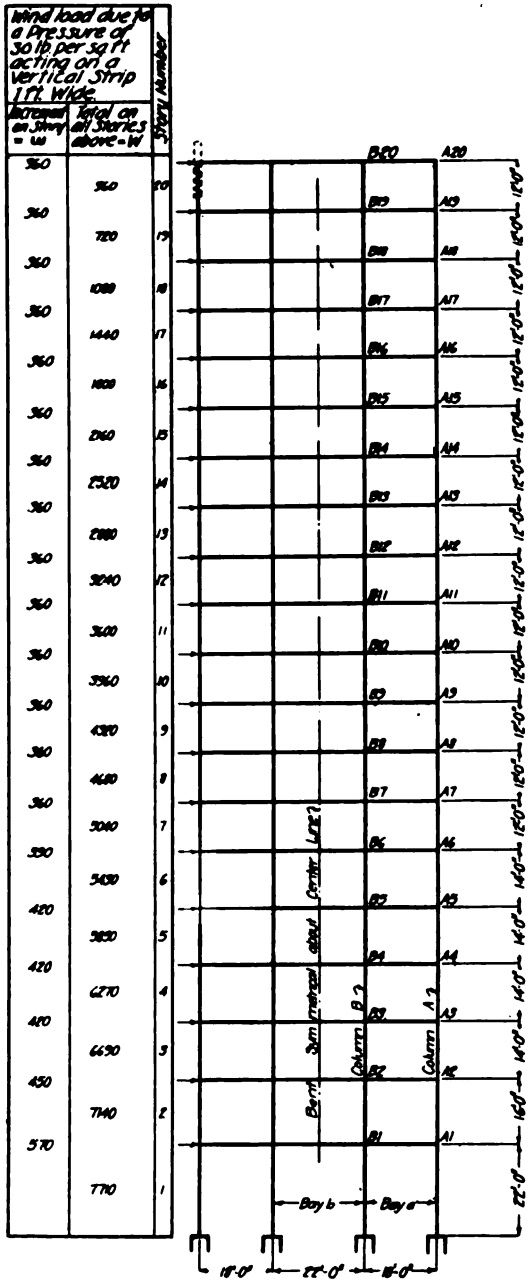

FIG. 13. SYMMETRICAL THREE-SPAN BENT TWENTY STORIES HIGH WITH LONG SPAN AT THE CENTER.

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mate methods when applied to bents in which the girders were designed to resist the bending moment due to the wind load as determined by the method which is to be compared with the slope-deflection method. The results of this investigation are given in Tables 28 to 31. In Table 28 the moments of inertia of the sections of the girders are proportional to the bending moments determined by method I. The sections of the columns are equal, and the moment of inertia of the sections of the columns is equal to the moment of inertia of the section of girder *a*. Table 28 shows that method I gives the moment in the columns and girders quite accurately when the spans are equal and the story height does not exceed the length of the span. This statement is true, in general, only when the two column sections are equal and when the moments of inertia of the sections of column *A* and girder *a* are equal.

In considering the merits of the approximate methods, it should be noted that the moment which should be determined with the greatest accuracy is the moment to which the joint that connects the girder to the column is subjected. This moment is the moment at the ends of the girders. If the moments of inertia of the sections are not made proportional to the bending moment determined by method I, the method will not be as accurate as Table 28 indicates.

The moments of inertia of the girder sections of Table 29 are proportional to the bending moments in the respective girders as determined by method II. The moments of inertia of the girder sections of Table 30 are proportional to the bending moments in the respective girders determined by method III. It is apparent from Tables 29-30 and Tables 23-26 that methods II and III are so inaccurate that they should never be used.

The moments of inertia of the girder sections of Table 31 are proportional to the bending moments in the respective girders as determined by method IV. For bents having equal spans and equal column sections this method gives quite accurate results.

While methods I and IV may be quite accurate in some cases, their accuracy depends upon the proportions of the bent. For example, by comparing Tables 27 and 28 it is apparent that if the moments of inertia of the girder sections are not proportional to the bending moments determined by method I, that method will not be as accurate as when the girder sections are so proportioned. Again, in Tables 28 and 31 the sections of the columns are equal and the moments of inertia of the sections of column *A* and girder *a* are equal; but if these relations

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do not exist, the methods will not in general be as accurate as Tables 28 and 31 seem to indicate.

Any inaccuracy in methods I and IV due to sudden changes in the members of the bent, are in addition to the inaccuracies shown in Tables 28 and 31.

For the bents shown in Tables 28 to 31, the moments determined by the proposed approximate method are exactly the same as the moments determined by the slope-deflection method.

X. Test of a Celluloid Model of a Bent.

23. *Description of Tests.*—In order to check the deflections and the changes in the slopes calculated by the slope-deflection method, a celluloid model of a bent was subjected to known shears, and the resulting deflections and changes in slopes were measured and compared with the calculated values. The model was made of celluloid 1/8 inch thick, and had the general dimensions shown in Fig. 14. A cord passing over a pulley and attached to a weight at one end and to the top of the model at the other, produced a uniform shear in all stories. Paper arms were fastened to the model at points where the members intersect. The movement of these arms indicated the changes in the slopes. One of these arms, A_2D, is shown in Fig. 14. The external force caused the point A_2 to move in approximately a horizontal line and at the same time to turn through a small angle, θ. The paper arm A_2D has the same motion as the point A_2; thus the vertical displacement of D measures the angle θ. The horizontal deflection of the model was obtained by measuring the displacement of a point at the middle of a girder. Paper arms were attached to all four columns at the tops of all stories simultaneously, and the changes in the slopes at all intersections were measured for each application of the load. Similarly, the horizontal deflection was measured at the middle of each of the three girders at the top of each story.

In the original model, known as model No. 1, members No. 1, 2, 3, and 4 were 1/2 inch wide; all other members were 1/4 inch wide. After this model had been tested, member No. 1 was reduced to 1/4 inch; and then the resulting model, now known as No. 2, was tested. Members No. 2, 3, and 4 were successively reduced to 1/4 inch; and the resulting models, now known as models No. 3, 4, and 5, were also tested.

In testing model No. 1, observations were made when loads of 2 1/2 lb., 5 lb., 7 1/2 lb., and 10 lb. were applied. The readings for the two sides of the model agreed very closely in the bottom three stories;

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but in the fourth story they did not agree. Thinking that this discrepancy might be due to excessive stresses, when testing model No. 2, a maximum load of only 7½ lb. was used; but there was still the same discrepancy in the readings. Thinking that the apparatus might be out of order, the loads were removed and then applied a second time but the second readings agreed very closely with the first. In testing models No. 3, 4, and 5, loads of 1½ lb., 3 lb., and 4½ lb. were applied. The loads were then removed and applied in the opposite direction. The readings for the two sides of the model continued to agree very closely in the bottom three stories; and in the fourth story they continued to

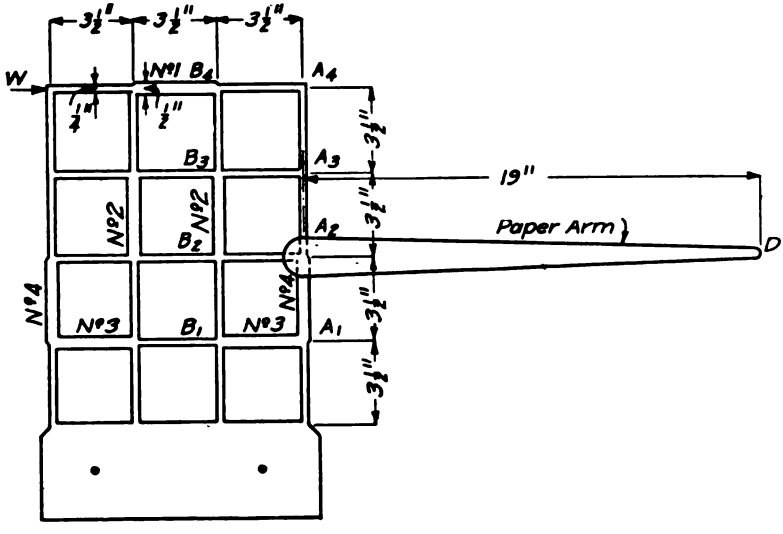

FIG. 14. CELLULOID MODEL.

disagree. It is therefore probable that either the material was not homogeneous or that there were internal stresses or local weaknesses in the upper part of the model.

The observations made in the test of model No. 4 are given in Table 32.

24. *Results of Tests.*—The results of the tests of the models are given in Fig. 15. The models are shown by the sketches at the left of the figure. In these sketches members represented by heavy lines are ½ inch wide, and all other members are ¼ inch wide. The upper diagram of Fig. 15 shows the changes in the slopes and the lower diagram shows the deflections.

In the upper diagram the first group of lines at the left represents

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the changes in the slope at the point A_1, the second group represents the changes at A_2, and similarly the third and fourth groups represent the changes at A_3 and A_4. The fifth group of lines from the left represents the changes in the slope at the point B_1, the sixth group at B_2, and similarly for the other groups. In the left-hand group of lines the change in the slope at the point A_1 of model No. 1 is laid off from the origin on a horizontal line opposite the sketch of model No. 1; the change in the slope at A_1 of model No. 2 is laid off from the origin on a horizontal line opposite model No. 2; the change in the slope at A_1 of model No. 3 is laid off from the origin on the horizontal line opposite model No. 3; and the change in the slope at A_1 of model No. 4 is laid off from the origin on the horizontal line opposite model No. 4. The full lines connect points which represent the quantities as measured; and the dotted lines connect points which represent the same quantities as computed by the slope-deflection method. The changes in the slopes at A_2, A_3, A_4, B_1, B_2, B_3, and B_4 are shown in a similar manner.

The lower diagram shows the ratios of the deflection to the story height in the different stories of the models. The method of representing the ratios of deflection to story height is similar to the method used to represent the changes in the slopes in the upper diagram.

It will be noticed that the calculated quantities are in general greater than the observed quantities. The reason for this is: In the computations, the length of a member was taken equal to the distance between center lines, whereas the length that is actually free to bend is the distance from outside to outside of the members. This accounts for the difference between the observed and the computed values.

Fig. 15 shows that the changes in the slopes and the ratios of the deflection to the story height as observed and as computed agree closely. In other words, the tests support the theory upon which the slope-deflection method is based.

XI. Discussion of the Assumptions.

25. *Preliminary.*—In making the analysis of the stresses the writers made certain assumptions in regard to the action of the frame when stressed. The effect of inaccuracies in these assumptions will now be considered.

If all of the columns of a story are taken together as a free body, the algebraic sum of the moments at the tops and bottoms of all the columns is equal to the shear on the story multiplied by the story height. As the moments in the columns are balanced by the moments in the

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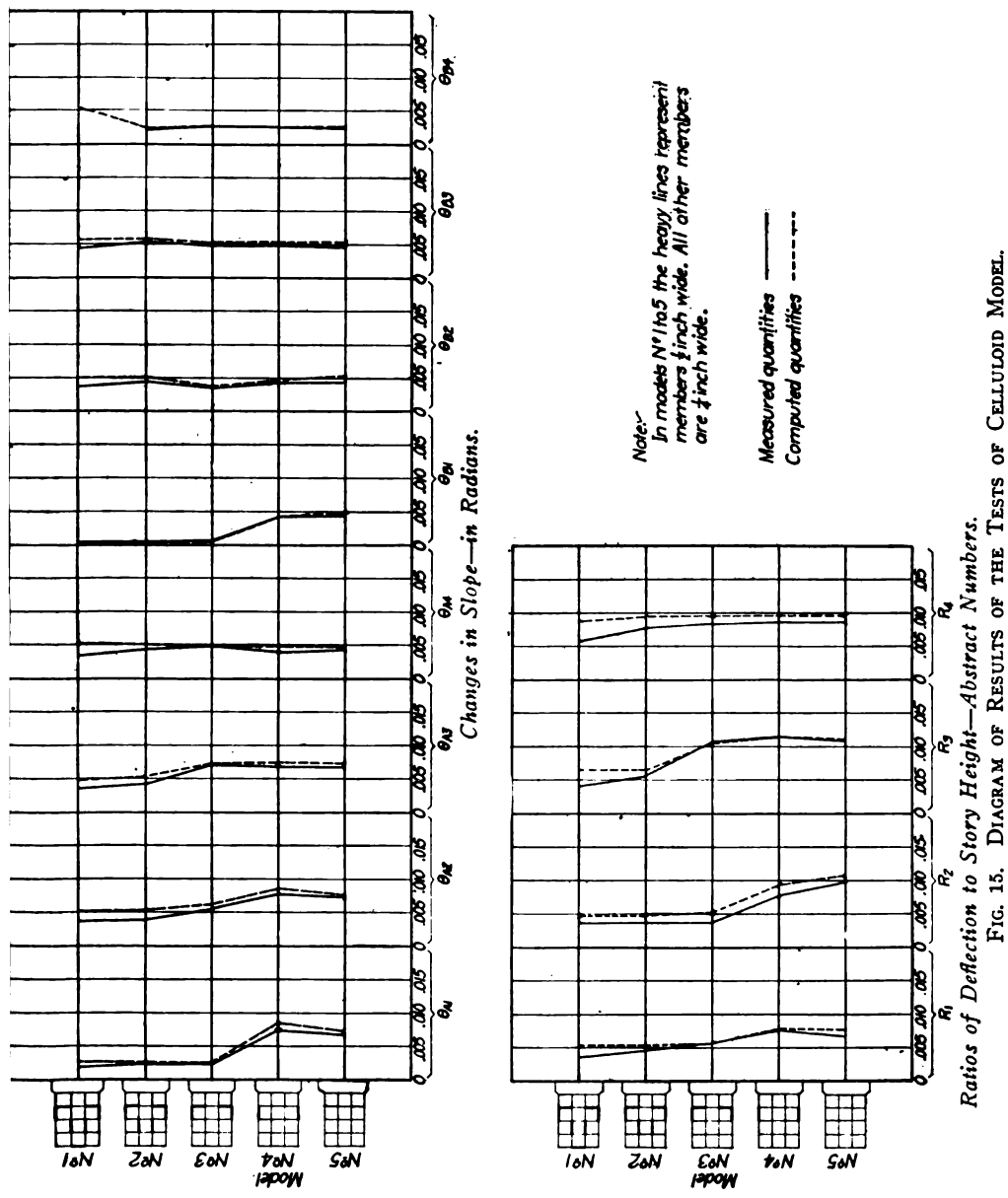

FIG. 15. DIAGRAM OF RESULTS OF THE TESTS OF CELLULOID MODEL.

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girders, the algebraic sum of the moments at the ends of the girders is dependent upon the product of the shear in the story and the story height. These facts should be borne in mind in discussing the effect of inaccuracies in the assumptions upon the moments in a bent.

26. *Assumption of Perfect Rigidity.*—According to assumption No. 1, the connections between the columns and girders are perfectly rigid. The truth of this assumption can be determined only by tests. As far as the writers are aware no such tests have been made. While it is undesirable to make a mathematical analysis which is based upon an unverified assumption, some assumption relative to the rigidity of the connections must be made before the stresses in a frame can be determined. The distribution of the stresses depends more upon the relative stiffness than upon the actual stiffness of the connections. In view of these facts, it seems that the assumption of perfect rigidity of the joints is the most logical one that can be made.

27. *Assumption of the Unchanged Length.*—According to assumption No. 2, the change in the length of a member due to the direct stress is equal to zero.

If columns A and B change in length, the moment at the right-hand end of girder a will be given by the equation

$$M_{AB} = 2EK_a\,(2\theta_A + \theta_B - 3R_a),$$

in which R_a is equal to the difference between the changes in the lengths of columns A and B divided by the length of girder a. In the derivation of the general equations in Section VI, R_a was assumed to be equal to zero. If the changes in the lengths of the columns do not alter the values of θ_A and θ_B, the change in the moment ΔM_{AB}, at the right-hand end of girder a due to the changes in the lengths of the columns is:

$$\Delta M_{AB} = 2EK_a\,(-3R_a).$$

The change in the moment at the left-hand end of girder a is equal to the change at the right-hand end. The difference between the changes in length of columns A and B, or the deflection of one end of girder a relative to the other end, is given by the equation:

$d = \dfrac{Pl}{AE}$, in which

d = the deflection,

P = the difference between the stress in column A and the stress in column B,

l = the length of the columns,

A = the area of the column section,

E = the modulus of elasticity of steel.

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From Table 19, page 75, the compressive stresses in the first story of columns A and B are 14,464 lb. and 4,587 lb. respectively.

Therefore $P = 14{,}464 - 4{,}587 = 9{,}877$ lb. From Table 11, page 48, $l = 264$ in., $K_a = 30.5$ in.^{3}, and $A = 112.8$ sq. in. $E = 29{,}000{,}000$ lb. per sq. in. Substituting these values in the above equation for d, gives:

$$d = .000798 \text{ in.}$$

By definition, $R_a = \frac{d}{l}$, therefore

$$R_a = \frac{.000798}{264} = .00000303, \text{ and}$$

$$\Delta M_{AB} = 2 \times 29{,}000{,}000 \times 30.5\ (-3 \times .00000303) = -16{,}050 \text{ in. lb.}$$

This change in the moment is 5.6 per cent of the moment at the right-hand end of girder a as given in Table 18, page 74. Likewise the moment at the left-hand end of girder a is decreased by 16,050 in. lb., which is 6.6 per cent of the moment as given in Table 18.

The moment at the end of girder b is affected by the changes in the lengths of two columns, B, one on each side of the center line of the bent. One column is subjected to a tensile stress of 4,587 lb., and the other to a compressive stress of 4,587 lb. The deflection of girder b is given by the equation

$$d = \frac{2 \times 4{,}587 \times 264}{112.8 \times 29{,}000{,}000} = .00074 \text{ inches.}$$

$$R_b = \frac{.00074}{216} = .00000343.$$

$$\Delta M_{BB} = 2 \times 29{,}000{,}000 \times 37.3 \times 3 \times .00000343 = 22{,}300 \text{ in. lb.,}$$

which is 9.0 per cent of the moment at the end of girder b as given in Table 18.

The change in the length of a column is a function of the unit direct stress and also a function of the story height. The first story of the building considered is much higher (22 ft.) than the other stories, and the unit direct stress in the columns in the first story is greater than in the stories above; hence the change in the length of the columns is very much less in the other stories than in the first story. Therefore the changes in the moments for the other stories will be less than those in the first story computed above.

The changes in the moments in the columns have been deter-

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mined and it remains to consider the effect of the direct stresses in the girders. The direct stress in a girder is very small in comparison with the direct stress in a column and therefore may be neglected.

If the changes in the slopes were not affected by the changes in the lengths of the columns, the moments at the ends of girders *a* and *b* would be decreased. As the sum of these moments is determined by the product of the shear in the story and the story height, they can not all be decreased simultaneously. Therefore all of the changes in the slopes must be increased until the sum of the moments at the ends of the girders will balance the moments in the columns. This will make the moments at the ends of the girders practically the same as they would have been if the columns had not changed in length. If the moments at the ends of the girders are not materially affected, the moments at the ends of the columns will not be materially affected. Therefore although the direct stresses in the columns do change the lengths of the columns, they do not affect the stresses in the frame to any great extent.

28. *Assumption as to Length of Members.*—In accordance with assumption No. 3, the length of a member was taken as the distance between the center lines of the members which it intersects. This makes the changes in the slopes and deflections, as calculated, greater than the actual values. The effect of the inaccuracy of this assumption upon the distribution of the moments is, however, not so apparent.

The curves in Fig. 6, 7, and 8 show that the distribution of the moment in a story depends upon the relative values of K of the members, but that it takes a comparatively large change in the K of a member to appreciably affect the distribution of the moments. The fact that the length of a member has been taken equal to the distance between center lines, has but little effect upon the relative values of K for the members; and therefore does not materially affect the distribution of the moments.

29. *Assumption as to Deflection Due to Shear.*—According to assumption No. 4, the internal shearing stresses in a member produce no deflection. The distribution of the stresses is dependent, not upon the actual deflection of the columns due to shear, but upon the differences between the deflections in the different columns. The shears on the columns are small and the differences between the shears are still smaller; and therefore the assumption that the deflection due to shear is equal to zero will not cause any appreciable error.

30. *Assumption as to Load.*—According to assumption No. 5, the

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entire wind load is resisted by the steel frame. The walls of a building no doubt help to resist the wind loads, but the resistance which they exert is uncertain. If a portion of the load is considered as being resisted by the walls the stresses in the steel frame are correspondingly reduced; but the method of determining the stresses is not affected.

XII. Conclusions.

As a result of the investigation described in this bulletin the following conclusions can be made relative to the methods used to determine the wind stresses in the steel frames of office buildings.

a. Methods II and III of Section II are so inaccurate that they should never be used; I and IV are quite accurate in some cases, but they may give results which are seriously in error.

b. The method presented in Sections VI and VII, and designated as the slope-deflection method, contains no approximations except those in the assumptions. It has been shown that the inaccuracies in the assumptions do not materially affect the results. Therefore the method is very accurate.

c. While the slope-deflection method is long, it could be used in the actual design of a building; but it has its greatest value as a standard by means of which the accuracy of the approximate methods may be determined.

d. The proposed approximate method is short; and, except at points where there are very large changes in the size of the members, gives results which are accurate enough to be used in the actual design of a building.

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Table 11.

Properties of the Columns and Girders in the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5.

	Story Number	Section of Member: Web Plate	Section of Member: 4 Angles	Section of Member: Cover Plate	Area of Section Sq. In.	Moment of Inertia—(Inches)⁴: Web	Moment of Inertia: 4 Angles: Primary	Moment of Inertia: 4 Angles: Secondary	Moment of Inertia: Cover Plate	Moment of Inertia: Total	Length of Columns=h of Girders=l Inches	K for Columns $=\frac{I}{h}$ for Girders $=\frac{I}{l}$ (Inches)³
Column A	1	17x$\frac{1}{4}$	8x8x$\frac{1}{4}$	2-18x1$\frac{1}{4}$	112.80	358	318	2190	3950	6816	264	25.8
	2	17x$\frac{1}{4}$	8x8x$\frac{1}{4}$	2-18x1$\frac{1}{4}$	112.80	358	318	2190	3950	6816	192	35.6
	3 and 4	17x$\frac{1}{4}$	8x8x$\frac{1}{4}$	2-18x1	103.80	358	318	2190	3080	5946	168	35.4
	5 and 6	17x$\frac{1}{4}$	8x8x$\frac{1}{4}$	2-18x$\frac{11}{16}$	92.55	358	318	2190	2060	4926	168	29.4
	7 and 8	17x$\frac{1}{4}$	8x8x$\frac{1}{4}$	2-18x$\frac{7}{16}$	83.55	358	318	2190	1266	4132	144	28.7
	9 and 10	17x$\frac{15}{16}$	8x8x$\frac{15}{16}$		72.42	384	337	2315		3036	144	21.1
	11 and 12	17x$\frac{1}{4}$	8x8x$\frac{7}{16}$		64.24	358	299	2050		2707	144	18.8
	13 and 14	17x$\frac{3}{8}$	8x6x$\frac{13}{16}$		53.79	308	131	2195		2634	144	18.3
	15 and 16	17x$\frac{1}{4}$	8x6x$\frac{5}{8}$		42.26	205	105	1745		2055	144	14.3
	17 and above	17x$\frac{1}{4}$	8x6x$\frac{9}{16}$		38.86	205	96	1590		1891	144	13.1
Column B	1	17x$\frac{1}{4}$	8x8x$\frac{1}{4}$	2-18x1$\frac{1}{4}$	112.80	358	318	2190	3950	6816	264	25.8
	2	17x$\frac{1}{4}$	8x8x$\frac{1}{4}$	2-18x1$\frac{1}{4}$	112.80	358	318	2190	3950	6816	192	35.6
	3 and 4	17x$\frac{1}{4}$	8x8x$\frac{1}{4}$	2-18x1	103.80	358	318	2190	3080	5946	168	35.5
	5 and 6	17x$\frac{1}{4}$	8x8x$\frac{1}{4}$	2-18x$\frac{9}{16}$	94.80	358	318	2190	2240	5106	168	30.4
	7 and 8	17x$\frac{1}{4}$	8x8x$\frac{1}{4}$	2-18x$\frac{1}{2}$	85.80	358	318	2190	1459	4325	144	30.0
	9 and 10	17x$\frac{1}{4}$	8x8x$\frac{1}{4}$	2-18x$\frac{5}{16}$	79.05	358	318	2190	892	3758	144	26.1
	11 and 12	17x$\frac{1}{4}$	8x8x$\frac{1}{4}$		67.80	358	318	2190		2886	144	19.9
	13 and 14	17x$\frac{1}{4}$	8x6x$\frac{13}{16}$		57.92	358	131	2195		2684	144	18.6
	15 and 16	17x$\frac{1}{4}$	8x6x$\frac{5}{8}$		44.39	256	105	1745		2106	144	14.6
	17 and above	17x$\frac{1}{4}$	8x6x$\frac{9}{16}$		41.00	256	96	1544		1896	144	13.2
Girders in Bay a	1	42x$\frac{3}{8}$	6x3$\frac{1}{2}$x$\frac{3}{8}$		29.43	2315	13	5730		8058	264	30.5
	2 and 3	36x$\frac{3}{8}$	6x3$\frac{1}{2}$x$\frac{3}{8}$		27.18	1458	13	4170		5641	264	21.4
	4 and 5	36x$\frac{3}{8}$	5x3$\frac{1}{2}$x$\frac{3}{8}$		25.70	1458	13	3690		5161	264	19.5
	6	30x$\frac{3}{8}$	6x3$\frac{1}{2}$x$\frac{3}{8}$		24.93	844	13	2860		3717	264	14.1
	7	30x$\frac{3}{8}$	5x3$\frac{1}{2}$x$\frac{3}{8}$		23.45	844	13	2530		3387	264	12.8
	8 and above	24x$\frac{3}{8}$	5x3$\frac{1}{2}$x$\frac{3}{8}$		21.20	432	13	1580		2025	264	7.7
Girders in Bay b	1	42x$\frac{3}{8}$	6x3$\frac{1}{2}$x$\frac{3}{8}$		29.43	2315	13	5730		8058	216	37.3
	2	36x$\frac{3}{8}$	6x3$\frac{1}{2}$x$\frac{7}{16}$		29.38	1458	15	4830		6303	216	29.2
	3 and 4	36x$\frac{3}{8}$	6x3$\frac{1}{2}$x$\frac{3}{8}$		27.18	1458	13	4170		5641	216	26.2
	5	36x$\frac{3}{8}$	5x3$\frac{1}{2}$x$\frac{3}{8}$		25.70	1458	13	3690		5161	216	23.8
	6	30x$\frac{3}{8}$	6x3$\frac{1}{2}$x$\frac{3}{8}$		24.93	844	13	2860		3717	216	17.2
	7	30x$\frac{3}{8}$	5x3$\frac{1}{2}$x$\frac{3}{8}$		23.45	844	13	2530		3387	216	15.7
	8 and above	24x$\frac{3}{8}$	5x3$\frac{1}{2}$x$\frac{3}{8}$		21.20	432	13	1580		2025	216	9.4

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TABLE 12.

NUMERICAL VALUES OF THE CONSTANTS IN THE EQUATIONS OF TABLE 3 FOR THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

All quantities are expressed in inches³

Story No.	Values of $K\left(=\frac{I}{h}\right)$ for		Values of $K\left(=\frac{I}{l}\right)$ for		Values of $J\left[=2\Sigma\left(\frac{I}{h}+\frac{I}{l}\right)\right]$ for all members intersecting at the		Values of $N\ (=2\Sigma K)$ for all Columns in a Story
	Column A	Column B	Girders in Bay a at Top of Story	Girders in Bay b at Top of Story	Top of Column A	Top of Column B	
1	25.8	25.8	30.5	37.3	183.8	258.4	206.8
2	35.6	35.6	21.4	29.2	184.8	243.4	284.8
3	35.4	35.5	21.4	26.2	184.4	237.2	283.6
4	35.4	35.5	19.5	26.2	168.6	223.2	283.6
5	29.4	30.4	19.5	23.8	156.6	208.2	239.2
6	29.4	30.4	14.1	17.2	144.4	183.4	239.2
7	28.7	30.0	12.8	15.7	140.4	177.0	234.8
8	28.7	30.0	7.7	9.4	115.0	146.4	234.8
9	21.1	26.1	7.7	9.4	99.8	138.6	188.8
10	21.1	26.1	7.7	9.4	95.2	126.2	188.8
11	18.8	19.9	7.7	9.4	90.6	113.8	154.8
12	18.8	19.9	7.7	9.4	89.6	111.2	154.8
13	18.3	18.6	7.7	9.4	88.6	108.6	147.6
14	18.3	18.6	7.7	9.4	80.6	100.6	147.6
15	14.3	14.6	7.7	9.4	72.6	92.6	115.6
16	14.3	14.6	7.7	9.4	70.2	89.8	115.6
17	13.1	13.2	7.7	9.4	67.8	87.0	105.2
18	13.1	13.2	7.7	9.4	67.8	87.0	105.2
19	13.1	13.2	7.7	9.4	67.8	87.0	105.2
20	13.1	13.2	7.7	9.4	41.6	60.6	105.2

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Table 13.

General Equations for the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5.

No. of Equation	Left-Hand Member of Equation															Right-Hand Member of Equation
	Story No. 1			Story No. 2			Story No. 3			Story No. 4			Story No. 5			
	R_1	θ_{A1}	θ_{B1}	R_2	θ_{A2}	θ_{B2}	R_3	θ_{A3}	θ_{B3}	R_4	θ_{A4}	θ_{B4}	R_5	θ_{A5}	θ_{B5}	
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height															Coefficient of .0001
A	—206.8	51.6	51.6													—117.2
B	— 77.4	183.8	30.5	—106.8	35.6											0.0
C	— 77.4	30.5	295.7	—106.8		35.6										0.0
D		71.2	71.2	—284.8	71.2	71.2										—78.9
E		35.6		—106.8	184.8	21.4	—106.2	35.4								0.0
F			35.6	—106.8	21.4	272.6	—106.5		35.5							0.0
G					70.8	71.0	—283.6	70.8	71.0							—64.5
H					35.4		—106.2	184.4	21.4	—106.2	35.4					0.0
I						35.5	—106.5	21.4	263.4	—106.5		35.5				0.0
J								70.8	71.0	—283.6	70.8	71.0				—60.5
K								35.4		—106.2	168.6	19.5	—88.2	29.4		0.0
L									35.5	—106.5	19.5	249.4	—91.2		30.4	0.0

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TABLE 13.—(Continued).

GENERAL EQUATIONS FOR THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

No. of Equation	Left-Hand Member of Equation															Right-Hand Member of Equation
	Story No. 4			Story No. 5			Story No. 6			Story No. 7			Story No. 8			
	R_4	θ_{A4}	θ_{B4}	R_5	θ_{A5}	θ_{B5}	R_6	θ_{A6}	θ_{B6}	R_7	θ_{A7}	θ_{B7}	R_8	θ_{A8}	θ_{B8}	Co-efficient of .0001
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height															
M		58.8	60.8	—239.2	58.8	60.8										—56.5
N		29.4		— 88.2	156.6	19.5	— 88.2	29.4								0.0
O			30.4	— 91.2	19.5	232.0	— 91.2		—30.4							0.0
P					58.8	60.8	—239.2	58.8	60.8							—52.5
Q					29.4		— 88.2	144.4	14.1	— 86.1	28.7					0.0
R						30.4	— 91.2	14.1	200.6	— 90.0		30.0				0.0
S								57.4	60.0	—234.8	57.4	60.0				—41.8
T								28.7		— 86.1	140.4	12.8	—86.1	28.7		0.0
U									30.0	— 90.0	12.8	192.7	—90.0		30.0	0.0

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TABLE 13.—(Continued).

GENERAL EQUATIONS FOR THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

No. of Equation	Left-Hand Member of Equation															Right-Hand Member of Equation
	Story No. 7			Story No. 8			Story No. 9			Story No. 10			Story No. 11			
	R_7	θ_{A7}	θ_{B7}	R_8	θ_{A8}	θ_{B8}	R_9	θ_{A9}	θ_{B9}	R_{10}	θ_{A10}	θ_{B10}	R_{11}	θ_{A11}	θ_{B11}	Co-efficient of .0001
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height															
V		57.4	60.0	—234.8	57.4	60.0										—38.7
W		28.7		— 86.1	115.0	7.7	— 63.3	21.1								0.0
X			30.0	— 90.0	7.7	155.8	— 78.3		26.1							0.0
Y					42.2	52.2	—188.8	42.2	52.2							—35.8
Z					21.1		— 63.3	99.8	7.7	— 63.3	21.1					0.0
a						26.1	— 78.3	7.7	148.0	— 78.3		26.1				0.0
b								42.2	52.2	—188.8	42.2	52.2				—32.8
c								21.1		— 63.3	95.2	7.7	—56.4	18.8		0.0
d									26.1	— 78.3	7.7	135.6	—59.7		19.9	0.0

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TABLE 13.—(Continued).

GENERAL EQUATIONS FOR THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

No. of Equation	Left-Hand Member of Equation															Right-Hand Member of Equation
	Story No. 10			Story No. 11			Story No. 12			Story No. 13			Story No. 14			
	R_{10}	θ_{A10}	θ_{B10}	R_{11}	θ_{A11}	θ_{B11}	R_{12}	θ_{A12}	θ_{B12}	R_{13}	θ_{A13}	θ_{B13}	R_{14}	θ_{A14}	θ_{B14}	
	Coefficient of Unknown Slopes and Ratios of Deflection to Story Height															Coefficient of .0001
e		37.6	39.8	—154.8	37.6	39.8										—29.8
f		18.8		— 56.4	90.6	7.7	— 56.4	18.8								0.0
g			19.9	— 59.7	7.7	123.2	— 59.7		19.9							0.0
h					37.6	39.8	—154.8	37.6	39.8							—26.8
i					18.8		— 56.4	89.6	7.7	— 54.9	18.3					0.0
j						19.9	— 59.7	7.7	120.6	— 55.8		18.6				0.0
k										—147.6	36.6	37.2				—23.8
l										— 54.9	88.6	7.7	—54.9	18.3		0.0
m										— 55.8	7.7	118.0	—55.8		18.6	0.0

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TABLE 13.—(Continued).

GENERAL EQUATIONS FOR THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

No. of Equation	Left-Hand Member of Equation															Right-Hand Member of Equation
	Story No. 13			Story No. 14			Story No. 15			Story No. 16			Story No. 17			
	R_{13}	θ_{A13}	θ_{B13}	R_{14}	θ_{A14}	θ_{B14}	R_{15}	θ_{A15}	θ_{B15}	R_{16}	θ_{A16}	θ_{B16}	R_{17}	θ_{A17}	θ_{B17}	Co-efficient of .0001
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height															
n		36.6	37.2	—147.6	36.6	37.2										—20.9
o		18.3		— 54.9	80.6	7.7	— 42.9	14.3								0.0
p			18.6	— 55.8	7.7	110.0	— 43.8		14.6							0.0
q					28.6	29.2	—115.6	28.6	29.2							—17.9
r					14.3		— 42.9	72.6	7.7	— 42.9	14.3					0.0
s						14.6	— 43.8	7.7	102.0	— 43.8		14.6				0.0
t								28.6	29.2	—115.6	28.6	29.2				—14.9
u								14.3		— 42.9	70.2	7.7	—39.3	13.1		0.0
v									14.6	— 43.8	7.7	99.2	—39.6		13.2	0.0

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TABLE 13.—(Continued).

GENERAL EQUATIONS FOR THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

No. of Equation	Left-Hand Member of Equation															Right-Hand Member of Equation
	Story No. 16			Story No. 17			Story No. 18			Story No. 19			Story No. 20			
	R_{16}	θ_{A16}	θ_{B16}	R_{17}	θ_{A17}	θ_{B17}	R_{18}	θ_{A18}	θ_{B18}	R_{19}	θ_{A19}	θ_{B19}	R_{20}	θ_{A20}	θ_{B20}	
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height															Co-efficient of .0001
w		26.2	26.4	—105.2	26.2	26.4										—11.9
x		13.1		— 39.3	67.8	7.7	— 39.3	13.1								0.0
y			13.2	— 39.6	7.7	96.4	— 39.6		13.2							0.0
z					26.2	26.4	—105.2	26.2	26.4							—8.94
A′					13.1		— 39.3	67.8	7.7	— 39.3	13.1					0.0
B′						13.2	— 39.6	7.7	96.4	— 39.6		13.2				0.0
C′								26.2	26.4	—105.2	26.2	26.4				—5.96
D′								13.1		— 39.3	67.8	7.7	— 39.3	13.1		0.0
E′									13.2	— 39.6	7.7	96.4	— 39.6		13.2	0.0
F′											26.2	26.4	—105.2	26.2	26.4	—2.98
G′											13.1		— 39.3	41.6	7.7	0.0
H′												13.2	— 39.6	7.7	70.0	0.0

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Table 14.

Elimination of the Unknown Quantities in the Equations for the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5.

No. of Equation	Left-Hand Member of Equation												Right-Hand Member of Equation
	Story No. 1			Story No. 2			Story No. 3			Story No. 4			
	R_1	θ_{A1}	θ_{B1}	R_2	θ_{A2}	θ_{B2}	R_3	θ_{A3}	θ_{B3}	R_4	θ_{A4}	θ_{B4}	
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height												Coefficient of .0001
A	1.0000	−0.2495	−0.2495										0.5667
B	1.0000	−2.3747	−0.3940	1.3798	−0.4599								0.0000
C	1.0000	−0.3940	−3.8204	1.3798		−0.4599							0.0000
A–B	0	2.1252	0.1445	−1.3798	0.4599								0.5667
B–C	0	−1.9806	3.4263	0	−0.4599	0.4599							0 0000
1		1 0000	0.0680	−0.6494	0.2164								0.2667
2		1.0000	−1.7300		0.2322	−0.2322							0.0000
D		1.0000	1.0000	−4.0000	1.0000	1.0000							−1.1062
E		1.0000		−3.0000	5.1912	0.6012	−2.9834	0.9944					0.0000
1–2		0	1.7980	−0.6494	−0.0158	0.2322							0 2667
2–D		0	−2.7300	4.0000	0.7678	−1.2322							1.1062
D–E		0	1.0000	−1.0000	−4.1912	0.3988	2.9834	−0.9944					−1.1062
3			1	−0.3611	−0.0068	0.1292							0.1483
4			1	−1.4652	0.2812	0.4513							−0.4059
5			1	−1.0000	−4.1912	0.3988	2.9834	−0.9941					−1.1062
F			1	−3.0000	0.6012	7.6575	−2.9915		0.9972				0.0000
3–4			0	1.1040	−0.2900	−0.3222							0.5543
4–5			0	−0.4652	4.4724	0.0525	−2.9834	0.9944					0.7023
5–F			0	2.0000	−4.7924	−7.2587	5.9749	−0.9944	−0.9972				−1.1062
6				1	−0.2627	−0.2918							0.5021
7				1	−9.6140	−0.1128	6.4127	−2.1378					−1.5095
8				1	−2.3962	−3.6293	2.9874	−0.4972	−0.4986				−0.5541
6–7				0	9.3513	−0.1790	−6.4127	2.1378					2.0116
7–8				0	−7.2178	3.5165	3.4252	−1.6406	0.4986				−0.9554
9					1	−0.0191	−0.6857	0.2286					0.2151
10					1	−0.4872	−0.4746	0.2273	−0.0691				0.1324
G					1	1.0028	−4.0055	1.0000	1.0028				−0.9110
H					1		−3.0000	5.2091	0.6045	−3.0000	1.0000		0.0000
9–10					0	0.4681	−0.2111	0.0013	0.0691				0.0827
10–G					0	−1.4900	3.5309	−0.7727	−1.0719				1.0434
G–H					0	1.0028	−1.0055	−4.2091	0.3983	3.0000	−1.0000		−0.9110
11						1	−0.4511	0.0027	0.1476				0.1768
12						1	−2.3700	0.5186	0.7194				−0.6990
13						1	−1.0028	−4.1971	0.3971	2.9915	−0.9972		−0.9084
I						1	−3.0000	0.6028	7.4200	−3.0000		1.0000	0.0000
11–12						0	1.0189	−0.5159	−0.5718				0 8758
12–13						0	−1.3672	4.7157	0.3223	−2.9915	0.9972		0.2094
13–I						0	1.9972	−4.7999	−7.0230	5.9915	−0.9972	−1.0000	−0.9084

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Table 14.—(Continued).

Elimination of the Unknown Quantities in the Equations for the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5.

No. of Equation	Left-Hand Member of Equation												Right-Hand Member of Equation
	Story No. 3			Story No. 4			Story No. 5			Story No. 6			
	R_3	θ_{A3}	θ_{B3}	R_4	θ_{A4}	θ_{B4}	R_5	θ_{A5}	θ_{B5}	R_6	θ_{A6}	θ_{B6}	Co-efficient of .0001
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height												
14	1	—0.2689	—0.2960										0.4564
15	1	—3.4490	—0.2357	2.1881	—0.7294								—0.1532
16	1	—2.4033	—3.5179	2.9999	—0.4993	—0.5007							—0.4548
14-15	0	3.1802	—0.0623	—2.1881	0.7294								0.6096
14-16	0	2.1344	3.2199	—2.9999	0.4993	0.5007							0.9112
17		1	—0.0196	—0.6881	0.2294								0.1917
18		1	1.5086	—1.4057	0.2339	0.2346							0.4270
J		1	1.0028	—4.0057	1.0000	1.0028							—0.8546
K		1		—3.0000	4.7628	0.5508	—2.4914	0.8302					0 0000
17-18		0	—1.5282	0.7176	—0.0046	—0.2346							—0.2353
18-K		0	1.5086	1.5943	—4.5290	—0.3162	2.4914	—0.8302					0.4270
J-K		0	1.0028	—1.0057	—3.7628	0.4520	2.4914	—0.8302					—0.8546
19			1	—0.4696	0.0030	0.1535							0.1540
20			1	1.0568	—3.0019	—0.2096	1.6514	—0.5503					0 2830
21			1	—1.0028	—3.7520	0.4507	2.4842	—0.8279					—0.8521
L			1	—3.0000	0.5496	7.0255	—2.5691		0.8564				0.0000
19-20			0	—1.5264	3.0049	0.3631	—1.6514	0.5503					—0.1290
20-21			0	2.0596	0.7501	—0.6603	—0.8328	0.2776					1.1352
20-L			0	4.0568	—3.5515	—7.2351	4.2205	—0.5503	—0.8564				0.2830
22				1	—1.9687	—0.2379	1.0819	—0.3605					0.0846
23				1	0.3642	—0.3206	—0.4044	0.1348					0.5512
24				1	—0.8754	—1.7836	1.0404	—0.1357	—0.2111				0.0697
22-23				0	—2.3329	0.0827	1.4863	—0.4953					—0.4666
23-24				0	1.2397	1.4630	—1.4448	0.2705	0.2111				0.4815
25					1	—0.0355	—0.6371	0.2123					0.2000
26					1	1.1803	—1.1657	0.2182	0.1703				0.3884
M					1	1.0340	—4.0684	1.0000	1.0340				—0.9609
N					1		—3 0000	5.3260	0.6632	—3.0000	1.0000		0.0000
25-26					0	—1.2158	0.5286	—0.0059	—0.1703				—0.1884
26-M					0	—1.0695	3.4313	—0.7877	—1.0340				1.1609
M-N					0	1.0340	—1.0684	—4.3260	0.3708	3.0000	—1.0000		—0.9609
27						1	—0.4348	0.0048	0.1401				0.1550
28						1	—3.2083	0.7365	0.9668				—1.0855
29						1	—1.0333	—4.1842	0.3586	2.9016	—0.9672		—0.9294
O						1	—3.0000	—0.6415	7.6321	—3.0000		1.0000	0.0000
27-28						0	2.7735	—0.7316	—0.8267				1.2405
28-29						0	—2.1750	4.9207	0.6082	—2.9016	0.9672		—0.1561
29-O						0	1.9667	—4.8257	—7.2735	5.9016	—0.9672	—1.0000	—0.9294
30							1	—0.2638	—0.2981				0.44
31							1	—2.2623	—0.2796	1.3341	—0.4447		0.0[illegible]
32							1	—2.4540	—3.6986	3.0004	—0.4918	—0.5085	—0.4[illegible]
30-31							0	1.9985	—0.0185	—1.3341	0.4447		0.3[illegible]
30-32							0	2.1902	3.4005	—3.0004	0.4918	0.5085	0.8[illegible]

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Table 14.—(Continued).

Elimination of the Unknown Quantities in the Equations for the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5.

No. of Equation	Left-Hand Member of Equation.												Right-Hand Member of Equation
	Story No. 5			Story No. 6			Story No. 7			Story No. 8			
	R_5	θ_{A5}	θ_{B5}	R_6	θ_{A6}	θ_{B6}	R_7	θ_{A7}	θ_{B7}	R_8	θ_{A8}	θ_{B8}	Co-efficient of .0001
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height.												
33		1	—0.0092	—0.6676	0.2225								0.1879
34		1	1.5526	—1.3698	0.2246	0.2322							0.4200
P		1	1.0341	—4.0682	1.0000	1.0341							—0.8929
Q		1		—3.0000	4.9120	0.4796	—2.9285	0.9762					0.0000
33-34		0	—1.5618	0.7022	—0.0020	—0.2322							—0.2321
33-P		0	—1.0433	3.4006	—0.7775	—1.0341							1.0608
P-Q		0	1.0341	—1.0682	—3.9120	0.5545	2.9285	—0.9762					—0.8929
35			1	—0.4497	0.0013	0.1487							0.1486
36			1	—3.2593	0.7452	0.9911							—1.0359
37			1	—1.0330	—3.7818	0.5362	2.8320	—0.9440					—0.8634
R			1	—3.0000	0.4638	6.5993	—2.9606		0.9870				0.0000
35-36			0	2.8097	—0.7439	—0.8425							1.1845
36-37			0	—2.2263	4.5270	0.4549	—2.8320	0.9440					—0.1724
37-R			0	1.9670	—4.2456	—6.0631	5.7926	—0.9440	—0.9870				—0.8634
38				1	—0.2648	—0.2998							0.4216
39				1	—2.0335	—0.2044	1.2721	—0.4240					0.0775
40				1	—2.1586	—3.0827	2.9451	—0.4799	—0.5018				—0.4290
38-39				0	1.7687	—0.0955	—1.2721	0.4240					0.3441
38-40				0	1.8938	2.7828	—2.9451	0.4799	0.5018				0.8606
41					1	—0.0540	—0.7192	0.2397					0.1946
42					1	1.4695	—1.5553	0.2534	0.2650				0.4544
S					1	1.0453	—4.0909	1.0000	1.0453				—0.7281
T					1		—3.0000	4.8920	0.4460	—3.0000	1.0000		0.0000
41-42					0	—1.5235	0.8361	—0.0137	—0.2650				—0.2599
41-S					0	—1.0993	3.3717	—0.7603	—1.0453				0.9227
S-T					0	1.0453	—1.0909	—3.8920	0.5993	3.0000	—1.0000		—0.7281
43						1	—0.5489	0.0090	0.1739				0.1706
44						1	—3.0672	0.6916	0.9510				—0.8394
45						1	—1.0436	—3.7234	0.5733	2.8700	—0.9567		0.6964
U						1	—3.0000	0.4267	6.4233	—3.0000		1.0000	0.0000
43-44						0	2.5183	—0.6826	—0.7781				1.0100
44-45						0	—2.0236	4.4150	0.3777	—2.8700	0.9567		—0.1430
45-U						0	1.9564	—4.1501	—5.8500	5.8700	—0.9567	—1.0000	—0.6964
46							1	—0.2711	—0.3090				0.4011
47							1	—2.1818	—0.1867	1.4184	—0.4728		0.0706
48							1	—2.1213	—2.9903	3.0006	—0.4890	—0.5111	—0.3560
46-47							0	1.9108	—0.1223	—1.4184	0.4728		0.3304
46-48							0	1.8502	2.6813	—3.0006	0.4890	0.5111	0.7570

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TABLE 14.—(Continued).

ELIMINATION OF THE UNKNOWN QUANTITIES IN THE EQUATIONS FOR THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

No. of Equation	Left-Hand Member of Equation												Right-Hand Member of Equation
	Story No. 7			Story No. 8			Story No. 9			Story No. 10			
	R_7	θ_{A7}	θ_{B7}	R_8	θ_{A8}	θ_{B8}	R_9	θ_{A9}	θ_{B9}	R_{10}	θ_{A10}	θ_{B10}	
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height												Coefficient of .0001
49		1	—0.0640	—0.7422	0.2474								0.1729
50		1	1.4491	—1.6219	0.2643	0.2763							0.4093
V		1	1.0453	—4.0909	1.0000	1.0453							—0.6741
W		1		—3.0000	4.0073	0.2683	—2.2056	0.7352					0.0000
49–50		0	—1.5131	0.8797	—0.0169	—0.2763							—0.2363
49–V		0	—1.1093	3.3487	—0.7526	—1.0453							0.8470
V–W		0	1.0453	—1.0909	—3.0073	—0.7770	2.2056	—0.7352					—0.6741
51			1	—0.5814	0.0112	0.1826							0.1562
52			1	—3.0186	0.6784	0.9423							—0.7636
53			1	—1.0436	—2.8769	0.7433	2.1100	—0.7036					—0.6449
X			1	—3.0000	0.2567	5.1933	—2.6100		0.8700				0.0000
51–52			0	2.4372	—0.6672	—0.7598							0.9198
52–53			0	—1.9750	3.5553	0.1990	—2.1100	0.7036					—0.1187
53–X			0	1.9564	—3.1336	—4.4500	4.7200	—0.7036	—0.8700				—0.6449
54				1	—0.2738	—0.3117							0.3774
55				1	—1.8002	—0.1008	1.0685	—0.3563					0.0601
56				1	—1 6019	—2.2747	2.4127	—0.3597	—0.4447				—0.3296
54–55				0	1.5265	—0.2110	—1.0685	0.3563					0.3173
54–56				0	1.3281	1.9630	—2.4127	0.3597	0.4447				0.7070
57					1	—0.1382	—0.7000	0.2334					0.2079
58					1	1.4779	—1.8165	0.2708	0.3348				0.5323
Y					1	1.2371	—4.4743	1.0000	1.2371				—0.8484
Z					1		—3.0000	4.7299	0.3649	—3.0000	1.0000		0.0000
57–58					0	—1.6161	1.1165	—0.0374	—0.3348				—0.3245
57–Y					0	—1.3753	3.7743	—0.7666	—1.2371				1.0562
Y–Z					0	1.2371	—1.4743	—3.7299	0.8722	3.0000	—1.0000		—0.8484
59						1	—0.6908	0.0231	0.2072				0.2008
60						1	—2.7443	0.5574	0.8995				—0.7681
61						1	—1.1918	—3.0151	0.7051	2.4253	—0.8084		—0.6859
a						1	—3.0000	0.2950	5.6705	—3.0000		1.0000	0.0000
59–60						0	2.0535	—0.5342	—0.6924				0.9689
60–61						0	—1.5525	3.5725	0.1945	—2.4253	0.8084		—0.0822
61–a						0	1.8082	—3.3102	—4.9654	5.4253	—0.8084	—1.0000	—0.6859
62							1	—0.2602	—0.3372				0.4719
63							1	—2.3013	—0.1253	1.5624	—0.5207		0.0529
64							1	—1.8308	—2.7462	3.0000	—0.4471	—0.5531	—0.3793
62–63							0	2.0411	—0.2119	—1.5624	0.5207		0.4189
62–64							0	1.5706	2.4090	—3.0000	0.4471	0.5531	0.8512

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Table 14.—(Continued).

Elimination of the Unknown Quantities in the Equations for the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5.

No. of Equation	Left-Hand Member of Equation												Right-Hand Member of Equation
	Story No. 9			Story No. 10			Story No. 11			Story No. 12			
	R_9	θ_{A9}	θ_{B9}	R_{10}	θ_{A10}	θ_{B10}	R_{11}	θ_{A11}	θ_{B11}	R_{12}	θ_{A12}	θ_{B12}	Coefficient of .0001
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height												
65		1	−0.1093	−0.7655	0.2551								0.2863
66		1	1.5340	−1.9103	0.2947	0.3522							0.5430
b		1	1.2371	−4.4743	1.0000	1.2371							−0.7771
c		1		−3.0000	4.5120	0.3649	−2.6720	0.8911					0.0000
65 66		0	−1.6375	1.1446	−0.0298	−0.3522							−0.3367
65 b		0	−1.3409	3.7098	−0.7449	−1.2371							0.9824
b c		0	1.2371	−1.4743	−3.5120	0.8722	2.6720	−0.8911					−0.7771
67			1	−0.6991	0.0180	0.2151							0.2056
68			1	−2.7660	0.5555	0.9226							−0.7337
69			1	−1.1918	−2.8392	0.7050	2.1610	−0.7204					−0.6283
d			1	−3.0000	0.2950	5.1954	−2.2873		0.7624				0.0000
67 68			0	2.0669	−0.5375	−0.7075							0.9383
68 69			0	−1.5742	3.3947	0.2176	−2.1610	0.7204					−0.1044
69 d			0	1.8092	−3.1342	−4.4904	4.4483	−0.7204	−0.7624				−0.6353
70				1	−0.2801	−0.3423							0.4540
71				1	−2.1566	−0.1382	1.3728	−0.4576					0.0863
72				1	−1.7333	−2.4833	2.4602	−0.3984	−0.4216				−0.3474
70 71				0	1.8565	−0.2041	−1.3728	0.4576					0.3877
70 72				0	1.4732	2.1310	−2.4602	0.3984	0.4216				0.8014
73					1	−0.1082	−0.7277	0.2426					0.2055
74					1	1.4464	−1.6700	0.2705	0.2862				0.5441
e					1	1.0585	−4.1170	1.0000	1.0585				−0.7926
f					1		−3.0000	4.8195	0.4096	−3.0000	1.0000		0.0000
73 74					0	−1.5547	0.9422	−0.0278	−0.2862				−0.3385
73 e					0	−1.1668	3.3993	−0.7574	−1.0585				0.9861
e f					0	1.0585	−1.1170	−3.8195	0.6499	3.0000	−1.0000		−0.7926
75						1	−0.6061	0.0179	0.1841				0.2178
76						1	−2.9051	0.6492	0.9076				−0.8555
77						1	−1.0553	−3.6083	0.6140	2.8343	−0.9449		−0.7485
g						1	−3.0000	0.3869	6.1913	−3.0000		1.0000	0.0000
75 76						0	2.2990	−0.6313	−0.7235				1.0733
76 77						0	−1.8498	4.2575	0.2936	−2.8343	0.9449		−0.1067
77 g						0	1.9447	−3.9952	−5.5773	5.8343	−0.9449	−1.0000	−0.7488
78							1	−0.2746	−0.3147				0.4669
79							1	−2.3018	−0.1587	1.5323	−0.5109		0.0577
80							1	−2.0547	−2.8682	3.0003	−0.4859	−0.5143	−0.3851
78 79							0	2.0272	−0.1560	−1.5323	0.5109		0.4092
78 80							0	1.7801	2.5535	−3.0000	0.4859	0.5143	0.8519

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TABLE 14.—(Continued).

ELIMINATION OF THE UNKNOWN QUANTITIES IN THE EQUATIONS FOR THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

No. of Equation	Left-Hand Member of Equation												Right-Hand Member of Equation
	Story No. 11			Story No. 12			Story No. 13			Story No. 14			
	R_{11}	θ_{A11}	θ_{B11}	R_{12}	θ_{A12}	θ_{B12}	R_{13}	θ_{A13}	θ_{B13}	R_{14}	θ_{A14}	θ_{B14}	Co-efficient of .0001
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height												
81		1	−0.0769	−0.7559	0.2520								0.2018
82		1	1.4344	−1.6854	0.2730	0.2889							0.4786
h		1	1.0585	−4.1170	1.0000	1.0585							−0.7128
i		1		−3.0000	4.7660	0.4096	−2.9205	0.9734					0.0000
81–82		0	−1.5113	0.9294	−0.0210	−0.2889							−0.2768
81–h		0	−1.1355	3.3611	−0.7480	−1.0585							0.9147
h–i		0	1.0585	−1.1170	−3.7660	0.6489	2.9205	−0.9734					−0.7128
83			1	−0.6150	0.0139	0.1911							0.1831
84			1	−2.9602	0.6587	0.9323							−0.8055
85			1	−1.0553	−3.5583	0.6131	2.7592	−0.9197					−0.6734
j			1	−3.0000	0.3869	6.0609	−2.8040		0.9347				0.0000
83–84			0	2.3452	−0.6449	−0.7412							0.9886
84–85			0	−1.9049	4.2170	0.3192	−2.7592	0.9197					−0.1321
85–j			0	1.9447	−3.9452	−5.4478	5.5632	−0.9197	−0.9347				−0.6734
86				1	−0.2750	−0.3160							0.4216
87				1	−2.2137	−0.1676	1.4484	−0.4828					0.0693
88				1	−2.0288	−2.8015	2.8609	−0.4730	−0.4807				−0.3463
86–87				0	1.9387	−0.1485	−1.4484	0.4828					0.3522
86–88				0	1.7539	2.4855	−2.8609	0.4730	0.4807				0.7679
89					1	−0.0766	−0.7471	0.2490					0.1816
90					1	1.4173	−1.6314	0.2697	0.2741				0.4378
k					1	1.0164	−4.0329	1.0000	1.0164				−0.6503
l					1		−3.0000	4.8417	0.4208	−3.0000	1.0000		0.0000
89–90					0	−1.4939	0.8842	−0.0207	−0.2741				−0.2562
89–k					0	−1.0930	3.2857	−0.7510	−1.0164				0.8320
k–l					0	1.0164	−1.0329	−3.8417	0.5956	3.0000	−1.0000		−0.6503
91						1	−0.5920	0.0138	0.1835				0.1715
92						1	−3.0063	0.6871	0.9299				−0.7612
93						1	−1.0163	−3.7802	0.5861	2.9520	−0.9840		−0.6399
m						1	−3.0000	0.4140	6.3443	−3.0000		1.0000	0.0000
91–92						0	2.4143	−0.6733	−0.7464				0.9327
92–93						0	−1.9900	4.4673	0.3438	−2.9520	0.9840		−0.1213
93–m						0	1.9837	−4.1942	−5.7582	5.9520	−0.9840	−1.0000	−0.6399
94							1	−0.2789	−0.3092				0.3863
95							1	−2.2413	−0.1728	1.4834	−0.4945		0.0610
96							1	−2.1143	−2.9030	3.0000	−0.4961	−0.5042	−0.3226
94–95							0	1.9624	−0.1364	−1.4834	0.4945		0.3254
94–96							0	1.8354	2.5938	−3.0000	0.4961	0.5042	0.7089

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Table 14.—(Continued).

Elimination of the Unknown Quantities in the Equations for the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5.

No. of Equation	Left-Hand Member of Equation												Right-Hand Member of Equation
	Story No. 13			Story No. 14			Story No. 15			Story No. 16			
	R_{13}	θ_{A13}	θ_{B13}	R_{14}	θ_{A14}	θ_{B14}	R_{15}	θ_{A15}	θ_{B15}	R_{16}	θ_{A16}	θ_{B16}	Co-efficient of .0001
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height												
97		1	−0.0695	−0.7559	0.2520								0.1658
98		1	1.4135	−1.6349	0.2704	0.2747							0.3963
n		1	1.0164	−4.0329	1.0000	1.0164							−0.5711
o		1		−3.0000	4.4044	0.4208	−2.3443	0.7814					0.0000
97–98		0	−1.4830	0.8790	−0.0184	−0.2747							−0.2205
97–n		0	−1.0859	3.2769	−0.7480	−1.0164							0.7369
n–o		0	1.0164	−1.0329	−3.4044	0.5956	2.3443	−0.7814					−0.5711
99			1	−0.5928	0.0124	0.1853							0.1487
100			1	−3.0177	0.6888	0.9360							−0.6786
101			1	−1.0163	−3.3497	0.5860	2.3067	−0.7689					−0.5619
p			1	−3.0000	0.4140	5.9148	−2.3551		0.7850				0.0000
99–100			0	2.4249	−0.6765	−0.7507							0.8274
100–101			0	−2.0014	4.0385	0.3499	−2.3067	0.7689					−0.1167
101–p			0	1.9837	−3.7637	−5.3287	4.6618	−0.7689	−0.7850				−0.5619
102				1	−0.2790	−0.3096							0.3412
103				1	−2.0179	−0.1748	1.1525	−0.3842					0.0553
104				1	−1.8974	−2.6867	2.3505	−0.3877	−0.3953				−0.2833
102–103				0	1.7389	−0.1348	−1.1525	0.3842					0.2829
102–104				0	1.6184	2.3771	−2.3505	0.3877	0.3958				0.6246
105					1	−0.0775	−0.6624	0.2209					0.1627
106					1	1.4690	−1.4525	0.2396	0.2446				0.3859
q					1	1.0210	−4.0419	1.0000	1.0210				−0.6258
r					1		−3.0000	5.0771	0.5385	−3.0000	1.0000		0.0000
105–106					0	−1.5465	0.7901	−0.0186	−0.2446				−0.2232
105–q					0	−1.0985	3.3795	−0.7790	−1.0210				0.7886
q–r					0	1.0210	−1.0419	−4.0771	0.4825	3.0000	−1.0000		−0.6258
107						1	−0.5109	0.0120	0.1581				0.1444
108						1	−3.0762	0.7092	0.9294				−0.7178
109						1	−1.0206	−3.9933	0.4726	2.9383	−0.9795		−0.6130
s						1	−3.0000	0.5274	6.9832	−3.0000		1.0000	0.0000
107–108						0	2.5653	−0.6972	−0.7713				0.8622
108–109						0	−2.0556	4.7025	0.4568	−2.9383	0.9795		−0.1048
109–s						0	1.9794	−4.5207	−6.5106	5.9383	−0.9795	−1.0000	−0.6130
110							1	−0.2718	−0.3006				0.3361
111							1	−2.2875	−0.2222	1.4294	−0.4765		0.0510
112							1	−2.2836	−3.2888	3.0000	−0.4946	−0.5052	−0.3097
110–111							0	2.0157	−0.0784	−1.4294	0.4765		0.2851
111–112							0	2.0118	2.9882	−3.0000	0.4946	5052	0.6457

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Table 14.—(Continued).

Elimination of the Unknown Quantities in the Equations for the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5.

No. of Equation	Left-Hand Member of Equation												Right-Hand Member of Equation
	Story No. 15			Story No. 16			Story No. 17			Story No. 18			
	R_{15}	θ_{A15}	θ_{B15}	R_{16}	θ_{A16}	θ_{B16}	R_{17}	θ_{A17}	θ_{B17}	R_{18}	θ_{A18}	θ_{B18}	
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height												Coefficient of .0001
113		1	—0.0389	—0.7091	0.2364								0.1414
114		1	1.4854	—1.4913	0.2459	0.2511							0.3210
t		1	1.0210	—4.0419	1.0000	1.0210							—0.5209
u		1		—3.0000	4.9092	0.5385	—2.7480	0.9161					0.0000
113–114		0	—1.5243	0.7822	—0.0095	—0.2511							—0.1796
113–t		0	—1.0599	3.3328	—0.7636	—1.0210							0.6624
t–u		0	1.0210	—1.0419	—3.9092	0.4825	2.7480	—0.9161					—0.5209
115			1	—0.5132	0.0062	0.1647							0.1178
116			1	—3.1444	0.7204	0.9633							—0.6249
117			1	—1.0206	—3.8290	0.4726	2.6916	—0.8973					—0.5102
v			1	—3.0000	0.5281	6.8055	—2.7161		0.9054				0.0000
115–116			0	2.6312	—0.7142	—0.7986							0.7427
116–117			0	—2.1238	4.5494	0.4907	—2.6916	0.8973					—0.1147
117–v			0	1.9794	—4.3571	—6.3329	5.4077	—0.8973	—0.9054				—0.5102
118				1	—0.2714	—0.3035							0.2823
119				1	—2.1422	—0.2310	1.2673	—0.4225					0.0540
120				1	—2.2013	—3.1993	2.7318	—0.4533	—0.4574				—0.2579
118–119				0	1.8708	—0.0724	—1.2673	0.4225					0.2282
118–120				0	1.9299	2.8958	—2.7318	0.4533	0.4574				0.5399
121					1	—0.0387	—0.6774	0.2259					0.1220
122					1	1.5027	—1.4177	0.2353	0.2374				0.2802
w					1	1.0092	—4.0214	1.0000	1.0092				—0.4542
x					1		—3.0000	5.1753	0.5878	—3.0000	1.0000		0.0000
121–122					0	—1.5414	0.7403	—0.0094	—0.2374				—0.1582
121–w					0	—1.0479	3.3440	—0.7741	—1.0092				0.5762
w–x					0	1.0092	—1.0214	—4.1755	0.4214	3.0000	—1.0000		—0.4542
123						1	—0.4810	0.0061	0.1542				0.1028
124						1	—3.1958	0.7398	0.9630				—0.5507
125						1	—1.0136	—4.1437	0.4182	2.9773	—0.9924		—0.4507
y						1	—3.0000	0.5833	7.3021	—3.0000		1.0000	0.0000
123–124						0	2.7148	—0.7337	—0.8088				0.6535
124–125						0	—2.1822	4.8835	0.5448	—2.9773	0.9924		—0.0999
125–y						0	1.9864	—4.7270	—6.8839	5.9773	—0.9924	—1.0000	—0.4507
126							1	—0.2703	—0.2979				0.2407
127							1	—2.2380	—0.2496	1.3644	—0.4548		0.0458
128							1	—2.3797	—3.4655	3.0093	—0.4996	—0.5035	—0.2269
126–127							0	1.9677	—0.0483	—1.3644	0.4548		0.1949
126–128							0	2.1094	3.1676	—3.0093	0.4996	0.5035	0.4677

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Table 14.—(Continued).

Elimination of the Unknown Quantities in the Equations for the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5.

No. of Equation	Left-Hand Member of Equation												Right-Hand Member of Equation
	Story No. 17			Story No. 18			Story No. 19			Story No. 20			
	R_{17}	θ_{A17}	θ_{B17}	R_{18}	θ_{A18}	θ_{B18}	R_{19}	θ_{A19}	θ_{B19}	R_{20}	θ_{A20}	θ_{B20}	
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height												Coefficient of .0001
129		1	−0.0245	−0.6935	0.2311								0.0891
130		1	1.5017	−1.4268	0.2369	0.2387							0.2217
s		1	1.0092	−4.0214	1.0000	1.0092							−0.3412
A′		1		−3.0000	5.1755	0.5878	−3.0000	1.0000					0.0000
129–130		0	−1.5262	0.7333	−0.0057	−0.2387							−0.1226
129–s		0	−1.0337	3.3279	−0.7689	−1.0092							0.4403
s–A′		0	1.0092	−1.0214	−4.1755	0.4214	3.0000	−1.0000					−0.3412
131			1	−0.4805	0.0037	0.1564							0.0804
132			1	−3.2197	0.7438	0.9764							−0.4260
133			1	−1.0120	−4.1373	0.4176	2.9727	−0.9909					−0.3351
B′			1	−3.0000	0.5833	7.3021	−3.0000		1.0000				0.0000
131–132			0	2.7392	−0.7401	−0.8200							0.5063
132–133			0	−2.2077	4.8811	0.5588	−2.9727	0.9909					−0.0879
133–B′			0	1.9680	−4.7206	−6.8846	5.9727	−0.9909	−1.0000				−0.3351
134				1	−0.2702	−0.2993							0.1849
135				1	−2.2110	−0.2531	1.3465	−0.4485					0.0398
136				1	−2.3745	−3.4629	3.0042	−0.4980	−0.5030				−0.1701
134–135				0	1.9408	−0.0462	−1.3465	0.4485					0.1451
134–136				0	2.1043	3.1635	−3.0042	0.4980	0.5030				0.3549
137					1	−0.0238	−0.6938	0.2311					0.0747
138					1	1.5034	−1.4277	0.2367	0.2390				0.1687
C′					1	1.0092	−4.0214	1.0000	1.0092				−0.2275
D′					1		−3.0000	5.1755	0.5878	−3.0000	1.0000		0.0000
137–138					0	−1.5272	0.7339	−0.0056	−0.2390				−0.0939
137–C′					0	−1.0330	3.3276	−0.7689	−1.0092				0.3022
C′–D′					0	1.0092	−1.0214	−4.1755	0.4214	3.0000	−1.0000		−0.2275
139						1	−0.4806	0.0036	0.1565				0.0615
140						1	−3.2214	0.7444	0.9769				−0.2926
141						1	−1.0121	−4.1377	0.4176	2.9727	−0.9909		−0.2254
E′						1	−3.0000	0.5833	7.3021	−3.0000		1.0000	0.0000
139–140						0	2.7408	−0.7407	−0.8204				0.3541
140–141						0	−2.2093	4.8821	0.5594	−2.9727	0.9909		−0.0872
141–E′						0	1.9879	−4.7210	−6.8846	5.9727	−0.9909	−1.0000	−0.2254
142							1	−0.2703	−0.2993				0.1292
143							1	−2.2099	−0.2532	1.3455	−0.4485		0.0304
144							1	−2.3749	−3.4632	3.0046	−0.4985	−0.5031	−0.1134
142–143							0	1.9396	−0.0461	−1.3455	0.4485		0.0988
142–144							0	2.1047	3.1639	−3.0046	0.4985	0.5031	0.2426
145								1	−0.0238	−0.6938	0.2313		0.0509
146								1	1.5034	−1.4278	0.2369	0.2390	0.1153
F′								1	1.0092	−4.0214	1.0000	1.0092	−0.1137
G′								1		−3.0000	3.1759	0.5878	0.0000
145–146								0	−1.5272	0.7340	−0.0056	−0.2390	−0.0643
145–F′								0	−1.0330	3.3276	−0.7687	−1.0092	0.1647
F′–G′								0	1.0092	−1.0214	−2.1759	0.4214	−0.1137
147									1	−0.4806	0.0037	0.1565	0.0421
148									1	−3.2217	0.7442	0.9770	−0.1504
149									1	−1.0122	−2.1563	0.4176	−0.1127
H′									1	−3.0000	0.5834	5.3033	0.0000

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TABLE 14.—(Continued).

ELIMINATION OF THE UNKNOWN QUANTITIES IN THE EQUATIONS FOR THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

No. of Equation	Left-Hand Member of Equation							Right-Hand Member of Equation
	Story No. 19			Story No. 20				
	R_{19}	θ_{A19}	θ_{B19}	R_{20}	θ_{A20}	θ_{B20}		
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height							Coefficient of .0001
147–148			0	2.7411	—0.7406	—0.8205		0.2015
148–149			0	—2.2095	2.9005	0.5595		—0.0467
149–H'			0	1.9878	—2.7397	—4.8857		—0.1127
150				1	—0.2702	—0.2993		0.0735
151				1	—1.3129	—0.2531		0.0211
152				1	—1.3783	—2.4580		—0.0567
150–151				0	1.0427	—0.0462		0.0524
150–152				0	1.1061	2.1587		0.1302
153					1	—0.0443		0.0502
154					1	1.9480		0.1175
153–154						—1.9923		—0.0673
155						1.0000		0.0338

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Table 15.

Determination of the Changes in the Slopes and of the Ratios of Deflection to Story Height in the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5.

The equations are taken from Table 14.

No. of Equation	Left-Hand Member of Equation The first line in each group is the algebraic form of the equation. The successive lines are the numerical values of the terms.			Right-Hand Member of Equation Coefficient of .0001	
155			θ_{B20} =		.0338
153	$\theta_{A20} - .0443\,\theta_{B20}$		=	.0502	
		.0443 x .00000338	=	.0015	
			θ_{A20} =		.0517
150	$R_{20} - .2702\,\theta_{A20} - .2993\,\theta_{B20}$		=	.0735	
		.2702 x .00000517	=	.0140	
		.2993 x .00000338	=	.0101	
			R_{20} =		.0976
147	$\theta_{B19} - .4806\,R_{20} + .0037\,\theta_{A20} + .1565\,\theta_{B20}$		=	.0421	
		.4806 x .00000976	=	.0469	.0890
		.0037 x .00000517	=	.0002	
		.1565 x .00000338	=	.0053	—.0055
			θ_{B19} =		.0835
145	$\theta_{A19} - .0238\,\theta_{B19} - .6938\,R_{20} + .2313\,\theta_{A20}$		=	.0509	
		.0238 x .00000835	=	.0020	
		.6938 x .00000976	=	.0676	.1205
		.2313 x .00000517	=		—.0120
			θ_{A19} =		.1085
142	$R_{19} - .2703\,\theta_{A19} - .2993\,\theta_{B19}$		=	.1292	
		.2703 x .00001085	=	.0294	
		.2993 x .00000835	=	.0250	
			R_{19} =		.1836
139	$\theta_{B18} - .4806\,R_{19} + .0037\,\theta_{A19} + .1566\,\theta_{B19}$		=	.0615	
		.4806 x .00001836	=	.0881	.1496
		.0037 x .00001085	=	.0004	
		.1565 x .00000835	=	.0131	—.0135
			θ_{B18} =		.1361
137	$\theta_{A18} - .0238\,\theta_{B18} - .6938\,R_{19} + .2311\,\theta_{A19}$		=	.0748	
		.0238 x .00001361	=	.0032	
		.6938 x .00001836	=	.1272	.2052
		.2311 x .00001085	=		—.0251
			θ_{A18} =		.1801
134	$R_{18} - .2702\,\theta_{A18} - .2994\,\theta_{B18}$		=	.1849	
		.2702 x .00001801	=	.0486	
		.2994 x .00001361	=	.0407	
			R_{18} =		.2742

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Table 15.—(Continued).

Determination of the Changes in the Slopes and of the Ratios of Deflection to Story Height in the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5.

No. of Equation	Left-Hand Member of Equation The first line in each group is the algebraic form of the equation. The successive lines are the numerical values of the terms.			Right-Hand Member of Equation Coefficient of .0001	
131	$\theta_{B17} - .4805\,R_{18} + .0038\,\theta_{A18} + .1564\,\theta_{B18}$		=	.0804	
		.4805 x .00002742	=	.1319	.2123
		.0038 x .00001801	=	.0007	
		.1564 x .00001361	=	.0213	— .0220
		θ_{B17}	=		.1903
129	$\theta_{A17} - .0246\,\theta_{B17} - .6935\,R_{18} + .2311\,\theta_{A18}$		=	.0991	
		.0246 x .00001903	=	.0047	
		.6935 x .00002742	=	.1905	.2943
		.2311 x .00001801	=		— .0416
		θ_{A17}	=		.2527
126	$R_{17} - .2703\,\theta_{A17} - .2979\,\theta_{B17}$		=	.2407	
		.2703 x .00002527	=	.0682	
		.2979 x .00001903	=	.0566	
		R_{17}	=		.3655
123	$\theta_{B16} - .4810\,R_{17} + .0061\,\theta_{A17} + .1542\,\theta_{B17}$		=	.1028	
		.4810 x .00003655	=	.1758	.2786
		.0061 x .00002527	=	.0015	
		.1542 x .00001903	=	.0294	— .0309
		θ_{B16}	=		.2477
121	$\theta_{A16} - .0387\,\theta_{B16} - .6774\,R_{17} + .2259\,\theta_{A17}$		=	.1220	
		.0387 x .00002477	=	.0096	
		.6774 x .00003655	=	.2473	.3789
		.2259 x .00002527	=		— .0570
		θ_{A16}	=		.3219
118	$R_{16} - .2714\,\theta_{A16} - .3035\,\theta_{B16}$		=	.2823	
		.2714 x .00003219	=	.0873	
		.3035 x .00002477	=	.0752	
		R_{16}	=		.4448
115	$\theta_{B15} - .5132\,R_{16} + .0062\,\theta_{A16} + .1647\,\theta_{B16}$		=	.1178	
		.5132 x .00004448	=	.2280	.3458
		.0062 x .00003219	=	.0020	
		.1647 x .00002477	=	.0408	— .0428
		θ_{B15}	=		.3030
113	$\theta_{A15} - .0389\,\theta_{B15} - .7091\,R_{16} + .2364\,\theta_{A16}$		=	.1414	
		.0389 x .00003030	=	.0118	
		.7091 x .00004448	=	.3180	.4712
		.2364 x .00003219	=		— .0762
		θ_{A15}	=		.3950
110	$R_{15} - .2718\,\theta_{A15} - .3006\,\theta_{B15}$		=	.3361	
		.2718 x .00003950	=	.1072	
		.3006 x .00003030	=	.0912	
		R_{15}	=		.5345
107	$\theta_{B14} - .5109\,R_{15} + .0120\,\theta_{A15} + .1581\,\theta_{B15}$		=	.1444	
		.5109 x .00005345	=	.2730	.4174
		.0120 x .00003950	=	.0047	
		.1581 x .00003030	=	.0480	— .0527
		θ_{B14}	=		.3647

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Table 15.—(Continued).

Determination of the Changes in the Slopes and of the Ratios of Deflection to Story Height in the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5.

No. of Equation	Left-Hand Member of Equation The first line in each group is the algebraic form of the equation. The successive lines are the numerical values of the terms.			Right-Hand Member of Equation Coefficient of .0001	
105	$\theta_{A14} - .0775\ \theta_{B14} - .6624\ R_{15} + .2210\ \theta_{A15}$		=	.1627	
		.0775 x .00003647	=	.0283	
		.6624 x .00005345	=	.3541	.5451
		.2210 x .00003950	=		—.0873
		θ_{A14}	=		.4578
102	$R_{14} - .2790\ \theta_{A14} - .3096\ \theta_{B14}$		=	.3412	
		.2790 x .00004578	=	.1278	
		.3096 x .00003647	=	.1160	
		R_{14}	=		.5850
99	$\theta_{B13} - 5928\ R_{14} + .0124\ \theta_{A14} + .1853\ \theta_{B14}$		=	.1487	
		.5928 x .00005850	=	.3470	.4957
		.0124 x .00004578	=	.0057	
		.1853 x .00003647	=	.0676	—.0733
		θ_{B13}	=		.4224
97	$\theta_{A13} - .0695\ \theta_{B13} - .7560\ R_{14} + .2520\ \theta_{A14}$		=	.1658	
		.0695 x .00004224	=	.0294	
		.7560 x .00005850	=	.4420	.6372
		.2520 x .00004578	=		— 1154
		θ_{A13}	=		.5218
94	$R_{13} - .2789\ \theta_{A13} - .3092\ \theta_{B13}$		=	.3864	
		.2789 x .00005218	=	.1458	
		.3092 x .00004224	=	.1308	
		R_{13}	=		.6630
91	$\theta_{B12} - .5920\ R_{13} + .0138\ \theta_{A13} + .1835\ \theta_{B13}$		=	.1715	
		.5920 x .00006630	=	.3921	.5636
		.0138 x .00005218	=	.0072	
		.1835 x .00004224	=	.0775	—.0847
		θ_{B12}	=		.4789
89	$\theta_{A12} - .0766\ \theta_{B12} - .7472\ R_{13} + .2490\ \theta_{A13}$		=	.1816	
		.0766 x .00004789	=	.0366	
		.7472 x .00006630	=	.4960	.7142
		.2490 x .00005218	=		—.1300
		θ_{A12}	=		.5842
86	$R_{12} - .2750\ \theta_{A12} - .3160\ \theta_{B12}$		=	.4216	
		.2750 x .00005842	=	.1603	
		.3160 x .00004789	=	.1511	
		R_{12}	=		.7330
83	$\theta_{B11} - .6150\ R_{12} + .0139\ \theta_{A12} + .1912\ \theta_{B12}$		=	.1831	
		.6150 x .00007330	=	.4512	.6343
		.0139 x .00005842	=	.0081	
		.1912 x .00004789	=	.0916	—.0997
		θ_{B11}	=		.5346
81	$\theta_{A11} - .0770\ \theta_{B11} - .7560\ R_{12} + .2520\ \theta_{A12}$		=	.2018	
		.0770 x .00005346	=	.0411	
		.7560 x .00007330	=	.5550	.7979
		.2520 x .00005842	=		—.1471
		θ_{A11}	=		.6508
78	$R_{11} - .2746\ \theta_{A11} - .3147\ \theta_{B11}$		=	.4669	
		.2746 x .00006508	=	.1790	
		.3147 x .00005346	=	.1681	
		R_{11}	=		.8140

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Table 15.—(Continued).

Determination of the Changes in the Slopes and of the Ratios of Deflection to Story Height in the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5.

No. of Equation	Left-Hand Member of Equation The first line in each group is the algebraic form of the equation. The successive lines are the numerical values of the terms.			Right-Hand Member of Equation Coefficient of .0001	
75	$\theta_{B10} - .6061\, R_{11} + .0179\, \theta_{A11} + .1841\, \theta_{B11}$		=	.2178	
		.6061 x .00008140	=	.4940	.7118
		.0179 x .00006508	=	.0116	
		.1841 x .00005346	=	.0984	—.1100
		θ_{B10}	=		.6018
73	$\theta_{A10} - .1082\, \theta_{B10} - .7278\, R_{11} + .2426\, \theta_{A11}$		=	.2055	
		.1082 x .00006018	=	.6651	
		.7278 x .00008140	=	.5925	.8631
		.2426 x .00006508	=		—.1580
		θ_{A10}	=		.7051
70	$R_{10} - .2601\, \theta_{A10} - .3423\, \theta_{B10}$		=	.4540	
		.2601 x .00007051	=	.1833	
		.3423 x .00006018	=	.2059	
		R_{10}	=		.8432
67	$\theta_{B9} - .6991\, R_{10} + .0181\, \theta_{A10} + .2151\, \theta_{B10}$		=	.2056	
		.6991 x .00008432	=	.5890	.7946
		.0181 x .00007051	=	.0127	
		.2151 x .00006018	=	.1293	—.1420
		θ_{B9}	=		.6526
65	$\theta_{A9} - .1038\, \theta_{B9} - .7655\, R_{10} + .2551\, \theta_{A10}$		=	.2053	
		.1038 x .00006526	=	.0677	
		.7655 x .00008432	=	.6460	.9190
		.2551 x .00007051	=		—.1800
		θ_{A9}	=		.7390
62	$R_9 - .2602\, \theta_{A9} - .3372\, \theta_{B9}$		=	.4719	
		.2602 x .00007390	=	.1922	
		.3372 x .00006526	=	.2200	
		R_9	=		.8841
59	$\theta_{B8} - .6908\, R_9 + .0231\, \theta_{A9} + .2072\, \theta_{B9}$		=	.2008	
		.6908 x .00008841	=	.6100	.8108
		.0231 x .00007390	=	.0171	
		.2072 x .00006526	=	.1351	—.1522
		θ_{B8}	=		.6586
57	$\theta_{A8} - .1382\, \theta_{B8} - .7000\, R_9 + .2334\, \theta_{A9}$		=	.2079	
		.1382 x .00006586	=	.0910	
		.7000 x .00008841	=	.6182	.9171
		.2334 x .00007390	=		—.1725
		θ_{A8}	=		.7446
54	$R_8 - .2738\, \theta_{A8} - .3118\, \theta_{B8}$		=	.3774	
		.2738 x .00007446	=	.2038	
		.3118 x .00006586	=	.2055	
		R_8	=		.7867
51	$\theta_{B7} - .5814\, R_8 + .0112\, \theta_{A8} + .1826\, \theta_{B8}$		=	.1562	
		.5814 x .00007867	=	.4579	.6132
		.0112 x .00007446	=	.0083	
		.1826 x .00006586	=	.1203	—.1286
		θ_{B7}	=		.4846

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Table 15.—(Continued).

Determination of the Changes in the Slopes and of the Ratios of Deflection to Story Height in the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5.

No. of Equation	Left-Hand Member of Equation The first line in each group is the algebraic form of the equation. The successive lines are the numerical values of the terms.			Right-Hand Member of Equation Coefficient of .0001	
49	$\theta_{A7} - .0640\ \theta_{B7} - .7422\ R_8 + .2474\ \theta_{A8}$		=	.1729	
		.0640 x .00004846	=	.0310	
		.7422 x .00007867	=	.5845	.7834
		.2474 x .00007446	=		—.1840
		θ_{A7}	=		.6044
46	$R_7 - .2711\ \theta_{A7} - .3090\ \theta_{B7}$		=	.4011	
		.2711 x .00006044	=	.1639	
		.3090 x .00004846	=	.1497	
		R_7	=		.7147
43	$\theta_{B6} - .5489\ R_7 + .0090\ \theta_{A7} + .1739\ \theta_{B7}$		=	.1706	
		.5489 x .00007147	=	.3920	.5626
		.0090 x .00006044	=	.0054	
		.1739 x .00004846	=	.0842	—.0896
		θ_{B6}	=		.4730
41	$\theta_{A6} - .0540\ \theta_{B6} - .7192\ R_7 + .2397\ \theta_{A7}$		=	.1946	
		.0540 x .00004730	=	.0256	
		.7192 x .00007147	=	.5140	.7342
		.2397 x .00006044	=		—.1448
		θ_{A6}	=		.5894
38	$R_6 - .2648\ \theta_{A6} - .3000\ \theta_{B6}$		=	.4216	
		.2648 x .00005894	=	.1561	
		.3000 x .00004730	=	.1420	
		R_6	=		.7197
35	$\theta_{B5} - .4496\ R_6 + .0013\ \theta_{A6} + .1487\ \theta_{B6}$		=	.1486	
		.4496 x .00007197	=	.3240	.4726
		.0013 x .00005894	=	.0008	
		.1487 x .00004730	=	.0704	—.0712
		θ_{B5}	=		.4014
33	$\theta_{A5} - .0092\ \theta_{B5} - .6676\ R_9 + .2225\ \theta_{A6}$		=	.1879	
		.0092 x .00004014	=	.0037	
		.6676 x .00007197	=	.4802	.6718
		.2225 x .00005894	=		—.1310
		θ_{A5}	=		.5408
30	$R_5 - .2638\ \theta_{A5} - .2981\ \theta_{B5}$		=	.4473	
		.2638 x .00005408	=	.1424	
		.2981 x .00004014	=	.1196	
		R_5	=		.7093
27	$\theta_{B4} - .4348\ R_5 + .0048\ \theta_{A5} + .1401\ \theta_{B5}$		=	.1550	
		.4348 x .00007093	=	.3080	.4630
		.0048 x .00005408	=	.0026	
		.1401 x .00004014	=	.0563	—.0589
		θ_{B4}	=		.4041
25	$\theta_{A4} - .0355\ \theta_{B4} - .6371\ R_5 + .2123\ \theta_{A5}$		=	.2000	
		.0355 x .00004041	=	.0143	
		.6371 x .00007093	=	.4515	.6658
		.2123 x .00005408	=		—.1150
		θ_{A4}	=		.5508

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Table 15.—(Continued).

Determination of the Changes in the Slopes and of the Ratios of Deflection to Story Height in the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5.

No. of Equation.	Left-Hand Member of Equation The first line in each group is the algebraic form of the equation. The successive lines are the numerical values of the terms.			Right-Hand Member of Equation Coefficient of .0001	
22	$R_4 + .3642\ \theta_{A4} - .3206\ \theta_{B4} - .4044\ R_5 + .1348\ \theta_{A5}$		=	.5512	
		.3206 x .00004041	=	.1295	
		.4044 x .00007093	=	.2862	.9669
		.3642 x .00005508	=	.2008	
		.1348 x .00005408	=	.0729	—.2737
		R_4	=		.6932
19	$\theta_{B3} - .4696\ R_4 + .0030\ \theta_{A4} + .1535\ \theta_{B4}$		=	.1540	
		.4696 x .00006932	=	.3257	.4797
		.0030 x .00005508	=	.0017	
		.1535 x .00004041	=	.0620	—.0637
		θ_{B3}	=		.4160
17	$\theta_{A3} - .0196\ \theta_{B3} - .6881\ R_4 + .2294\ \theta_{A4}$		=	.1917	
		.0196 x .00004160	=	.0082	
		.6881 x .00006932	=	.4770	.6769
		.2294 x .00005508	=		—.1263
		θ_{A3}	=		.5506
14	$R_3 - .2689\ \theta_{A3} - .2980\ \theta_{B3}$		=	.4564	
		.2689 x .00005506	=	.1481	
		.2980 x .00004160	=	.1240	
		R_3	=		.7285
11	$\theta_{B2} - .4511\ R_3 + .0027\ \theta_{A3} + .1476\ \theta_{B3}$		=	.1768	
		.4511 x .0000 285	=	.3285	.5053
		.0027 x .00005506	=	.0015	
		.1476 x .00004160	=	.0614	—.0629
		θ_{B2}	=		.4424
9	$\theta_{A2} - .0191\ \theta_{B2} - .6857\ R_3 + .2286\ \theta_{A3}$		=	.2151	
		.0191 x .00004424	=	.0085	
		.6857 x .00007285	=	.4990	.7226
		.2286 x .00005506	=		—.1259
		θ_{A2}	=		.5967
6	$R_2 - .2627\ \theta_{A2} - .2918\ \theta_{B2}$		=	.5021	
		.2627 x .00005967	=	.1565	
		.2918 x .00004424	=	.1291	
		R_2	=		.7877
3	$\theta_{B1} - .3612\ R_2 - .0088\ \theta_{A2} + .1292\ \theta_{B2}$		=	.1483	
		.3612 x .00007877	=	.2845	
		.0088 x .00005967	=	.0053	.4381
		.1292 x .00004424	=		—.0572
		θ_{B1}	=		.3809
1	$\theta_{A1} + .0680\ \theta_{B1} - .6494\ R_2 + .2164\ \theta_{A2}$		=	.2667	
		.6494 x .00007877	=	.5110	.7777
		.0680 x .00003809	=	.0259	
		.2164 x .00005967	=	.1291	—.1550
		θ_{A1}	=		.6227
A	$R_1 - .2495\ \theta_{A1} - .2495\ \theta_{B1}$		=	.5667	
		.2495 x .00006227	=	.1553	
		.2495 x .00003809	=	.0951	
		R_1	=		.8171

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Table 16.

Values of R and θ for the Symmetrical Three-span Twenty-Story Bent Shown in Fig. 5, and the Functions of these Values that Occur in the Equations Used to Determine the Moments in the Columns and Girders.

Story No.	$10{,}000 \times R$	$10{,}000 \times \theta_A$	$10{,}000 \times \theta_B$	$10{,}000 \times 3R$	$10{,}000 \times 2\theta_A$	$10{,}000 \times 2\theta_B$	$10{,}000 \times (2\theta_A+\theta_B)$	$10{,}000 \times (2\theta_B+\theta_A)$	$10{,}000 \times 3\theta_B$	For Column A* $10{,}000 \times (2\theta_{N-1}+\theta_N-3R)$	For Column A* $10{,}000 \times (2\theta_N+\theta_{N-1}-3R)$	For Column B $10{,}000 \times (2\theta_{N-1}+\theta_N-3R)$	For Column B $10{,}000 \times (2\theta_N+\theta_{N-1}-3R)$	Story No.
1	.8171	.6227	.3903	2.4413	1.2454	.7618	1.6263	1.3845	1.1427	1.8186	1.1959	2.0604	1.6795	1
2	.7877	.5967	.4424	2.3631	1.1934	.8848	1.6358	1.4815	1.3372	.5210	.5470	1.1589	1.0974	2
3	.7285	.5503	.4160	2.1855	1.1012	.8320	1.5172	1.3826	1.2480	.4415	.4876	.8847	.9111	3
4	.6932	.5508	.4041	2.0796	1.1016	.8082	1.5057	1.3590	1.2123	.4276	.4274	.8435	.8554	4
5	.7093	.5403	.4014	2.1279	1.0816	.8028	1.4830	1.3436	1.2042	.4855	.4955	.9183	.9210	5
6	.7197	.5894	.4730	2.1591	1.1788	.9460	1.6518	1.5354	1.4190	.4881	.4395	.8833	.8117	6
7	.7147	.6044	.4846	2.1441	1.2088	.9692	1.6934	1.5736	1.4538	.3609	.3459	.7135	.7019	7
8	.7867	.7446	.6586	2.3601	1.4892	1.3172	2.1478	2.0618	1.9758	.4067	.2665	.7323	.5583	8
9	.8841	.7390	.6526	2.6523	1.4780	1.3052	2.1306	2.0442	1.9578	.4241	.4297	.6825	.6885	9
10	.8432	.7051	.6018	2.5296	1.4102	1.2036	2.0120	1.9087	1.8054	.3465	.3804	.6226	.6734	10
11	.8140	.6508	.5346	2.4420	1.3016	1.0692	1.8362	1.7200	1.6038	.3810	.4353	.7038	.7710	11
12	.7330	.5842	.4789	2.1990	1.1684	.9578	1.6473	1.5420	1.4367	.3132	.3798	.6509	.7066	12
13	.6630	.5218	.4224	1.9890	1.0436	.8448	1.4660	1.3666	1.2682	.2988	.3612	.6088	.6653	13
14	.5850	.4578	.3647	1.7550	.9156	.7294	1.2803	1.1872	1.0941	.2536	.3176	.5455	.6032	14
15	.5345	.3950	.3030	1.6035	.7900	.6060	1.0930	1.0010	.9090	.2929	.3557	.5711	.6328	15
16	.4448	.3219	.2477	1.3344	.6438	.4954	.8915	.8173	.7431	.2225	.2956	.4807	.5360	16
17	.3655	.2527	.1903	1.0965	.5054	.3806	.6957	.6333	.5709	.2000	.2692	.4108	.4682	17
18	.2742	.1801	.1361	.8226	.3602	.2722	.4963	.4523	.4083	.1371	.2097	.3059	.3601	18
19	.1836	.1085	.0335	.5503	.2170	.1670	.3005	.2755	.2505	.0621	.1537	.1951	.2477	19
20	.0976	.0517	.0338	.2923	.1034	.0676	.1372	.1193	.1014	.0231	.0809	.0920	.1417	20

* N represents the number of the story in question, and (N—1) the story below.

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Table 17.

Values of K for the Columns and the Girders of the Symmetrical Three-span Twenty-story Bent Shown in Fig. 5, and the Functions of these Values that Occur in the Equations Used to Determine the Moments in the Columns and Girders.

Story No.	K for Girder at Top of Story in		K for		.0001 x $2EK$ for Girder at Top of Story in		.0001 x $2EK$ for	
	Bay *a*	Bay *b*	Column *A*	Column *B*	Bay *a*	Bay *b*	Column *A*	Column *B*
1	30.5	37.3	25.8	25.8	177 000	216 200	149 600	149 600
2	21.4	29.2	35.6	35.6	124 100	169 400	206 300	206 300
3	21.4	26.2	35 4	35.5	124 100	152 000	205 500	205 800
4	19.5	26.2	35.4	35.5	113 050	152 000	205 500	205 800
5	19.5	23.8	29.4	30.4	113 050	138 050	170 500	176 300
6	14.1	17.2	29.4	30.4	81 750	99 700	170 500	176 300
7	12.8	15.7	28.7	30 0	74 250	91 100	166 500	174 000
8	7.7	9.4	28.7	30.0	44 600	54 500	166 500	174 000
9	7.7	9.4	21.1	26.1	44 600	54 500	122 400	151 300
10	7.7	9.4	21.1	26.1	44 600	54 500	122 400	151 300
11	7.7	9.4	18.8	19.9	44 600	54 500	109 000	115 400
12	7.7	9.4	18.8	19.9	44 600	54 500	109 000	115 400
13	7.7	9.4	18.3	18.6	44 600	54 500	106 100	107 900
14	7.7	9.4	18.3	18.6	44 600	54 500	106 100	107 900
15	7.7	9.4	14.3	14.6	44 600	54 500	82 900	84 600
16	7.7	9.4	14 3	14.6	44 600	54 500	82 900	84 600
17	7.7	9 4	13.1	13.2	44 600	54 500	76 000	76 500
18	7.7	9.4	13.1	13.2	44 600	54 500	76 000	76 500
19	7.7	9.4	13.1	13.2	44 600	54 500	76 000	76 500
20	7.7	9.4	13.1	13.2	44 600	54 500	76 000	76 500

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TABLE 18.

MOMENTS AT THE ENDS OF THE COLUMNS AND GIRDERS OF THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

Moments are expressed in inch-pounds.

No. of Story	Moment in Column A*		Moment in Column B		Moment in Girder at Top of Story in Bay a		Moment in Girder at Top of Story in Bay b
	At Top = $2EK\,(2\theta_N+\theta_{N-1}-3R)$	At Bottom = $2EK\,(2\theta_{N-1}+\theta_N-3R)$	At Top = $2EK\,(2\theta_N+\theta_{N-1}-3R)$	At Bottom = $2EK\,(2\theta_{N-1}+\theta_N-3R)$	At Right-hand End = $2EK\,(2\theta_A+\theta_B)$	At Left-hand End = $2EK\,(2\theta_B+\theta_A)$	At Each End = $2EK\,(3\theta_B)$
1	178 800	272 000	251 000	308 200	287 800	245 000	247 500
2	113 000	107 500	226 500	239 000	203 000	184 000	226 000
3	100 300	90 700	187 500	182 000	187 100	171 700	189 600
4	87 800	87 800	176 100	173 500	170 500	153 600	184 400
5	84 500	82 800	162 300	162 000	167 700	152 000	166 000
6	75 000	83 200	143 300	155 700	135 000	125 600	141 500
7	57 600	60 000	122 000	124 000	125 900	117 000	132 500
8	44 400	67 700	97 200	127 500	95 800	92 000	107 800
9	52 550	51 900	104 200	103 300	95 000	91 100	106 800
10	46 500	42 400	102 000	94 200	89 600	85 000	98 500
11	47 500	41 500	89 000	81 200	81 900	76 600	87 500
12	41 400	34 100	81 500	75 100	73 400	68 600	78 300
13	38 300	31 700	71 800	65 600	65 300	60 800	69 100
14	33 700	26 900	65 100	58 900	5 700	52 800	59 600
15	29 420	24 300	53 600	48 350	48 700	49 600	49 600
16	24 450	18 450	45 400	40 600	39 700	36 400	40 500
17	20 450	15 400	35 800	31 400	31 000	28 200	31 100
18	15 920	10 430	27 600	23 400	22 100	20 150	23 300
19	11 680	6 240	18 900	14 930	13 380	12 250	13 650
20	6 150	1 750	10 830	7 040	6 110	5 320	5 530

*N represents the number of the story in question, and (N—1) the story below.

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Table 19.

Direct Stresses in the Columns, and the Shears in the Columns and Girders of the Symmetrical Three-span Twenty-story Bent shown in Fig. 5.

All quantities are in pounds.

Story No.	Shear in		Shear in Girder at Top of Story in		Direct Stress in	
	Column *A*	Column *B*	Bay *a*	Bay *b*	Column *A*	Column *B*
1	1 709	2 115	2 020	2 290	14 464	4 587
2	1 149	2 421	1 467	2 095	12 444	4 317
3	1 136	2 200	1 360	1 755	10 977	3 689
4	1 044	2 080	1 228	1 709	9 617	3 294
5	995	1 931	1 210	1 536	8 389	2 813
6	940	1 780	986	1 310	7 179	2 489
7	816	1 709	920	1 228	6 193	2 163
8	779	1 560	712	996	5 273	1 855
9	725	1 440	705	988	4 561	1 571
10	617	1 363	661	912	3 856	1 288
11	618	1 180	600	810	3 195	1 037
12	524	1 088	538	725	2 595	827
13	486	954	478	640	2 057	640
14	421	861	416	552	1 579	478
15	373	710	354	459	1 163	342
16	298	597	288	375	809	237
17	249	466	221	288	521	150
18	182	354	160	206	300	83
19	124	235	97	126	140	37
20	55	124	43	51	43	8

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Table 20.

Check on the Numerical Values of the Moments at the Ends of the Columns and Girders of the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5.

Quantities are in inch-pounds.

For a perfect check the corresponding quantities in the two columns of each group of columns should be identical.

Story No.	Total Shear in a Story Multiplied by the Story Height	Sum of the Moments at the Tops and the Bottoms of all of the Columns in a Story	Sum of the Moments in Column *A* just above and below Girder *a* at the Top of a Story	Moment in Girder *a* at the Top of a Story, at the Section adjacent to Column *A*	Sum of the Moments in Column *B* just above and below Girders *a* and *b* at the Top of a Story	Sum of the Moments in Girders *a* and *b* at the Top of a Story at Sections adjacent to Column *B*
1	2 035 000	2 020 000	286 300	287 800	490 000	492 500
2	1 370 000	1 372 000	203 700	203 000	408 500	410 000
3	1 123 000	1 121 000	187 100	188 100	361 000	361 300
4	1 053 000	1 050 400	170 500	170 600	338 100	338 000
5	983 000	983 200	167 700	167 700	318 000	318 000
6	913 000	914 400	135 000	135 000	267 300	267 100
7	725 000	727 200	125 900	125 300	249 500	249 500
8	673 000	673 600	95 800	96 300	200 500	199 800
9	622 000	624 000	95 000	94 950	198 400	197 900
10	570 000	570 200	89 600	88 000	183 200	183 500
11	518 000	518 400	81 900	81 600	164 100	164 100
12	466 000	464 200	73 400	72 100	147 100	146 900
13	414 000	414 800	65 300	65 200	130 700	129 900
14	363 000	369 200	57 000	58 000	113 450	112 400
15	311 000	311 400	48 700	47 870	94 200	94 200
16	259 000	259 800	39 700	39 850	76 800	76 900
17	207 500	206 200	31 000	30 880	59 200	59 300
18	155 300	154 800	22 100	22 160	42 530	42 450
19	103 600	103 500	13 380	13 430	25 940	25 900
20	51 800	51 540	6 110	6 150	10 830	10 850

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Table 21.

Elimination of the Unknown Quantities in the Equations Used to Determine the Slopes and the Deflections in the Bottom Story of the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5, by a Modification of the Slope-Deflection Method.

No. of Equation	Left-Hand Member of Equation							Right-Hand Member of Equation
	Story No. 1			Story No. 2			Story No. 3	
	R_1	θ_{A1}	θ_{B1}	R_2	θ_{A2}	θ_{B2}	R_3	Co-efficient of .0001
	Coefficients of Unknown Slopes and Ratios of Deflection to Story Height							
A	−206.8	51.6	51.6					−117.2
B	− 77.4	183.8	30.5	−106.8	35.6			0.0
C	− 77.4	30.5	295.7	−106.8		35.6		0.0
D		71.2	71.2	−284.8	71.2	71.2		− 78.9
E		35.6		−106.8	220.2	21.4	−106.2	0.0
F			35.6	−106.8	21.4	308.1	−106.5	0.0
G					141.6	142.0	−283.6	− 64.5
A	1	−0.2495	−0.2495					0.5667
B	1	−2.3447	−0.3941	1.3798	−0.4599			0.0000
C	1	−0.3941	−3.8204	1.3798		−0.4599		0.0000
A–B	0	2.1252	0.1445	−1.3798	0.4599			0.5667
B–C	0	−1.9807	3.4264		−0.4599	0.4599		0.0000
1		1	0.0680	−0.6494	0.2164			0.2667
2		1	−1.7300		0.2322	−0.2322		0.0000
D		1	1.0000	−4.0000	1.0000	1.0000		−1.1082
E		1		−3.0000	6.1856	0.6012	−2.9834	0.0000
1–2		0	1.7980	−0.6494	−0.0158	0.2322		0.2667
2–D		0	−2.7300	4.0000	−0.7678	−1.2322		1.1082
D–E		0	1.0000	−1.0000	−5.1856	0.3988	2.9834	−1.1082
3			1	−0.3612	−0.0088	0.1292		0.1483
4			1	−1.4552	0.2812	0.4513		−0.4059
5			1	−1.0000	−5.1856	0.3988	2.9834	−1.1082
F			1	−3.0000	0.6012	8.6547	−2.9915	0.0000
3–4			0	1.1041	−0.2900	−0.3222		0.5543
4–5			0	−0.4552	5.4668	0.0525	−2.9834	0.7023
5–F			0	2.0000	−5.7868	8.2559	5.9749	−1.1082
6				1	−0.2627	−0.2918		0.5021
7				1	−11.7518	−0.1128	6.4127	−1.5095
8				1	−2.8934	−4.1279	2.9875	−0.5541
6–7				0	11.4891	−0.1790	−6.4127	2.0116
7–8				0	−8.8554	4.0151	3.4252	−0.9554
9					1	−0.0156	−0.5600	0.1750
10					1	−0.4540	−0.3862	0.1079
G					1	1.0030	−2.0010	−0.4555
9–10					0	0.4384	−0.1738	0.0671
10–G					0	−1.4570	1.6148	0.5634
11						1	−0.3961	0.1530
12						1	−1.1090	−0.3865
11–12						0	0.7131	0.5395
13							1	0.7560

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TABLE 22.

DETERMINATION OF THE CHANGES IN THE SLOPES AND THE RATIO OF THE DEFLECTION TO STORY HEIGHT IN THE BOTTOM STORY OF THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5, BY A MODIFICATION OF THE SLOPE-DEFLECTION METHOD.

No. of Equation	Left-Hand Member of Equation The first line in each group is the algebraic form of the equation. The successive lines are the numerical values of the terms.			Right-Hand Member of Equation Coefficient of .0001	
13			R_3 =	.7560	
11	$\theta_{B2} - .3961\, R_3$		=	.1530	
		.3961 x .00007560	=	.2995	
			θ_{B2} =		.4525
9	$\theta_{A2} - .0156\ \theta_{B2} - .5600\ R_3$		=	.1750	
		.0156 x .00004525	=	.0071	
		.5600 x .00007560	=	.4240	
			θ_{A2} =		.6061
6	$R_2 - .2627\ \theta_{A2} - .2918\ \theta_{B2}$		=	.5021	
		.2627 x .00006061	=	.1593	
		.2918 x .00004525	=	.1320	
			R_2 =		.7934
3	$\theta_{B1} - .3612\ R_2 - .0088\ \theta_{A2} + .1292\ \theta_{B2}$		=	.1483	
		.3612 x .00007934	=	.2865	
		.0088 x .00006061	=	.0053	.4401
		.1292 x .00004525	=		—.0585
			θ_{B1} =		.3816
1	$\theta_{A1} + .0680\ \theta_{B1} - .6494\ R_2 + .2164\ \theta_{A2}$		=	.2667	
		.6494 x .00007934	=	.5150	.7817
		.0680 x .00003816	=	.0260	
		.2164 x .00006061	=	.1313	—.1573
			θ_{A1} =		.6244
A	$R_1 - .2495\ \theta_{A1} - .2495\ \theta_{B1}$		=	.5667	
		.2495 x .00006244	=	.1556	
		.2495 x .00003816	=	.0951	
			R_1 =		.8174

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TABLE 23.

COMPARISON OF THE APPROXIMATE METHODS WITH THE SLOPE-DEFLECTION METHOD WHEN APPLIED TO THE SYMMETRICAL THREE-SPAN TWENTY-STORY BENT SHOWN IN FIG. 5.

MOMENTS IN THE COLUMNS.

For each story the upper line is the moment in 1000 in. lb., and the lower line is the moment in per cent of the moment determined by the slope-deflection method.

Story No.	Moment at Top of Column A					Moment at Bottom of Column A					Moment at Top of Column B					Moment at Bottom of Column B					Story No.
	Slope-Deflection Method	Proposed Approximate Method	Fleming's Method			Slope-Deflection Method	Proposed Approximate Method	Fleming's Method			Slope-Deflection Method	Proposed Approximate Method	Fleming's Method			Slope-Deflection Method	Proposed Approximate Method	Fleming's Method			
			I	II	III			I	II	III			I	II	III			I	II	III	
1	178.8	191.5	166.9	254.5	254.5	272.0	191.5	166.9	254.5	254.5	251.0	319.0	343.2	254.5	254.5	308.2	319.0	343.2	254.2	254.5	1
	100.0	*106.8*	*93.7*	*145.0*	*145.0*	*100.0*	*70.3*	*61.3*	*93.6*	*93.6*	*100.0*	*126.8*	*136.9*	*101.4*	*101.4*	*100.0*	*103.5*	*111.3*	*82.5*	*82.5*	
2	113.0	113.8	111.0	171.4	171.4	107.5	113.8	111.0	171.4	171.4	226.5	227.5	228.6	171.4	171.4	239.0	227.5	228.6	171.4	171.4	2
	100.0	*100.7*	*98.3*	*152.0*	*152.0*	*100.0*	*106.0*	*103.4*	*159.4*	*159.4*	*100.0*	*100.4*	*101.0*	*75.7*	*75.7*	*100.0*	*95.3*	*95.8*	*71.7*	*71.7*	
3	100.3	96.8	91.5	140.5	140.5	90.7	96.8	91.5	140.5	140.5	187.5	184.1	187.6	140.5	140.5	182.0	184.1	187.6	140.5	140.5	3
	100.0	*96.5*	*91.2*	*140.3*	*140.3*	*100.0*	*106.8*	*101.0*	*155.0*	*155.0*	*100.0*	*98.3*	*100.0*	*75.0*	*75.0*	*100.0*	*101.2*	*103.0*	*77.2*	*77.2*	
4	87.8	87.1	87.1	131.7	131.7	87.8	87.1	87.1	131.7	131.7	176.1	175.0	175.3	131.7	131.7	173.5	175.0	175.3	131.7	131.7	4
	100.0	*99.3*	*99.2*	*150.0*	*150.0*	*100.0*	*99.3*	*99.2*	*150.0*	*150.0*	*100.0*	*99.4*	*99.5*	*74.6*	*74.6*	*100.0*	*101.0*	*101.2*	*75.9*	*75.9*	
5	84.5	85.6	80.9	120.4	120.4	82.8	85.6	80.9	120.4	120.4	162.3	160.0	163.1	125.3	125.3	162.0	160.0	163.1	125.3	125.3	5
	100.0	*101.3*	*95.6*	*142.3*	*142.3*	*100.0*	*103.3*	*97.6*	*145.3*	*145.3*	*100.0*	*98.5*	*100.6*	*77.2*	*77.2*	*100.0*	*98.8*	*100.8*	*77.5*	*77.5*	
6	75.0	76.8	74.3	111.7	111.7	83.2	76.8	74.3	111.7	111.7	143.3	151.8	153.0	116.3	116.3	155.7	151.8	153.0	116.3	116.3	6
	100.0	*102.0*	*99.0*	*149.0*	*149.0*	*100.0*	*92.2*	*89.5*	*134.2*	*134.2*	*100.0*	*106.0*	*106.8*	*81.1*	*81.1*	*100.0*	*97.5*	*98.3*	*74.8*	*74.8*	
7	57.6	60.3	59.2	88.6	88.6	60.0	60.3	59.2	88.6	88.6	122.0	120.7	122.0	92.8	92.8	124.0	120.7	122.0	92.8	92.8	7
	100.0	*105.0*	*102.8*	*154.0*	*154.0*	*100.0*	*100.5*	*98.5*	*147.6*	*147.6*	*100.0*	*98.8*	*100.0*	*76.1*	*76.1*	*100.0*	*97.3*	*98.3*	*74.8*	*74.8*	
8	44.4	54.7	55.9	82.3	82.3	67.7	54.7	55.9	82.3	82.3	97.2	114.0	113.0	86.2	86.2	127.5	114.0	113.0	86.2	86.2	8
	100.0	*125.0*	*126.0*	*185.5*	*185.5*	*100.0*	*80.7*	*82.5*	*121.8*	*121.8*	*100.0*	*117.2*	*116.2*	*88.7*	*88.7*	*100.0*	*89.5*	*88.6*	*67.6*	*67.6*	
9	52.6	49.8	51.4	69.5	69.5	51 9	49.8	51.4	69.5	69.5	104.2	105.5	104.5	86.0	86.0	103.3	105.5	104.5	86.0	86.0	9
	100.0	*94.8*	*97.7*	*132.0*	*132.0*	*100.0*	*96.1*	*99.1*	*134.0*	*134.0*	*100.0*	*100.8*	*100.3*	*82.5*	*82.5*	*100.0*	*102.1*	*101.0*	*83.2*	*83.2*	
10	46.5	45.6	46.0	63.7	63.7	42.4	45.6	46.0	63.7	63.7	102.2	96.3	96.8	78.8	78.8	94.2	96.3	96.8	78.8	78.8	10
	100.0	*98.0*	*99.0*	*137.0*	*137.0*	*100.0*	*107.8*	*108.5*	*150.4*	*150.4*	*100.0*	*94.3*	*94.9*	*77.3*	*77.3*	*100.0*	*102.1*	*102.7*	*83.6*	*83.6*	
11	47.5	43.0	41.9	62.9	62.9	41.5	43.0	41.9	62.9	62.9	89.0	86.4	88.1	66.7	66.7	81.2	86.4	88.1	66.7	66.7	11
	100.0	*90.6*	*88.1*	*132.3*	*132.3*	*100.0*	*103.8*	*101.0*	*152.8*	*152.8*	*100.0*	*97.1*	*99.0*	*74.9*	*74.9*	*100.0*	*106.5*	*108.5*	*82.0*	*82.0*	
12	41.4	38.7	38.2	56.7	56.7	34.1	38.7	38.2	56.7	56.7	81.5	77.9	79.1	60.0	60.0	75.1	77.9	79.1	60.0	60.0	12
	100.0	*93.6*	*92.4*	*137.0*	*137.0*	*100.0*	*113.7*	*112.0*	*166.4*	*166.4*	*100.0*	*95.6*	*97.0*	*73.6*	*73.6*	*100.0*	*103.5*	*105.4*	*79.9*	*79.9*	

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Table 24.

Comparison of the Approximate Methods with the Slope-Deflection Method When Applied to the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 5.

Moments in the Girders.

For each story the upper line is the moment in 1000 in. lb., and the lower line is the moment in per cent of the moment determined by the slope-deflection method.

Story No.	Moment at Right End of Girder *a*					Moment at Left End of Girder *a*					Moment at End of Girder *b*					Story No.
	Slope-Deflection Method	Proposed Approximate Method	Fleming's Method			Slope-Deflection Method	Proposed Approximate Method	Fleming's Method			Slope-Deflection Method	Proposed Approximate Method	Fleming's Method			
			I	II	III			I	II	III			I	II	III	
1	287.8	305.3	279.3	425.9	425.9	245.0	274.5	279.3	143.7	130.9	247.5	272.0	295.0	282.2	295.0	1
	100.0	*106.1*	*97.1*	*148.0*	*148.0*	*100.0*	*112.0*	*114.0*	*58.6*	*53.4*	*100.0*	*110.0*	*119.0*	*114.0*	*119.0*	
2	203.0	210.6	199.8	311.9	311.9	184.0	185.6	199.8	104.4	100.7	226.0	226.0	211.2	207.5	211.2	2
	100.0	*103.5*	*98.5*	*153.6*	*153.6*	*100.0*	*100.8*	*108.5*	*56.7*	*54.8*	*100.0*	*100.0*	*93.4*	*91.9*	*93.4*	
3	187.1	183.9	178.2	272.2	272.2	171.7	173.1	178.2	90.7	84.3	189.6	186.0	187.9	181.5	187.9	3
	100.0	*98.2*	*95.2*	*145.8*	*145.8*	*100.0*	*100.8*	*104.0*	*52.9*	*49.1*	*100.0*	*98.0*	*99.0*	*95.8*	*99.0*	
4	170.5	172.7	169.2	252.1	252.1	153.6	152.0	169.2	87.3	84.9	184.4	183.0	172.1	169.7	172.1	4
	100.0	*101.0*	*99.5*	*148.0*	*148.0*	*100.0*	*99.0*	*110.3*	*56.9*	*55.3*	*100.0*	*99.0*	*93.5*	*92.0*	*93.5*	
5	167.7	162.4	154.8	232.1	232.1	152.0	151.8	154.8	83.7	77.3	166.0	160.0	164.4	157.9	164.4	5
	100.0	*97.0*	*92.4*	*138.7*	*138.7*	*100.0*	*99.9*	*101.8*	*55.0*	*50.9*	*100.0*	*96.5*	*99.0*	*95.0*	*99.0*	
6	135.0	137.1	133.2	200.3	200.3	125.6	128.5	133.2	73.0	68.3	141.5	144.0	140.8	136.1	140.8	6
	100.0	*101.2*	*98.8*	*148.2*	*148.2*	*100.0*	*102.6*	*112.2*	*58.1*	*54.4*	*100.0*	*102.0*	*99.5*	*96.3*	*99.5*	
7	125.9	115.0	114.5	170.9	170.9	117.0	110.9	114.5	62.4	57.4	132.5	123.8	121.6	116.6	121.6	7
	100.0	*91.5*	*91.0*	*135.9*	*135.9*	*100.0*	*94.9*	*98.0*	*53.3*	*49.1*	*100.0*	*93.0*	*91.8*	*88.0*	*91.8*	
8	95.8	104.5	108.7	151.8	151.8	92.0	103.5	108.7	64.2	56.0	107.8	116.0	106.2	108.0	106.2	8
	100.0	*109.3*	*113.3*	*158.4*	*158.4*	*100.0*	*112.5*	*118.0*	*69.8*	*60.9*	*100.0*	*108.0*	*95.8*	*100.3*	*95.8*	
9	95.0	95.4	96.8	133.2	133.2	91.1	95.8	96.8	65.5	60.5	106.8	103.0	104.3	99.3	104.3	9
	100.0	*100.4*	*101.9*	*140.3*	*140.3*	*100.0*	*105.0*	*112.4*	*71.9*	*66.4*	*100.0*	*99.4*	*97.6*	*93.0*	*97.6*	
10	89.6	88.6	87.1	126.6	126.6	85.0	85.7	87.1	54.8	48.0	98.5	97.0	97.5	90.7	97.5	10
	100.0	*98.9*	*97.3*	*141.2*	*141.2*	*100.0*	*100.8*	*102.4*	*64.5*	*56.5*	*100.0*	*98.5*	*99.0*	*92.1*	*99.0*	
11	81.9	81.7	80.3	119.6	119.6	76.6	77.0	80.3	44.6	40.8	87.5	87.3	85.9	82.1	85.9	11
	100.0	*99.8*	*98.2*	*146.2*	*146.2*	*100.0*	*100.4*	*104.8*	*58.2*	*53.3*	*100.0*	*99.8*	*98.1*	*94.0*	*98.1*	
12	73.4	73.1	72.3	108.0	108.0	68.6	68.5	72.3	38.9	36.4	78.3	78.1	75.9	73.4	75.9	12
	100.0	*99.6*	*98.7*	*147.2*	*147.2*	*100.0*	*100.0*	*105.5*	*56.8*	*53.0*	*100.0*	*99.8*	*96.9*	*93.8*	*96.9*	

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Table 25.

Comparison of the Approximate Methods with the Slope-Deflection Method When Applied to the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 13.

Moments in the Columns.

For each story the upper line is the moment in 1000 in. lb., and the lower line is the moment in per cent of the moment determined by the slope-deflection method.

Story No.	Moment at Top of Column *A*					Moment at Bottom of Column *A*					Moment at Top of Column *B*					Moment at Bottom of Column *B*					Story No.
	Slope-Deflection Method	Proposed Approximate Method	Fleming's Method			Slope-Deflection Method	Proposed Approximate Method	Fleming's Method			Slope-Deflection Method	Proposed Approximate Method	Fleming's Method			Slope-Deflection Method	Proposed Approximate Method	Fleming's Method			
			I	II	III			I	II	III			I	II	III			I	II	III	
1	193.2	203.5	138.0	254.4	254.4	275.9	203.5	138.0	254.4	254.4	248.0	305.0	370.5	254.4	254.4	302.8	305.0	370.5	254.4	254.4	1
	100.0	*105.5*	*71.4*	*131.5*	*131.5*	*100.0*	*73.8*	*50.0*	*92.0*	*92.0*	*100.0*	*122.8*	*149.5*	*102.2*	*102.2*	*100.0*	*100.5*	*122.0*	*84.0*	*80.4*	
2	125.9	127.5	92.5	171.4	171.4	123.8	127.5	92.5	171.4	171.4	212.7	214.0	248.5	171.4	171.4	223.7	214.0	248.5	171.4	171.4	2
	100.0	*101.3*	*73.5*	*136.0*	*136.0*	*100.0*	*102.8*	*75.0*	*148.2*	*148.2*	*100.0*	*100.5*	*117.0*	*80.5*	*80.5*	*100.0*	*95.8*	*111.3*	*76.8*	*76.8*	
3	110.6	108.0	75.9	140.5	140.5	101.6	108.0	75.9	140.5	140.5	177.1	174.0	203.9	140.5	140.5	171.6	174.0	203.9	140.5	140.5	3
	100.0	*97.5*	*68.7*	*127.0*	*127.0*	*100.0*	*106.5*	*74.7*	*138.2*	*138.2*	*100.0*	*98.2*	*114.9*	*79.3*	*79.3*	*100.0*	*101.5*	*118.6*	*82.0*	*82.0*	
4	99.4	97.8	71.5	131.7	131.7	99.4	97.8	71.5	131.7	131.7	165.3	164.1	191.9	131.7	131.7	163.7	164.1	191.9	131.7	131.7	4
	100.0	*98.3*	*72.0*	*132.2*	*132.2*	*100.0*	*98.3*	*72.0*	*132.0*	*132.0*	*100.0*	*99.5*	*116.0*	*79.6*	*79.6*	*100.0*	*100.3*	*117.5*	*80.5*	*80.5*	

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Table 26

Comparison of the Approximate Methods with the Slope-Deflection Method When Applied to the Symmetrical Three-Span Twenty-Story Bent Shown in Fig. 13.

Moments in the Girders.

For each story the upper line is the moment in 1000 in. lb., and the lower line is the moment in per cent of the moment determined by the slope-deflection method.

Story No.	Moment at Right End of Girder *a*					Moment at Left End of Girder *a*					Moment at End of Girder *b*					Story No.
	Slope-Deflection Method	Proposed Approximate Method	Fleming's Method			Slope-Deflection Method	Proposed Approximate Method	Fleming's Method			Slope-Deflection Method	Proposed Approximate Method	Fleming's Method			
			I	II	III			I	II	III			I	II	III	
1	316.9	331.0	230.9	425.8	425.8	277.4	309.0	230.9	138.5	36.0	194.5	210.0	389.2	287.3	389.8	1
	100.0	*104.5*	*73.0*	*135.2*	*135.2*	*100.0*	*111.5*	*83.3*	*50.0*	*13.0*	*100.0*	*108.0*	*200.0*	*148.0*	*200.0*	
2	227.6	235.5	167.7	311.9	311.9	209.4	212.2	167.7	103.0	23.6	174.6	175.8	282.8	208.8	288.2	2
	100.0	*103.8*	*73.5*	*137.0*	*137.0*	*100.0*	*101.9*	*80.0*	*49.4*	*11.3*	*100.0*	*100.5*	*162.0*	*119.6*	*165.0*	
3	210.1	205.8	147.7	272.2	272.2	194.7	194.1	147.7	90.7	23.2	146.4	144.0	249.1	181.4	249.0	3
	100.0	*98.0*	*70.0*	*129.5*	*129.5*	*100.0*	*99.7*	*75.8*	*46.6*	*11.9*	*100.0*	*98.2*	*170.0*	*124.0*	*170.0*	
4	191.0	192.3	138.1	254.5	254.5	175.6	173.3	138.1	84.8	21.7	143.3	143.0	232.9	169.7	232.8	4
	100.0	*100.8*	*72.3*	*133.0*	*133.0*	*100.0*	*98.8*	*79.0*	*48.3*	*12.4*	*100.0*	*98.8*	*162.2*	*118.1*	*162.2*	

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TABLE 27.

EFFECT OF THE PROPORTIONS OF A BENT UPON THE ACCURACY OF METHOD I.

ALL GIRDER SECTIONS ARE THE SAME.

All stories of a bent are identical, and the shears on all stories are equal. All column sections are equal, all girder sections are equal, and the column sections are equal to the girder sections.

For each bent the upper line is the moment in per cent of $W \times h$, and the lower line is the moment in per cent of the moment determined by the slope-deflection method.

Bent No.	Proportions of the Bent		Moment at Top and Bottom of Column *A*		Moment at Top and Bottom of Column *B*		Moment at Right End of Girder *a*		Moment at Left End of Girder *a*		Moment at End of Girder *b*	
	Ratio of Story Height to Width of Bay *a*	Ratio of Width of Bay *a* to Width of Bay *b*	Slope-Deflection Method	Method I	Slope-Deflection Method	Method I	Slope-Deflection Method	Method I	Slope-Deflection Method	Method I	Slope-Deflection Method	Method I
1	2	2	9.09 *100.0*	9.62 *105.9*	15.91 *100.0*	15.38 *96.5*	18.20 *100.0*	19.24 *105.9*	13.67 *100.0*	19.24 *140.7*	18.20 *100.0*	11.52 *63.2*
2	2	1	10.00 *100.0*	7.50 *75.0*	15.00 *100.0*	17.50 *116.5*	20.00 *100.0*	15.00 *75.0*	16.66 *100.0*	15.0 *90.0*	13.33 *100.0*	20.00 *150.0*
3	2	0.5	10.87 *100.0*	5.00 *46.0*	14.13 *100.0*	20.00 *141.5*	21.70 *100.0*	10.00 46.1	19.54 *100.0*	10.00 *51.2*	8.70 *100.0*	30.00 *345.0*
4	1	2	8.09 *100.0*	9.62 *119.00*	16.91 *100.0*	15.38 *90.9*	16.18 *100.0*	19.24 *119.3*	13.23 *100.0*	19.24 *145.5*	20.60 *100.0*	11.52 *55.9*
5	1	1	9.38 *100.0*	7.50 *79.9*	15.62 *100.0*	17.50 *111.9*	18.75 *100.0*	15.00 *80.0*	16.66 *100.0*	15.00 *90.0*	14.60 *100.0*	20.00 *137.0*
6	1	0.5	10.53 *100.0*	5.00 *47.5*	14.47 *100.0*	20.00 *138.0*	21.04 *100.0*	10.00 *47.5*	19.74 *100.0*	10.00 *50.6*	9.20 *100.0*	30.00 *326.0*
7	0.5	2	7.32 *100.0*	9.62 *131.3*	17.67 *100.0*	15.38 *87.0*	14.66 *100.0*	19.24 *131.0*	12.93 *100.0*	19.24 *149.0*	22.40 *100.0*	11.52 *51.3*
8	0.5	1	8.93 *100.0*	7.50 *83.9*	16.06 *100.0*	17.50 *108.7*	17.84 *100.0*	15.00 *83.9*	16.66 *100.0*	15.00 *90.00*	15.45 *100.0*	20.00 *129.5*
9	0.5	0.5	10.33 *100.0*	5.00 *48.4*	14.68 *100.0*	20.00 *136.1*	20.61 *100.0*	10.00 *48.5*	19.76 *100.0*	10.0 *50.6*	9.60 *100.0*	30.00 *312.0*

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Table 28.

Effect of the Proportions of a Bent Upon the Accuracy of Method I.

All Girders Proportional to the Bending Moments.

All stories of a bent are identical and the shears on all stories are equal. All column sections are equal, the moments of inertia of column *A* and girder *a* are equal, and the ratio of the moment of inertia of girder *a* to the moment of inertia of girder *b* equals the ratio of the bending moment in girder *a* to the bending moment in girder b, as determined by method I.

For each bent the upper line is the moment in per cent of $W \times h$, and the lower line is the moment in per cent of the moment as determined by the slope-deflection method.

Bent No.	Proportions of the Bent		Moment at Top and Bottom of Column *A*		Moment at Top and Bottom of Column *B*		Moment at Right End of Girder *a*		Moment at Left End of Girder *a*		Moment at End of Girder *b*	
	Ratio of Story Height to Width of Bay *a*	Ratio of Width of Bay *a* to Width of Bay *b*	Slope-Deflection Method	Method I	Slope-Deflection Method	Method I	Slope-Deflection Method	Method I	Slope-Deflection Method	Method I	Slope-Deflection Method	Method I
1	2	2	9.75	9.62	15.25	15.38	19.50	19.24	16.05	19.24	14.45	11.52
			100.0	*98.8*	*100.0*	*100.5*	*100.0*	*98.8*	*100.0*	*120.0*	*100.0*	*79.6*
2	2	1	9.60	7.50	15.40	17.50	19.2	15.00	15.60	15.00	15.20	20.00
			100.0	*78.1*	*100.0*	*113.6*	*100.0*	*78.1*	*100.0*	*96.1*	*100.0*	*131.5*
3	2	0.5	9.45	5.00	15.55	20.00	18.9	10.00	15.10	10.00	16.00	30.00
			100.0	*52.9*	*100.0*	*129.0*	*100.0*	*52.9*	*100.0*	*66.3*	*100.0*	*187.5*
4	1	2	9.05	9.62	15.95	15.38	18.10	19.24	15.70	19.24	16.20	11.52
			100.0	*106.2*	*100.0*	*96.2*	*100.0*	*106.2*	*100.0*	*122.5*	*100.0*	*92.6*
5	1	1	8.85	7.50	16.15	17.50	17.70	15.00	15.20	15.00	17.10	20.00
			100.0	*84.8*	*100.0*	*108.3*	*100.0*	*84.8*	*100.0*	*98.8*	*100.0*	*117.0*
6	1	0.5	8.60	5.00	16.40	20.00	17.20	10.00	14.7	10.00	18.1	30.00
			100.0	*58.0*	*100.0*	*122.0*	*100.0*	*58.1*	*100.0*	*68.0*	*100.0*	*165.8*
7	0.5	2	8.55	9.62	16.45	15.38	17.10	19.24	15.80	19.24	17.1	11.52
			100.0	*112.5*	*100.0*	*93.2*	*100.0*	*112.5*	*100.0*	*122.0*	*100.0*	*67.3*
8	0.5	1	8.30	7.50	16.70	17.50	16.60	15.00	15.20	15.00	18.2	20.00
			100.0	*90.3*	*100.0*	*105.0*	*100.0*	*90.4*	*100.0*	*98.8*	*100.0*	*110.0*
9	0.5	0.5	8.00	5.00	17.00	20.00	16.00	10.00	14.60	10.00	19.4	30.00
			100.0	*62.5*	*100.0*	*117.5*	*100.0*	*62.5*	*100.0*	*68.5*	*100.0*	*154.8*

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TABLE 29.

EFFECT OF THE PROPORTIONS OF A BENT UPON THE ACCURACY OF METHOD II.

All stories of a bent are identical and the shears on all stories are equal. All column sections are equal, the moments of inertia of column *A* and girder *a* are equal, and the ratio of the moment of inertia of girder *a* to the moment of inertia of girder *b* equals the ratio of the bending moment in girder *a* to the bending moment in girder *b*, as determined by method II.

For each bent the upper line is the moment in per cent of $W \times h$, and the lower line is the moment in per cent of the moment as determined by the slope-deflection method.

Bent No.	Proportions of the Bent		Moment at Top and Bottom of Column *A*		Moment at Top and Bottom of Column *B*		Moment at Right End of Girder *a*		Moment at Left End of Girder *a*		Moment at End of Girder *b*	
	Ratio of Story Height to Width of Bay *a*	Ratio of Width of Bay *a* to Width of Bay *b*	Slope-Deflection Method	Method II	Slope-Deflection Method	Method II	Slope-Deflection Method	Method II	Slope-Deflection Method	Method II	Slope-Deflection Method	Method II
1	2	2	9.65	12.50	15.35	12.50	19.30	25.00	15.45	8.33	15.25	16.66
			100.0	*129.5*	*100.0*	*81.5*	*100.0*	*129.5*	*100.0*	*53.9*	*100.0*	*109.3*
2	2	1	10.55	12.50	14.45	12.50	21.10	25.00	18.30	8.33	10.60	16.66
			100.0	*118.5*	*100.0*	*86.5*	*100.0*	*118.5*	*100.0*	*45.6*	*100.0*	*157.1*
3	2	0.5	11.25	12.50	13.75	12.50	22.50	25.00	20.95	8.33	6.55	16.66
			100.0	*111.0*	*100.0*	*91.0*	*100.0*	*111.1*	*100.0*	*39.8*	*100.0*	*255.0*
4	1	2	8.82	12.50	16.18	12.50	17.64	25.00	15.26	8.33	17.10	16.66
			100.0	*141.8*	*100.0*	*77.3*	*100.0*	*141.8*	*100.0*	*54.6*	*100.0*	*97.5*
5	1	1	10.10	12.50	14.90	12.50	20.20	25.00	18.45	8.33	11.35	16.66
			100.0	*123.8*	*100.0*	*83.9*	*100.0*	*124.0*	*100.0*	*45.20*	*100.0*	*146.8*
6	1	0.5	11.00	12.50	14.00	12.50	22.00	25.00	21.20	8.33	6.80	16.66
			100.0	*113.5*	*100.0*	*89.3*	*100.0*	*113.7*	*100.0*	*39.3*	*100.0*	*245.0*
7	0.5	2	8.30	12.50	16.70	12.50	16.60	25.00	15.20	8.33	18.20	16.66
			100.0	*150.6*	*100.0*	*74.9*	*100.0*	*150.7*	*100.0*	*54.9*	*100.0*	*91.6*
8	0.5	1	9.80	12.50	15.20	12.50	19.60	25.00	18.55	8.33	11.85	16.66
			100.0	*127.5*	*100.0*	*82.3*	*100.0*	*127.0*	*100.0*	*44.9*	*100.0*	*140.5*
9	0.5	0.5	10.85	12.50	14.15	12.50	21.70	25.00	21.40	8.33	6.9	16.66
			100.0	*115.2*	*100.0*	*88.4*	*100.0*	*115.0*	*100.0*	*38.9*	*100.0*	*241.5*

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Table 30.

Effect of the Proportions of a Bent Upon the Accuracy of Method III.

All stories of a bent are identical and the shears on all stories are equal. All column sections are equal, the moments of inertia of column *A* and girder *a* are equal, and the ratio of the moment of inertia of girder *a* to the moment of inertia of girder *b* equals the ratio of the bending moment in girder *a* to the bending moment in girder *b*, as determined by method III.

For each bent the upper line is the moment in per cent of W x h, and the lower line is the moment in per cent of the moment as determined by the slope-deflection method.

Bent No.	Proportions of the Bent		Moment at Top and Bottom of Column *A*		Moment at Top and Bottom of Column *B*		Moment at Right End of Girder *a*		Moment at Left End of Girder *a*		Moment at End of Girder *b*	
	Ratio of Story Height to Width of Bay *a*	Ratio of Width of Bay *a* to Width of Bay *b*	Slope-Deflection Method	Method III	Slope-Deflection Method	Method III	Slope-Deflection Method	Method III	Slope-Deflection Method	Method III	Slope-Deflection Method	Method III
1	2	2	10.12 *100.0*	12.50 *123.5*	14.88 *100.0*	12.50 *84.1*	20.24 *100.0*	25.00 *123.5*	17.06 *100.0*	13.48 *79.0*	12.70 *100.0*	11.52 *90.8*
2	2	1	10.30 *100.0*	12.50 *121.5*	14.70 *100.0*	12.50 *85.2*	20.60 *100.0*	25.00 *121.5*	17.60 *100.0*	5.0 *28.4*	11.80 *100.0*	20.00 *170.0*
3	2	0.5	10.67 *100.0*	12.50 *117.4*	14.33 *100.0*	12.50 *87.3*	21.34 *100.0*	25.00 *117.3*	18.79 *100.0*	—5.0	9.87 *100.0*	30.00 *304.5*
4	1	2	9.55 *100.0*	12.50 *131.0*	15.45 *100.0*	12.50 *80.9*	19.10 *100.0*	25.00 *131.0*	17.00 *100.0*	13.48 *79.3*	13.90 *100.0*	11.52 *83.0*
5	1	1	9.80 *100.0*	12.50 *127.7*	15.20 *100.0*	12.50 *82.3*	19.60 *100.0*	25.00 *127.6*	17.60 *100.0*	5.0 *28.4*	12.80 *100.0*	20.00 *156.3*
6	1	0.5	10.26 *100.0*	12.50 *122.0*	14.74 *100.0*	12.50 *84.9*	20.52 *100.0*	25.00 *122.0*	18.98 *100.0*	—5.0	10.50 *100.0*	30.00 *286.1*
7	0.5	2	9.15 *100.0*	12.50 *136.9*	15.85 *100.0*	12.50 *78.8*	18.30 *100.0*	25.00 *136.8*	17.10 *100.0*	13.48 *78.8*	14.60 *100.0*	11.52 *79.0*
8	0.5	1	9.46 *100.0*	12.50 *132.2*	15.54 *100.0*	12.50 *80.5*	18.92 *100.0*	25.00 *132.1*	17.63 *100.0*	5.0 *28.4*	13.45 *100.0*	20.00 *148.8*
9	0.5	0.5	10.05 *100.0*	12.50 *124.5*	14.95 *100.0*	12.50 *83.70*	20.10 *100.0*	25.00 *124.5*	18.95 *100.0*	—5.0	10.95 *100.0*	30.00 *274.5*

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Table 31.

Effect of the Proportions of a Bent Upon the Accuracy of Method IV.

All stories of a bent are identical and the shears on all stories are equal. All column sections are equal, the moments of inertia of column *A* and girder *a* are equal, and the ratio of the moment of inertia of girder *a* to the moment of inertia of girder *b* equals the ratio of the bending moment in girder *a* to the bending moment in girder *b*, as determined by method IV.

For each bent the upper line is the moment in per cent of *W* x *h*, and the lower line is the moment in per cent of the moment as determined by the slope-deflection method.

Bent No.	Proportions of the Bent		Moment at Top and Bottom of Column *A*		Moment at Top and Bottom of Column *B*		Moment at Right End of Girder *a*		Moment at Left End of Girder *a*		Moment at End of Girder *b*	
	Ratio of Story Height to Width of Bay *a*	Ratio of Width of Bay *a* to Width of Bay *b*	Slope-Deflection Method	Method IV	Slope-Deflection Method	Method IV	Slope-Deflection Method	Method IV	Slope-Deflection Method	Method IV	Slope-Deflection Method	Method IV
1	2	2	9.09	8.33	15.91	16.66	18.18	16.66	13.62	16.66	18.20	16.66
			100.0	*91.8*	*100.0*	*104.5*	*100.0*	*91.7*	*100.0*	*122.1*	*100.0*	*91.5*
2	2	1	10.0	8.33	15.00	16.66	20.00	16.66	16.66	16.66	13.33	16.66
			100.0	*83.3*	*100.0*	*111.1*	*100.0*	*83.3*	*100.0*	*100.0*	*100.0*	*125.0*
3	2	0.5	10.89	8.33	14.13	16.66	21.74	16.66	19.56	16.66	8.70	16.66
			100.0	*76.8*	*100.0*	*117.8*	*100.0*	*76.6*	*100.0*	*85.20*	*100.0*	*191.5*
4	1	2	8.09	8.33	16.91	16.66	16.18	16.66	13.22	16.66	20.60	16.66
			100.0	*103.2*	*100.0*	*98.3*	*100.0*	*103.0*	*100.0*	*126.0*	*100.0*	*80.9*
5	1	1	9.38	8.33	15.62	16.66	18.76	16.66	16.64	16.66	14.60	16.66
			100.0	*88.8*	*100.0*	*106.4*	*100.0*	*88.9*	*100.0*	*100.0*	*100.0*	*114.0*
6	1	0.5	10.53	8.33	14.47	16.66	21.06	16.66	19.74	16.66	9.20	16.66
			100.0	*79.2*	*100.0*	*115.0*	*100.0*	*79.1*	*100.0*	*84.4*	*100.0*	*181.0*
7	0.5	2	7.32	8.33	17.68	16.66	14.64	16.66	12.96	16.66	22.40	16.66
			100.0	*113.9*	*100.0*	*94.2*	*100.0*	*113.8*	*100.0*	*128.7*	*100.0*	*74.4*
8	0.5	1	8.93	8.33	16.07	16.66	17.86	16.66	16.69	16.66	15.45	16.66
			100.0	*93.3*	*100.0*	*103.7*	*100.0*	*93.3*	*100.0*	*100.0*	*100.0*	*107.8*
9	0.5	0.5	10.33	8.33	14.67	16.66	20.66	16.66	19.74	16.66	9.60	16.66
			100.0	*80.7*	*100.0*	*113.5*	*100.0*	*80.6*	*100.0*	*84.4*	*100.0*	*173.5*

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TABLE 32.

LOG OF THE TEST OF CELLULOID MODEL NO. 4.

Story No.	Shear W in Lb.	Horizontal Deflection at Center of Girder at Top of Story in Inches			Change in Slope Measured on an Arc Having a 19-inch Radius, in Inches.							
					Force at Top of Model Acting Toward the Right				Force at Top of Model Acting Toward the Left			
					θ at Top of Column A		θ at Top of Column B		θ at Top of Column A		θ at Top of Column A	
		Right-Hand Span	Middle Span	Left-Hand Span	Right-Hand Side of Model	Left-Hand Side of Model	Right-Hand Side of Model	Left-Hand Side of Model	Right-Hand Side of Model	Left-Hand Side of Model	Right-Hand Side of Model	Left-Hand Side of Model
1	1.5	.04	.04	.04	.20	.20	.09	.10	.19	.18	.09	.10
	3.0	.08	.08	.08	.40	.40	.20	.21	.40	.40	.19	.23
	4.5	.11	.11	.11	.61	.59	.31	.33	.61	.63	.30	.38
2	1.5	.08	.08	.08	.21	.21	.10	.10	.19	.19	.11	.10
	3.0	.16	.16	.16	.43	.43	.21	.20	.41	.41	.24	.22
	4.5	.25	.24	.24	.66	.66	.34	.33	.63	.65	.37	.37
3	1.5	.14	.14	.14	.18	.17	.11	.12	.17	.16	.13	.14
	3.0	.28	.29	.28	.37	.35	.26	.26	.36	.35	.26	.29
	4.5	.42	.42	.42	.59	.54	.39	.40	.56	.55	.40	.42
4	1.5	.18	.18	.18	.13	.10	.06	.07	.13	.09	.06	.07
	3.0	.36	.36	.37	.25	.19	.13	.15	.26	.20	.13	.15
	4.5	.55	.55	.55	.40	.30	.20	.22	.39	.27	.18	.25

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PUBLICATIONS OF THE ENGINEERING EXPERIMENT STATION

Bulletin No. 1. Tests of Reinforced Concrete Beams, by Arthur N. Talbot. 1904 *None available.*

Circular No. 1. High-Speed Tool Steels, by L. P. Breckenridge. 1905. *None available.*

Bulletin No. 2. Tests of High-Speed Tool Steels on Cast Iron, by L. P. Breckenridge and Henry B. Dirks. 1905. *None available.*

Circular No. 2. Drainage of Earth Roads, by Ira O. Baker. 1906. *None available.*

Circular No. 3. Fuel Tests with Illinois Coal (Compiled from tests made by the Technologic Branch of the U. S. G. S., at the St. Louis, Mo., Fuel Testing Plant, 1904-1907), by L. P. Breckenridge and Paul Diserens. 1909. *Thirty cents.*

Bulletin No. 3. The Engineering Experiment Station of the University of Illinois, by L. P. Breckenridge. 1906. *None available.*

Bulletin No. 4. Tests of Reinforced Concrete Beams, Series of 1905, by Arthur N. Talbot. 1906. *Forty-five cents.*

Bulletin No. 5. Resistance of Tubes to Collapse, by Albert P. Carman and M. L. Carr. 1906. *None available.*

Bulletin No. 6. Holding Power of Railroad Spikes, by Roy I. Webber. 1906. *None available.*

Bulletin No. 7. Fuel Tests with Illinois Coals, by L. P. Breckenridge, S. W. Parr, and Henry B. Dirks. 1906. *None available.*

Bulletin No. 8. Tests of Concrete: I, Shear; II, Bond, by Arthur N. Talbot. 1906. *None available.*

Bulletin No. 9. An Extension of the Dewey Decimal System of Classification Applied to the Engineering Industries, by L. P. Breckenridge and G. A. Goodenough. 1906. Revised Edition 1912. *Fifty cents.*

Bulletin No. 10. Tests of Concrete and Reinforced Concrete Columns. Series of 1906, by Arthur N. Talbot. 1907. *None available.*

Bulletin No. 11. The Effect of Scale on the Transmission of Heat Through Locomotive Boiler Tubes, by Edward C. Schmidt and John M. Snodgrass. 1907. *None available.*

Bulletin No. 12. Tests of Reinforced Concrete T-Beams, Series of 1906, by Arthur N. Talbot. 1907. *None available.*

Bulletin No. 13. An Extension of the Dewey Decimal System of Classification Applied to Architecture and Building, by N. Clifford Ricker. 1907. *None available.*

Bulletin No. 14. Tests of Reinforced Concrete Beams, Series of 1906, by Arthur N. Talbot. 1907. *None available.*

Bulletin No. 15. How to Burn Illinois Coal Without Smoke, by L. P. Breckenridge. 1908. *Twenty-five cents.*

Bulletin No. 16. A Study of Roof Trusses, by N. Clifford Ricker. 1908. *Fifteen cents.*

Bulletin No. 17. The Weathering of Coal, by S. W. Parr, N. D. Hamilton, and W. F. Wheeler. 1908. *None available.*

Bulletin No. 18. The Strength of Chain Links, by G. A. Goodenough and L. E. Moore. 1908. *Forty cents.*

Bulletin No. 19. Comparative Tests of Carbon, Metallized Carbon and Tantalum Filament Lamps, by T. H. Amrine. 1908. *None available.*

Bulletin No. 20. Tests of Concrete and Reinforced Concrete Columns, Series of 1907, by Arthur N. Talbot. 1908. *None available.*

Bulletin No. 21. Tests of a Liquid Air Plant, by C. S. Hudson and C. M. Garland. 1908. *Fifteen cents.*

Bulletin No. 22. Tests of Cast-Iron and Reinforced Concrete Culvert Pipe, by Arthur N. Talbot. 1908. *None available.*

Bulletin No. 23. Voids, Settlement, and Weight of Crushed Stone, by Ira O. Baker. 1908. *Fifteen cents.*

**Bulletin No. 24.* The Modification of Illinois Coal by Low Temperature Distillation, by S. W. Parr and C. K. Francis. 1908. *Thirty cents.*

Bulletin No. 25. Lighting Country Homes by Private Electric Plants, by T. H. Amrine. 1908. *Twenty cents.*

Bulletin No. 26. High Steam-Pressures in Locomotive Service. A Review of a Report to the Carnegie Institution of Washington, by W. F. M. Goss. 1908. *Twenty-five cents.*

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